Test 01 Solutions

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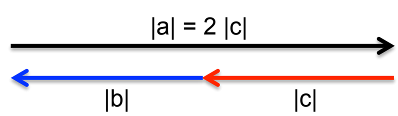
Problem 02: $\boxed{2\ pt}$ Three vectors a, b, and c add together to yield zero: a+b+c=0. The vectors a and c point in opposite directions and their magnitudes are related by the expression |a|=2|c|. Which one of the following conclusions is correct?



$$\vec{a} = -2\vec{c} \qquad \vec{a} + \vec{b} + \vec{c} = 0 \;\Rightarrow\; -2\vec{c} + \vec{b} + \vec{c} = 0 \;\Rightarrow\; \vec{b} = \vec{c}$$

E. b and c have equal magnitudes and point in the same direction. **True**

Problem 03: $\boxed{2\ pt}$ Complete the following statement: A displacement vector

D. is directed from an object's initial position toward its final position. **Correct**

Problems 04-09: $\boxed{6\ pt}$ The velocity vector $\mathbf{V}_1$ has a magnitude of 4.0 m/s and is directed along the +x-axis. The velocity vector $\mathbf{V}_2$ has a magnitude of 3.0 m/s. The sum of the two is $\mathbf{V}_3$, so that $\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2$.

To solve this problem, you have to consider possible arrangements of vector $\mathbf{V}_2$, since $\mathbf{V}_1$ is fixed. If $\mathbf{V}_2$ is along +x (parallel to $\mathbf{V}_1$) we get maximum possible sum and therefore $\mathbf{V}_3$ will have a magnitude of (4+3) = 7 m/s. At the other extreme, if $\mathbf{V}_2$ is anti-parallel (directed along -x), then we get the minumum possible sum of (4 - 3) = 1 m/s. Every other arrangement of $\mathbf{V}_2$ will yield a value in between these two extremes. ($1 \le |\mathbf{V_{3x}}| \le 7$) and ($1 \le |\mathbf{V_3}| \le 7$). Once you notice this, the answers below can be easily arrived at.

04. The magnitude of $\mathbf{V}_3$ can be -6.0 m/s **False**

05. The x-component of $\mathbf{V}_3$ can be 0.0 m/s **False**

06. The magnitude of $\mathbf{V}_3$ can be 8.0 m/s **False**

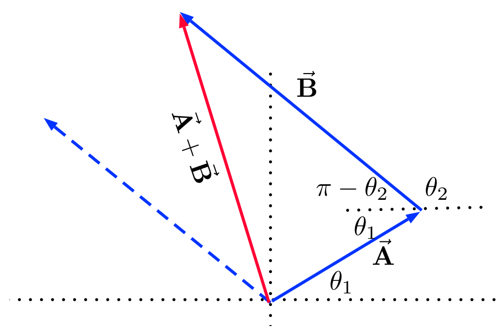
07. The magnitude of $\mathbf{V}_3$ can be 7.0 m/s **True**

08. The magnitude of $\mathbf{V}_3$ can be 0.0 m/s **False**

09. The magnitude of $\mathbf{V}_3$ can be 5.0 m/s **True**

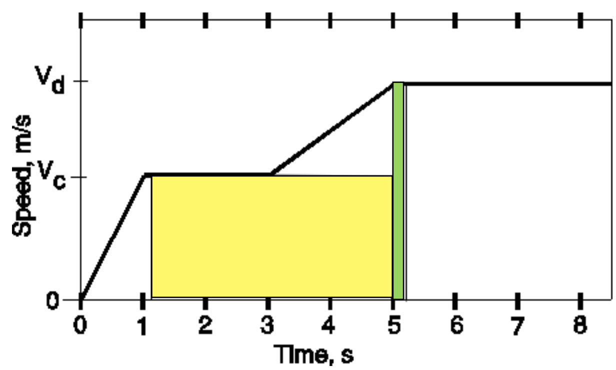
Problem 10: $\boxed{3\ pt}$ With the diagram and the data answer the question: What is the magnitude of **A** + **B**?

