Test 03 Solutions

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Problem 02: $\boxed{2\ pt}$ Which one of the statements concerning the center of mass of an object is true?

The velocity of the center of mass of a system of objects is constant when the sum of the external forces acting on the system is zero.

Problem 03: $\boxed{2\ pt}$ Use conservation of energy to find out the initial velocity. In order to find the change in momentum, use the following. I have chosen $\hat{j}$ to be pointing straight up from the ground.

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow v = \sqrt{2gh} \quad \vec{p}_i = -mv\ \hat{j}; \quad \vec{p}_f = +m(0.75v)\ \hat{j}$$
$$\Delta p = \vec{p}_f - \vec{p}_i = +m(0.75v)\ \hat{j} - (-mv\ \hat{j}) = 1.75mv\ \hat{j} = \frac{7m}{4}\sqrt{2gh}\ \hat{j}$$

Problem 04: $\boxed{2\ pt}$ Angular momentum and rotational kinetic energy are both conserved. Since she rotates faster after she pulls her arms in, the final angular velocity is larger than the initial, that's evident from the problem statement. Find the ratio of the final to initial rotational kinetic energy (as shown below) and it's clear that her final rotational kinetic energy is larger.

Correct: larger because she is rotating faster.

$$\text{Given } : \frac{\omega_f}{\omega_i} > 1 \quad I_i\omega_i = I_f\omega_f$$
$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} = \frac{(I_f\omega_f)\omega_f}{(I_i\omega_i)\omega_i} = \frac{\omega_f}{\omega_i} > 1$$
$$\Rightarrow KE_f > KE_i$$

Problem 05: $\boxed{3\ pt}$ To find the mass, we have to integrate the expression for linear mass density

$$m = \int_{x=1.0}^{4.0} dm = \int_{1.0}^{4.0} \lambda dx = \int_{1.0}^{4.0} (0.2x^2 + 0.6)\, dx = \left| \frac{0.2}{3}x^3 + 0.6x \right|_{1.0}^{4.0} \quad \boxed{= 6.0\ kg}$$

Problem 06: $\boxed{3\ pt}$ Use conservation of linear momentum (l=linebacker, q=quarterback)

$$p_i = m_l v_l \quad p_f = (m_l + m_q)v_f$$
$$p_f = p_i$$
$$v_l = \frac{(m_l + m_q)v_f}{m_l} = \frac{(116 + 84\ kg)(3.3\ m/s)}{116\, kg} \qquad \boxed{v_l = 5.69\ m/s}$$

Problem 07: $\boxed{2\ pt}$ The "second" hand takes 60 seconds to make one complete revolution (2π radians).

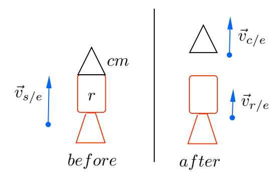
$$\omega = \frac{2\pi\ rad}{60\ s} \qquad\qquad \boxed{\omega = 0.105\ rad/s}$$

Problems 08-09: $\boxed{4\ pt}$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{7(0) + 3(2) + 3(1)}{(7 + 3 + 3)} = 0.692\ m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{7(0) + 3(1) + 3(2)}{(7 + 3 + 3)} = 0.692\ m$$

Problem 10: The rocket and command module (CM) together form the space vehicle. Their masses are in the ratio 4:1. No external forces act on the system, the momentum of the space vehicle must be identical before and after the rocket disengages. Denote the following subscripts: s=space vehicle (ie rocket+CM), r=rocket, c=CM and e=earth. Given $\vec{v}_{s/e}$= 4250 km/h and $\vec{v}_{r/c}$ = -88 km/h (minus sign is because rocket is sent backward relative to the command module). The center-of-mass (CoM) continues to move without any change in direction or magnitude after the rocket disengages, because there are no external forces to alter it's motion. $\boxed{4\ pt}$



- Initial velocity of CoM = $\vec{v}_{CoM} = \dfrac{(4M + M)\,\vec{v}_{s/e}}{(4M + M)} = \vec{v}_{s/e}$
- Final velocity of CoM = $\vec{v}_{CoM} = \dfrac{(4M)\,\vec{v}_{r/e} + (M)\,\vec{v}_{c/e}}{(4M + M)} = \dfrac{4\,\vec{v}_{r/e} + \vec{v}_{c/e}}{5}$

$$\vec{v}_{CoM}\ \text{(before)} = \vec{v}_{CoM}\ \text{(after)}$$

$$\vec{v}_{s/e} = \frac{4\,\vec{v}_{r/e} + \vec{v}_{c/e}}{5} = \frac{4(\vec{v}_{r/c} + \vec{v}_{c/e}) + \vec{v}_{c/e}}{5}$$

$$\Rightarrow \quad \vec{v}_{c/e} = (\vec{v}_{s/e} - \frac{4}{5}\vec{v}_{r/c}) = 4250 - \frac{4}{5}(-88) = 4320\ km/h$$

