Test 02 Solutions

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Problems 02-06: $\boxed{5\ pt}$

2. During the collision of a car with a large truck, the car exerts a lesser force on the truck than the truck exerts on the car. **False**. Because the contact force (which is the normal force between the two objects) is the same.
3. The net force which acts on an object which maintains a constant velocity is zero. **True**. $\vec{F}_{net} = m\vec{a}$ and $\vec{a} = 0$ which means $\vec{F}_{net} = 0$
4. During the collision of a car with a large truck, the truck exerts an equal size force on the car as the car exerts on the truck. **True**. Same reasoning, the normal force is the same no matter what the size of each object is.
5. In order not to slow down, an object moving at a constant velocity needs a small net force applied. **False**. An object moving at constant velocity will continue to move at constant velocity indefinitely. It does not need any force to change it's state of motion (or inertial state).
6. If two objects are under the influence of equal forces, they have the same acceleration. **False**. We do not know anything about the inertia (mass) of each object. Equal forces could cause different masses to have different accelerations.

Problems 07-12: $\boxed{5\ pt}$

7. Work is done when the form of energy changes. **True** $\Delta KE = \Sigma W$

3. More power is required while slowly lifting a box than while lifting it up quickly. **False**.

9. A source of energy is required to do work. **True**.

10. Work can be done in the absence of motion. **False**.

11. Energy conservation law for a projectile (no friction): Po- tential energy increase equals the kinetic energy decrease. **True**.

12. Without friction or work by an external force, the sum of the potential and kinetic energies of a body is constant. **True**.

Problem 13: $\boxed{2\ pt}$ Consider the three cases shown in the drawing in which the same force is applied to a box of mass m. In which cases will the magnitude of the normal force be equal to (F$\sin\theta$)+mg)?

C. Case Three Only

Problem 14: $\boxed{2\ pt}$ A bicycle is moving at a speed v = 3.76 m/s. If the radius of the front wheel is 0.450 m, how long does it take for that wheel to make a complete revolution? Look at the formula sheet and you can just plug in the numbers.

$$v = \frac{2\pi r}{T} \quad \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi \cdot (0.450\ m)}{3.76\ m/s} = 0.752\ s$$

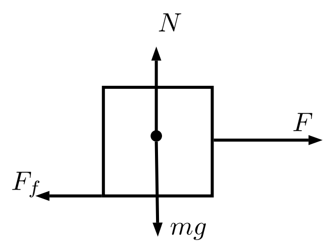
Problem 15: $\boxed{3\ pt}$ Two forces parallel to the x axis do W = 14.9 J of work on a small tray while moving it d = 22.0 m in the x direction across a gym floor. One of the forces has a value of $F_1 = +3.94$ N in the x direction. What is the other force F_2?

We do not know either the magnitude or the direction of the other force, except one detail stated in the problem. The other force is parallel to x axis. So it could be along +x or -x. Let's assume it's along +x. If we are right, the answer will be positive validating our choice of direction. If we are wrong, the answer will be negative (invalidating our choice of direction, but the magnitude in both cases will be the same).

$$\begin{aligned}
W &= \sum \vec{F} \cdot \vec{x} \\
14.9 &= (3.94\,N)\,\hat{i} \cdot (22.0\,m)\,\hat{i} + (F_2\,N)\,\hat{i} \cdot (22.0\,m)\,\hat{i} \\
F_2 &= \frac{14.9\,J}{22.0\,m} - 3.94\,N \\
&\boxed{F_2 = -3.26\ N}
\end{aligned}$$

Our choice of direction (+x) was wrong. I deliberately chose it to illustrate the solution "fixes itself". So $\vec{F} = -3.26\,\hat{i}$

Problems 16-18: $\boxed{6\ pt}$ A 21.8 kg block is initially at rest on a horizontal surface. A horizontal force of 88.0 N is required to set the block in motion, after which a horizontal force of 62.0 N is required to keep the block moving with constant speed. Use g = 9.8 m/s^2 Free body diagram is shown. For the case of static



friction, we will use the maximum force that will cause the block to just start moving. After that, to keep the object moving at a constant velocity, the force required is smaller (and balances out the opposing kinetic friction force).

