CAPA 05: Solutions

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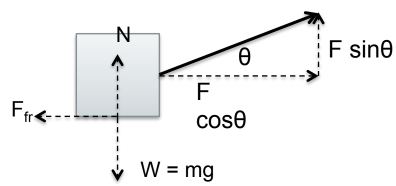
••21 An initially stationary box of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1100 N. The coefficient of static friction between the box and the floor is 0.35. (a) What should be the angle between the cable and the horizontal in order to pull the greatest possible amount of sand, and (b) what is the weight of the sand and box in that situation?



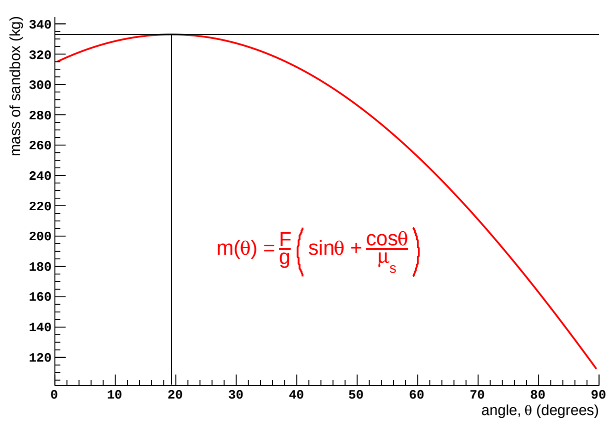
Solution In order to pull the greatest possible amount of sand (m) as function of the angle, we have to first express $m = f(\theta)$. Setup the free body diagram and solve for m as a function of θ. Then you can either use calculus or graphical solution to figure out the greatest possible angle.

$$F\cos\theta - F_{fr} = ma \tag{1}$$

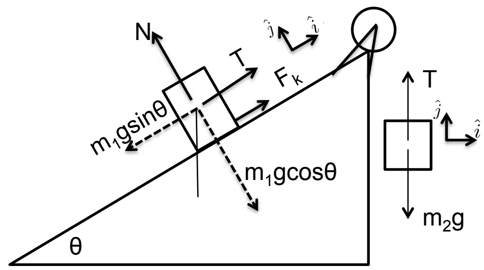
$$F\sin\theta + N - mg = 0 \quad F_{fr} = \mu_s N \tag{2}$$

$$m = \frac{F}{g}\left(\sin\theta + \frac{\cos\theta}{\mu_s}\right)$$

$$\frac{dm}{d\theta} = \frac{F}{g}\left(\cos\theta - \frac{\sin\theta}{\mu_s}\right) = 0 \quad \Rightarrow \boxed{\theta = \tan^{-1}(\mu_s) = 19.3°} \qquad \text{Verify } \left.\frac{d^2m}{d\theta^2}\right|_{\theta=19.3°} \le 0$$



••28 In Fig. 6-33, two blocks are connected over a pulley. The mass of block A is 10 kg, and the coefficient of kinetic friction between A and the incline is 0.20. Angle θ of the incline is 30°. Block A slides down the incline at constant speed. What is the mass of block B? Assume the connecting rope has negligible mass. (The pulley's function is only to redirect the rope.)



Solution Given mass of block A $m_1 = 10$ kg, coefficient of static friction $\mu_s = 0.2$ and $\theta = 30°$. If block A slides down the incline at constant speed, its acceleration must be zero. F_k is the force of kinetic friction. Free body diagram of A and B are shown. Equations of motion are as follows:

$$(T + F_k - m_1 g \sin\theta)\,\hat{\imath} = 0\;; \quad (N - m_1 g \cos\theta)\,\hat{\jmath} = 0$$

$$(T - m_2 g)\,\hat{\jmath} = 0\;; \quad F_k = \mu_s\, N$$

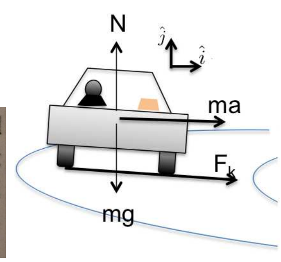
Solve for m_2 by re-arranging the equations above, substituting for T and F_k.

$$0 = m_2 g + \mu_k m_1 g \cos\theta - m_1 g \sin\theta$$

$$\Rightarrow \; m_2 = m_1(\sin\theta - \mu_k \cos\theta) \; = 10\,(\sin 30° - 0.2\,\cos 30°)$$

$$\boxed{m_2 = 3.26\ kg}$$

•42 Suppose the coefficient of static friction between the road and the tires on a car is 0.60 and the car has no negative lift. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius?



Solution Given r = 30.5 m, $\mu_k = 0.6$. For the horizontal forces - the centripetal force (effect) is provided by friction (cause), which keeps the car from sliding or having any negative lift. For vertical forces - the normal force and weight of the car are equal and opposite. We can write the equations of motion. Solve for v by re-arranging the equations below, substituting for a_c and F_k.

$$F_k\,\hat{\imath} = m\,a_c\,\hat{\imath} \quad a_c = \frac{v^2}{r}$$

$$(N - mg)\,\hat{\jmath} = 0 \quad F_k = \mu_k\, N$$

$$\mu_k m g = m\frac{v^2}{r}$$

$$v = \sqrt{\mu_k g r} = \sqrt{0.6 \times 9.8 \times 30.5}$$

$$\boxed{v = 13.4\ m/s}$$

Notice that if the road is icy/slick (ie dangerous), the coefficient of friction becomes smaller, so the maximum (safe) speed at which one can negotiate a turn also reduces. In this case if the driver turns at high speed, the vehicle is more likely to overturn and/or slide. The slide/overturn can be reduced by increasing the turning radius (if possible).