CAPA 04: Solutions

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Solution Reference frame: \hat{i} : up the incline. Free body diagram consists of essentially three forces acting on the block. It's weight, normal force and the tension in the cord. For parts (a, b) we know it's not moving, therefore $\vec{a} = 0$. When the cord is cut, it accelerates down the incline (T = 0).

$$\begin{split} (T - mg\sin\theta)\,\hat{i} + (N - mg\cos\theta)\,\hat{j} &= 0\\ \Rightarrow \ T &= mg\sin\theta = (8.5\,kg)(9.8\,m/s^2)(\sin30^\circ) \approx 42\,N\\ \Rightarrow \ N &= mg\cos\theta = (8.5\,kg)(9.8\,m/s^2)(\cos30^\circ) \approx 72\,N\\ \qquad -mg\sin\theta\,\hat{i} &= -ma\,\hat{i}\\ \Rightarrow \ a &= g\sin\theta = (9.8\,m/s^2)(\sin30^\circ) \approx 4.9\,m/s^2 \end{split}$$

••41 Using a rope that will snap if the tension in it exceeds 387 N, you need to lower a bundle of old roofing material weighing 449 N from a point 6.1 m above the ground. Obviously if you hang the bundle on the rope, it will snap. So, you allow the bundle to accelerate downward. (a) What magnitude of the bundle's acceleration will put the rope on the verge of snapping? (b) At that acceleration, with what speed would the bundle hit the ground?



Solution

$$m = \frac{W}{g} = \frac{449 N}{9.8 m/s^2} = 45.82 kg; \quad (T - mg) \hat{j} = -(ma) \hat{j} \quad \Rightarrow \ a = \frac{(mg - T)}{m} = \frac{(449 - 387) N}{45.82 kg} \approx 1.4 m/s^2$$
$$v = at; \ h = \frac{1}{2}at^2 = \frac{1}{2}a\left(\frac{v}{a}\right)^2 \quad \Rightarrow \ v = \sqrt{2ah} = \sqrt{2 \cdot 1.35 \cdot 6.1} = 4.1 m/s^2$$

••61 SSM ILW A hot-air balloon of mass M is descending vertically with downward acceleration of magnitude a. How much mass (ballast) must be thrown out to give the balloon an upward acceleration of magnitude a? Assume that the upward force from the air (the lift) does not change because of the decrease in mass.



Solution When the ballast is thrown out, the downward force (weight) reduces and the balloon starts to accelerate upward. Let a mass "m" be thrown out. The free body diagrams before and after are shown. The tension in the ropes will remain the same since the upward force (lift) does not change - ie they are burning fuel (to generate hot air) at the same rate. Write down the equations of motion and solve for m (by eliminating T).

$$\{Mg - T\}\hat{j} = Ma\hat{j}$$
$$\{T - (M - m)g\}\hat{j} = (M - m)a\hat{j}$$
$$\boxed{m = \frac{2Ma}{(a + g)} kg}$$



Solution Free body diagrams are shown above. Acceleration of the system is a (all bodies move with the same acceleration).

$$T_1 = m_1 a \quad T_2 - T_1 = m_2 a \quad T_3 - T_2 = m_3 a$$

$$\Rightarrow T_2 = m_1 a + m_2 a \quad T_3 = (m_1 + m_2 + m_3)a \Rightarrow a = \frac{65 N}{(12 + 24 + 31) kg} = 0.970 m/s^2$$

$$\Rightarrow T_1 = (12 kg)(0.970 m/s^2) = 11.64 N \quad T_2 = (12 + 24) kg (0.970 m/s^2) = 34.93 N$$