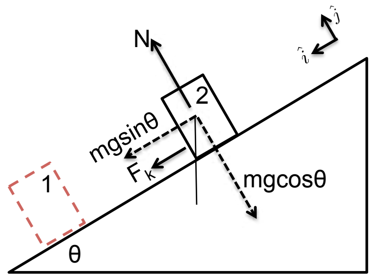
$$F_s \leqslant \mu_s\,N \quad \Rightarrow \mu_s \geqslant \frac{F_s}{N} = \frac{88.0\,N}{(21.8\,kg)(9.8\,m/s^2)} = 0.412$$

$$F_k = \mu_k\,N = F \quad \Rightarrow \mu_k = \frac{F}{N} = \frac{62.0\,N}{(21.8\,kg)(9.8\,m/s^2)} = 0.290$$

When the surface is tilted there are no external (applied forces), gravity does the bidding and causes the block to slide down on the surface.

$$N = mg\cos\theta \quad F_s = mg\sin\theta \quad F_s = \mu_s N$$
$$\Rightarrow \tan\theta = \mu_s \quad \Rightarrow \theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.412) = 22.4°$$

Problem 19: $\boxed{3\ pt}$ A 4.9 kg bundle starts up a 30° incline with 158 J of kinetic energy. How far will it slide up the plane if the coefficient of friction is 0.36?



Work is done by (a) horizontal component of weight (b) force of friction both acting along the incline. No work is done by forces perpendicular to the incline (since the motion is along the incline). And all of the kinetic energy is "used up" to bring the block to rest as it works it's way up by a distance (let's call it) "d" up the incline

$$\begin{aligned}(N - mg\cos\theta) &= 0\\ F_k &= \mu_k N\\ W_{F_k} &= (F_k\,\hat{i})\cdot(-d\,\hat{i})\\ &= -\mu_k mgd\cos\theta\\ W_{F_k} &= -14.971\,d\ J \quad ......(1)\end{aligned}$$

$$\begin{aligned}W_{gr} &= (mg\sin\theta\,\hat{i})\cdot(-d\,\hat{i})\\ W_{gr} &= -24.01\,d\ J \quad ......(2)\end{aligned}$$

$$\begin{aligned}KE_1 = 158\,J\,,\ \ KE_2 &= 0\\ KE_2 - KE_1 &= (W_{gr} + W_{F_k})\\ (0 - 158) &= -14.971\,d - 24.01\,d\\ &\boxed{d = 4.05\,m}\end{aligned}$$

Problem 20: $\boxed{2\ pt}$ A missile 52 cm in diameter is cruising at constant speed of 260 m/s at low altitude where the density of air is 1 kg/m^3. Assume the drag coefficient, C = 0.7.

"A" is the cross-sectional area of the missile. If you slice a missile (which a cylinder) perpendicular to it's axis you get a circular cross section. Note the diameter is given, radius is half of diameter. Also convert into SI units.

$$A = \pi r^2 = (3.1415)(0.52/2)^2 = 0.2124\,m^2$$

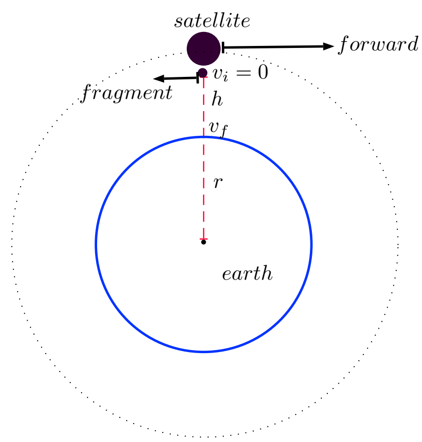
$$D = \frac{1}{2}\rho A C v^2 = \frac{1}{2}(1\,kg/m^3)(0.2124\,m^2)(0.7)(260\,m/s)^2 = 5025.34\,N \quad \Rightarrow \boxed{d = 5.03\ kN}$$