Quickest way to solve this problem is to arrange the vectors tip-to-tail. Keeping $\vec{A}$ as is, move the tail of $\vec{B}$ to the tip of $\vec{A}$ parallel to itself (as shown by the dotted $\rightarrow$ solid blue line). The sum $\vec{C} = \vec{A} + \vec{B}$ is the vector that completes the triangle. Using the cosine rule, we can solve for the length of the third side.



$$
\begin{aligned}
\alpha &= \theta_1 + (\pi - \theta_2) = (31.4 + 180 - 133.3) = 78.1^\circ \\
|\vec{C}| &= \sqrt{|\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\alpha} \\
&= \sqrt{4.1^2 + 6.3^2 - 2 \cdot 4.1 \cdot 6.3 \cdot \cos(78.1^\circ)} = 6.771\ cm
\end{aligned}
$$

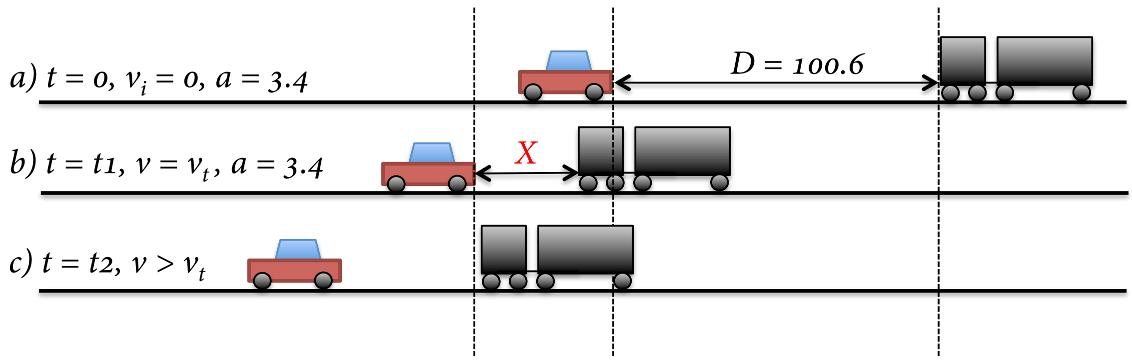
Problem 11: $\boxed{4\ pt}$ The graph shows the speed of a car traveling in a straight line as a function of time. The value of V_c is 3.60 m/s and the value of V_d is 5.30 m/s. Calculate the distance traveled by the car from a time of 1.20 to 5.20 seconds.



Find the area under the curve. I have split the area into three regions. Add the area of the big rectangle, the long strip, and the triangle. That will give you the total distance traveled.

$$
\begin{aligned}
\square\,\#1 &: (3.6)\, m/s \cdot (5 - 1.2)\, s \ = 13.68\, m \\
\square\,\#2 &: (5.3)\, m/s \cdot (5.2 - 5)\, s \ = 1.06\, m \\
\Delta\,\#3 &: \frac{1}{2}(5.3 - 3.6)\, m/s \cdot (5 - 3.0)\, s \ = 1.7\, m \\
\text{Total} &: 14.4 + 1.87 = 16.43\, m
\end{aligned}
$$

Problems 12, 13: $\boxed{4\ pt}$ To save fuel, some truck drivers try to maintain a constant speed when possible. A truck traveling at 26.0 km/hr approaches a car stopped at the red light. When the truck is 100.6 meters from the car the light turns green and the car immediately begins to accelerate at 3.4 m/s^2. How close (in m) does the truck come to the car assuming the truck does not slow down? How far from the stop light has the car travelled when the truck reaches its closest distance (in m)?



<u>Method 1:</u> The truck keeps approaching closer to the car (at constant speed) until the car accelerates from rest to reach the truck's speed. At that instant, the truck is closest to the rear of the car. After that, the car speeds up (faster than the truck) and the truck can no longer catch it. (b) captures the instant when the car's speed is exactly equal to that of the truck. Let's say the car moved a distance X_c during time t=t$_1$. Truck's speed is always v_t = 26 km/hr = 26(1000 m / 3600s) = 7.22 m/s and at t=t_1 the truck has moved X_t.

$$a)\quad \text{Car } X_c = \frac{1}{2}(3.4)t_1^2$$
$$v_t = 7.22 = 3.4t_1$$
$$\boxed{t_1 = 2.12\ s \quad X_c = 7.66\ m}$$

$$b)\quad \text{Truck } X_t = v_t t_1$$
$$= (7.22)(2.12)\ m = 15.306\ m$$
$$\boxed{\Delta X = 100.6 - 15.306 + 7.66 = 92.93\ m}$$

So the distance of closest approach is $(D - X_t + X_c)$ = 92.93 m and the car has moved a distance of X_c = 7.66 m.

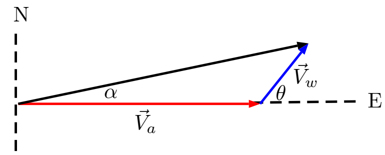
<u>Method 2:</u> This is a more elegant approach to solving this problem using relative acceleration, relative speed. Imagine the car remains stationary instead of accelerating. For this to happen, we have to add (-3.4 m/s^2) to it's acceleration. Same treatment for the truck (ie, add -3.4 m/s^2 to it's acceleration). Since the car is at rest, its velocity is zero. The truck's velocity relative to the car is just it's initial velocity (7.22 m/s).

