CAPA 08: Solutions

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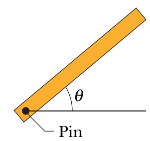
Solution Consider the motion of the center-of-mass. It's located at a height $(\ell/2)\sin\theta$ initially. When it passes through horizontal position, all it's potential energy is converted into kinetic energy of rotation about the pin (axis). Use PE = KE and solve for ω

$$PE = mg(\ell/2)\sin\theta$$

$$KE = \frac{1}{2}I_{pin}\omega^2 = \frac{\omega^2}{2}\left(\frac{1}{12}m\ell^2 + m(\ell/2)^2\right) = \frac{\omega^2}{2}\frac{m\ell^2}{3}$$

$$\Rightarrow \ \omega = \sqrt{\frac{3g\sin\theta}{\ell}} = \sqrt{\frac{3(9.8)\sin 40^\circ}{2.0}} = 3.1 \ rad/s$$

81 The thin uniform rod in Fig. 10-50 has length 2.0 m and can pivot about a horizontal, frictionless pin through one end. It is released from rest at angle $\theta = 40^\circ$ above the horizontal. Use the principle of conservation of energy to determine the angular speed of the rod as it passes through the horizontal position.



•**47** SSM A small ball of mass 0.75 kg is attached to one end of a 1.25-m-long massless rod, and the other end of the rod is hung from a pivot. When the resulting pendulum is 30° from the vertical, what is the magnitude of the gravitational torque calculated about the pivot?

Solution Gravitational torque is the product of the gravitational force and the moment arm about a vertical axis passing through the pivot.

$$r_\perp = \ell\sin\theta \quad F = mg$$

$$\Rightarrow \ \tau = mg\ell\sin\theta = (0.75)(9.8)(1.25)\sin 30^\circ = 4.6 \ Nm$$

Solution The latitude is a great circle (in navigation parlance) which rotates about the axis of the earth. The radius of this circle is $r = R\cos\theta$ where R is the equatorial radius of the earth (will be given). Earth takes 24 hrs = 86400 s to complete one rotation (2π radians). At the equator, just use r_{eq} = R. Angular speed will not change no matter where you are - earth sweeps the same angle about it's polar axis.

$$(a) \ \omega = \frac{2\pi}{86400} = 7.3 \cdot 10^{-5} \ rad/s$$

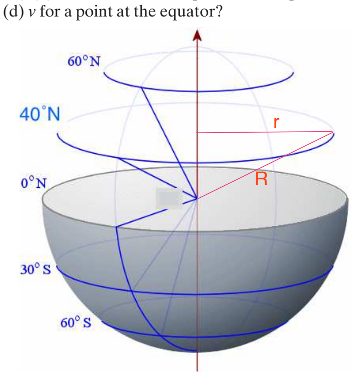
$$r = 6.4 \cdot 10^{6m}\cos 40^\circ$$

$$(b) \ v = r\omega = (6.4 \cdot 10^{6m}\cos 40^\circ)(7.3 \cdot 10^{-5}) = 350 \ m/s$$

$$(d) \ v = r_{eq}\omega = (6.4 \cdot 10^{6m})(7.3 \cdot 10^{-5}) = 545 \ m/s$$

That speed is faster than the fastest human being (or any other animal on the planet). Some passenger airlines travel at that speed or grater. Check out some of the cities that are on this latitude https://en.wikipedia.org/wiki/40th_parallel_north

••**25** SSM (a) What is the angular speed ω about the polar axis of a point on Earth's surface at latitude 40° N? (Earth rotates about that axis.) (b) What is the linear speed v of the point? What are (c) ω and (d) v for a point at the equator?



Solution Consider the motion of center-of-mass first. It's at a height 0.5 m and when the meter stick lands on the floor, all it's PE is converted into KE. The linear speed of any point on the meter stick is equal to the angular speed about the pivot times the distance of that point from the pivot.

••63 SSM ILW A meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end just before it hits the floor, assuming that the end on the floor does not slip. (*Hint:* Consider the stick to be a thin rod and use the conservation of energy principle.)

$$PE = mg(\ell/2) \quad v = \ell\omega$$

$$KE = \frac{1}{2} I_{pivot}\omega^2 = \frac{\omega^2}{2}\left(\frac{1}{12}m\ell^2 + m(\ell/2)^2\right) = \frac{\omega^2}{2}\frac{m\ell^2}{3}$$

$$PE = KE \;\Rightarrow\; v = \sqrt{3g\ell} = \sqrt{3(9.8)(1)} = 5.42\ m/s$$