Problem 21: $\boxed{4\ pt}$ A constant force of 51.1 N, directed at 26.9° from horizontal, pulls a mass of 15.1 kg horizontally a distance of 2.69 m. Calculate the work done by the force.

$$\vec{F} = (51.1\cos 26.9°\,N)\,\hat{i} + (51.1\sin 26.9°\,N)\,\hat{j} \quad \vec{d} = (2.69\,m)\,\hat{i}$$

$$W = \vec{F}\cdot\vec{d} = (51.1\cos 26.9°\,N)\,(2.69\,m)\ \boxed{W = 122.6\,J}$$

Problem 22-23: $\boxed{4\ pt}$ A satellite in a circular orbit around the earth with a radius 1.013 times the mean radius of the earth is hit by an incoming meteorite. A large fragment M = 60.0 kg is ejected in the backwards direction so that it is stationary with respect to the earth and falls directly to the ground. Its speed just before it hits the ground is 369.0 m/s. Find the total work done by gravity on the satellite fragment. The radius of the Earth = 6.37 $\times 10^3$ km; The mass of the Earth = 5.98 $\times 10^2 4$ kg.



Some imagination is required to solve this problem. It is not hard and does not require anything other than what you have learned in work/energy theorem.

Look at the diagram. Assume the satellite is going around in circular orbit around the earth (dotted line) in a clockwise direction. Due to a meteorite hit, a fragment separates. Now at any instant (such as the one shown) the speed of the satellite is along the tangent to the circle. Let's call that the "forward" direction. The ejected fragment is in the "backward" direction (also tangential, but pointing the other way). If the speed of ejection in the backward direction is equal to the speed of the statellite (of which it was a part of) in the forward direction, they both cancel out (ie there is no net velocity in the horizontal direction). Then earth's gravity sucks that fragment right into it and to someone on the earth watching the fragment, it appears to drop out of the sky directly to the ground. Initial velocity is zero and final velocity is given. Let the radius of the earth be **r** and the height through which the fragment falls be **h**. The radius of the circular orbit is (r+h). I will choose the unit vector $\hat{j}$ to point straigt down toward the center of the earth along the dotted red line.

$$(r+h) = 1.013\,r \quad \Rightarrow \; h = 0.013\,r = 0.013\,(6.37 \times 10^3 \times 10^3 \; m) = 82810.0 \; m$$
$$\Rightarrow \; \vec{d} = (82810.0\,m)\;\hat{j} \quad \vec{F} = mg\,N\;\hat{j} = (60.0\,kg)(9.8\,m/s^2)\;\hat{j} = (588\,N)\;\hat{j}$$
$$W_{gravity} = \vec{F} \cdot \vec{d} = (588\,N)\;\hat{j} \cdot (82810.0\,m)\;\hat{j} = 4.87 \times 10^7 \; J$$

Next, calculate the net change in kinetic energy and notice something odd. It is **much less** than the work done by gravity. But work energy theorem requires the change in KE be equal to the total work done. So we are missing a piece of work (no pun intended). Let's call it W_{other}

$$\Delta KE = KE_f - KE_i = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\,(60.0\,kg)(369.0\,m/s)^2 = 0.41 \times 10^7 \, J$$
$$\Delta KE = \Sigma W = W_{gravity} + W_{other}$$
$$0.41 \times 10^7 \; J = 4.87 \times 10^7 \; J + W_{other} \quad \Rightarrow \; W_{other} = -4.46 \times 10^7 \; J$$

But what is this W_{other}? The work done by gravity does not equal the change in kinetic energy. There must be other (non conservative) forces at play. When the fragment drops through the atmosphere, it burns up and W_{other} must have been used up in heating up the fragment. Therefore the answer to the last part is $4.46 \times 10^7 \; J$.

On a different note, an interesting incident happened in 2013 where a meteorite streaked through the earth's atmosphere landing in Siberia, Russia. Check out this video: `https://www.youtube.com/watch?v=dpmXyJrs7iU`.