Then the problem simplifies to solving for the truck's motion with the car being stationary. We see that the truck has to <u>slow down or decelerate</u> at the same rate as the car would accelerate. Eventually the truck has to come to a halt since it's slowing down. So let's solve for the time and distance of the truck's subsequent motion.

$$\vec{v}_{truck/car} = \vec{v}_{truck} - \vec{v}_{car} = 7.22 - 0 = 7.22\ m/s$$
$$\vec{a}_{truck/car} = \vec{a}_{truck} - \vec{a}_{car} = 0 - 3.4 = -3.4\ m/s^2$$
$$v_f = v_0 + at \ \Rightarrow\ 0 = 7.22 - 3.4(t) \ \Rightarrow\ t = (7.22/3.4) = 2.12\,s$$
$$X = v_0(t) + \frac{1}{2}at^2 \ \Rightarrow\ 7.22(2.12) - \frac{1}{2}(-3.4)(2.12^2) = 7.66\,m$$

This implies the truck will cover 7.66 m before coming to a halt. So the distance of closest approach is (100.6 - 7.66) = 92.93 m.

Problems 14, 15: $\boxed{4\ pt}$ A jet airliner moving at $\vec{V}_a = 120$ m/s due east (measured relative to the ground) suddenly encounters a wind blowing at $\vec{V}_w = 38$ m/s toward the direction $\theta = 24$ degrees north of east. What is the new speed of the aircraft relative to the ground? (in m/s)

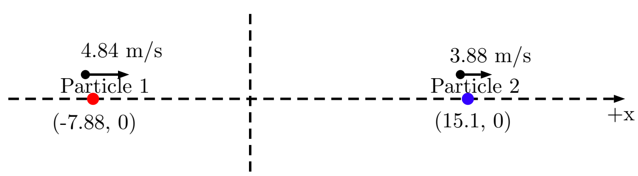


The new speed is the magnitude of the resulting vector ($\vec{V}_a + \vec{V}_w$) and the new direction is α degrees north of east as shown in the figure.

$$
\begin{aligned}
|\vec{V}_{a/w}| &= \sqrt{|\vec{V}_a|^2 + |\vec{V}_w|^2 - 2|\vec{V}_a||\vec{V}_w|\cos(\pi - \theta)} \\
&= \sqrt{120^2 + 38^2 - 2 \cdot 120 \cdot 38 \cdot \cos(180 - 24)^\circ} = 155.48\ m/s
\end{aligned}
$$

$$
\begin{aligned}
\frac{\sin\alpha}{V_w} &= \frac{\sin(\pi - \theta)}{V_{a/w}} \\
\sin\alpha &= \frac{\sin(\pi - \theta) \cdot V_w}{V_{a/w}} \\
\alpha &= \sin^{-1}\left[\frac{\sin(180 - 24)^\circ \cdot 38}{155.48}\right] = 0.0995 \text{ radians} = \ 5.7^\circ
\end{aligned}
$$

Problems 16, 17: $\boxed{4\ pt}$ Two particles are moving uniformly along x-axis in the +x direction. At the initial moment, Particle 1 is located at -7.88 m (left of origin) and has a speed 4.84 m/s and particle 2 is located at 15.1 m (right of origin) and has a speed of 3.88 m/s. After what time from the initial moment will particle 1 catch up with particle 2? Where are they located on the x-axis when they catch up?



This problem is easier to solve if we choose relative velocity of **Particle 1** with respect to **Particle 2** (we chose 1 because it's moving faster and will catch up with 2. Also 2 is at rest relative to 1 and we just have to deal with motion of 1).

$$
\begin{aligned}
\vec{v}_{1/2} &= \vec{v}_1 - \vec{v}_2 = 4.84 - 3.88 = 0.96\,m/s \\
\Delta\vec{r} &= \vec{r}_2 - \vec{r}_1 = 15.1 - (-7.88) = 22.98\,m \\
\Delta\vec{r} &= \vec{v}_{1/2}\,t \ \Rightarrow\ t = 22.98/0.96 = \underline{23.9375\,s}
\end{aligned}
$$