Problem 11: $\boxed{3\ pt}$ Notice radius = half the diameter.

$$\alpha_{avg} = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(v_f - v_i)}{r\Delta t} = \frac{(26 - 13)\,m/s}{(0.35\,m \cdot 8.5\,s)} = 4.37\ rad/s^2$$

Problems 12-13: $\boxed{4\ pt}$

$$F + F_{fr} = ma \quad \Rightarrow\ |F_{fr}| = |ma - F| = |10(0.474) - 6| = 1.26\ N$$

$$\tau = rF_{fr} = (I\alpha)_{com} \quad \Rightarrow\ I_{com} = \frac{rF_{fr}}{(a/r)} = \frac{0.25\,m(1.26\,N)}{(0.474\,m/s^2/0.25\,m)} = 0.166\ kg\,m^2$$

Problem 14: $\boxed{4\ pt}$ Apply momentum conservation before and after impact. The rotational inertia after the impact is the sum of moments of inertia of the rod and the bullet since the bullet gets lodged in the rod.

$$
\begin{aligned}
I_r &= \frac{1}{12} M\ell^2 = \frac{1}{12}(5)(0.65^2) = 0.176\ kg\,m^2\,, \quad I_b = m(\ell/2)^2 = 3\times 10^{-3}(0.65/2)^2 = 3.17\times 10^{-4}\ kg\,m^2 \\
L_i &= L_f \quad \Rightarrow \ (\ell/2) mv \sin\theta = (I_r + I_b)\omega \\
v &= \frac{(I_r + I_b)\omega}{m(\ell/2)\sin\theta} == \frac{(0.176 + 3.17\times 10^{-4}\,kg\,m^2)(11\,rad/s)}{(3\times 10^{-3}\,kg)(0.325\,m)\sin 60^\circ} \ \Rightarrow \qquad \boxed{v = 2.29\times 10^3\ m/s}
\end{aligned}
$$

Problem 15: $\boxed{4\ pt}$: The wheel has eight spokes. The arrow must pass through the hole before a spoke hits it, that is within one-eighth of a revolution, *i.e.,* $\theta = \frac{2\pi}{8} = 0.7854\ rad$. The best scenario for the arrow is for it's head to enter just after one spoke crosses it's path and the tail of the arrow finishes crossing before the next spoke crosses it's path. The timing must be just right for this. Find out the time it takes to make one-eighth of a revolution first. Use that time to figure out the speed for the full length of the arrow to cross.

$$
\begin{aligned}
\omega &= 3\ rev/s = 3(2\pi)\,rad/s = 18.85\,rad/s \\
t &= \frac{\theta}{\omega} = \frac{0.7854\,rad}{18.85\,rad/s} = 0.0417\,s \quad v = \frac{\ell}{t} = \frac{22\times 10^{-2}\,m}{0.0417\,s} = 5.28\ m/s
\end{aligned}
$$

Problems 16-17: $\boxed{4\ pt}$: Use conservation of kinetic energy and linear momentum.

$$
\begin{aligned}
2mv &= mv_1 + mv_2 \quad \text{where } m = 6\,kg \quad v = 4.4\,m/s \quad \Rightarrow (v_1 + v_2) = 8.8 \\
KE_i &= \frac{1}{2}(2m)v^2 = \frac{1}{2}(12\,kg)(4.4\,m/s)^2 = 116.2\ J \quad KE_f = KE_i + 16\,J = 132.2\,J \\
132.2\,J &= \frac{1}{2}(m)v_1^2 + \frac{1}{2}(m)v_2^2 \quad \Rightarrow \ v_1^2 + v_2^2 = 44.1 \\
(v_1 + v_2)^2 &= v_1^2 + v_2^2 + 2v_1v_2 \\
8.8^2 &= 44.1 + 2v_1v_2 \quad \Rightarrow \ v_1v_2 = \frac{(8.8^2 - 44.1)}{2} = 16.67 \\
v_1 + \frac{16.67}{v_1} &= 8.8 \quad \Rightarrow \ v_1^2 - 8.8v_1 + 16.67 = 0 \\
\Rightarrow v_1 &= \frac{8.8 \pm \sqrt{8.8^2 - 4(16.67)(1)}}{2} = \{2.76, 6.04\} \\
\Rightarrow v_2 &= \{6.04, 2.76\}
\end{aligned}
$$