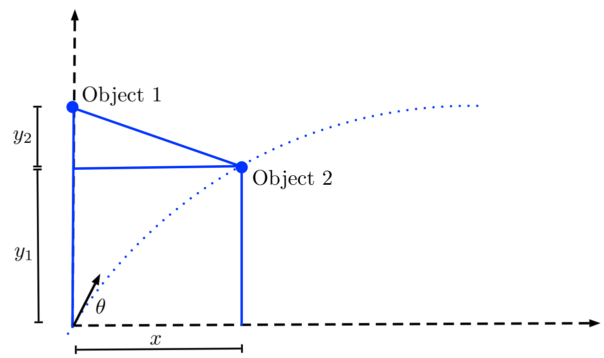
It takes 23.9375 s for Particle 1 to catch up with Particle 2. Now if we suppose Particle 1 moved with it's speed for 23.9375 s, how far would it have traveled? That is easy to calculate.

$$\vec{r}\,'_1 = \vec{v}_1\,t = (3.88)(23.9375) = 92.8775\,m$$

So it would be located at $\vec{r}_1 + \vec{r}\,'_1 = (15.1 + 92.8775) = \underline{107.97\text{ m}}$ when Particle 2 catches up with it.

Problems 18 - 24: $\boxed{13\ pt}$ Two objects are thrown simultaneously from the ground, object 1 straight up and object 2 at an angle of $\theta = 38$ degrees to the horizontal. The initial velocity of each object is 20.9 m/s. Choose g = 9.8 m/s^2 to answer the following questions. Neglect air resistance and assume both objects started from the same horizontal level (ground).

Write down the equations governing the motion of objects 1 and 2. If you get the equations right, 80% of the problem is solved. Time of flight is **twice the time taken to reach maximum height** since the motion up (to max height) and down (to reach the ground) are symmetrical.



Object 1 at any instant t :

$$x = 0$$

$$y = v_0 t - \frac{1}{2} g t^2$$

$$\text{At max height} : 0 = v_0 - g t'$$

Object 2 at any instant t :

$$x = (v_0 \cos\theta)\, t$$

$$y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$\text{At max height} : 0 = (v_0 \sin\theta) - g t'$$

Problem 18 : $y = (y_1 + y_2) = (20.9)(0.36) - \frac{1}{2}(9.8)(0.36^2) = \boxed{6.889\, m}$

Problem 19 : $y = y_1 = (20.9 \sin 38^\circ)(0.36) - \frac{1}{2}(9.8)(0.36^2) = \boxed{3.997\, m}$

Problem 20 : $x = (20.9 \cos 38^\circ)(0.36) = \boxed{5.93\, m}$

Problem 21 : $r^2 = (y_2^2 + x^2) \;\Rightarrow\; r = \sqrt{(6.889 - 3.997)^2 + 5.93^2} = \boxed{6.596\, m}$

Problem 22 : $t = 2t' = 2(20.9)/9.8 = \boxed{4.265\, s}$

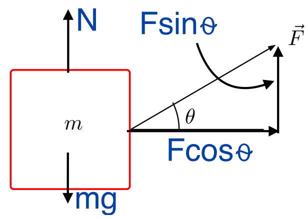
Problem 23 : $t = 2t' = 2(20.9 \sin 38^\circ)/9.8 = \boxed{2.626\, s}$

Problem 24 : Obviously Object 2

Problems 25 - 29: $\boxed{10\ pt}$ An box of mass m = 126 kg is being dragged on a frictionless horizontal surface with a force F = 0.24 kN using a rope. The rope is taut and makes an angle θ=37 degrees with the horizontal. Choose g=9.8 m/s^2, assume rope is inextensible.

Free body diagram is shown with forces acting on the box.

For the remaining parts, we have to write down the equations of motion using Newton's second law. The box will lose contact with the floor when the net force in the +y is slightly larger than the weight when the contact force (or the normal force in this case tends to vanish). For this to happen a larger force F' is required (problem states the angle remains the same).



$$\sum \vec{F}_{ext} = m\vec{a}$$

$$\text{y component} : N + F\sin\theta = mg \tag{1}$$

$$\text{x component} : F\cos\theta = ma \tag{2}$$

Use the above equations to solve the problems

Problem 25: The tension in the rope is equal to the applied force (since rope is inextensible) = 240 N

$$\text{Problem 26} : N = mg - F\sin\theta = 126(9.8) - 240(\sin 37°) = \boxed{1090.36\ N}$$

$$\text{Problem 27} : a = F\cos\theta/m = 240(\cos 37°)/126 = \boxed{1.52\ m/s^2}$$

$$\text{Problem 28} : N \to 0 \ \Rightarrow \ mg - F'\sin\theta = 126(9.8) - F'(\sin 37°) = 0$$

$$\Rightarrow \ F' = mg/(\sin\theta) = 126(9.8)/(\sin 37°) = \boxed{2051.8\ N}$$

$$\text{Problem 29} : a = F'\cos\theta/m = 2051.8(\cos 37°)/126 = \boxed{13.0\ m/s^2}$$
