CAPA 03: Solutions

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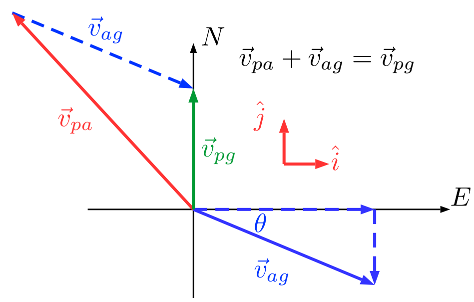
74 After flying for 15 min in a wind blowing 42 km/h at an angle of 20° south of east, an airplane pilot is over a town that is 55 km due north of the starting point. What is the speed of the airplane relative to the air?



Solution

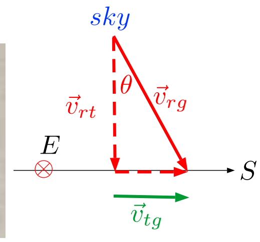
- $\vec{v}_{ag}$: velocity of air relative to ground
- $\vec{v}_{pg}$: velocity of plane relative to ground
- $\vec{v}_{pa}$: velocity of plane relative to air

If there was no wind (still air) and the plane flew for 15 mins = 0.25 h for distance of 55 km (due north), it's velocity would be $\vec{v}_{pg}$ = 55 km / 0.25 h = 220 $\hat{j}$ km/h (vector shown in green). Velocity of air relative to ground is 42 km/h at $\theta = 20°$ south of east which means $\vec{v}_{ag} = 42\cos 20°\hat{i} - 42\sin 20°\hat{j}$ km/h (see blue dotted vectors that are it's components)

$$\begin{aligned}
\vec{v}_{pg} &= \vec{v}_{pa} + \vec{v}_{ag} \\
\Rightarrow \quad \vec{v}_{pa} &= \vec{v}_{pg} - \vec{v}_{ag} \\
&= 220\hat{j} - \left\{42\cos 20°\hat{i} - 42\sin 20°\hat{j}\right\} \\
&= -39.5\hat{i} + 234.4\hat{j} \\
\Rightarrow \quad |\vec{v}_{pa}| &= \sqrt{(-39.5)^2 + (234.4)^2} \\
|\vec{v}_{pa}| &\approx 238.0\ km/h \\
\angle\vec{v}_{pa} &= \tan^{-1}(234.4/-39.5) = -80.4°
\end{aligned}$$

Notice the airplane has to fly at a greater speed than in still air. Also notice that the airplane is compensating for the wind (look at the direction of the red line which denotes the velocity relative to air is <u>80.4° north of west</u>) in order for the airplane to arrive directly north of where it started.

••75 SSM A train travels due south at 30 m/s (relative to the ground) in a rain that is blown toward the south by the wind. The path of each raindrop makes an angle of 70° with the vertical, as measured by an observer stationary on the ground. An observer on the train, however, sees the drops fall perfectly vertically. Determine the speed of the raindrops relative to the ground.



Solution Note the easterly direction is pointing into the page. You are observing the rain from the platform (for instance) facing east with your ourstretched right hand pointing south. Since the rain appears vertical from inside the train (ie the horizontal component of the rain vanishes for an observer inside the train), the only way that happens is when it's horizontal component is equal to the velocity of train. $\theta = 70°$

- $\vec{v}_{rg}$: velocity of rain relative to ground
- $\vec{v}_{tg}$: velocity of train relative to ground
- $\vec{v}_{rt}$: velocity of rain relative to train

$$
\begin{aligned}
\vec{v}_{rg} &= \vec{v}_{rt} + \vec{v}_{tg} \\
|\vec{v}_{tg}| &= 30\, m/s \\
\tan\theta &= \frac{|\vec{v}_{tg}|}{|\vec{v}_{rt}|} \quad \Rightarrow \quad \vec{v}_{rt} = \frac{|\vec{v}_{tg}|}{\tan\theta} = \frac{30}{\tan 70°} = 10.92\ m/s \\
|\vec{v}_{rg}| &= \sqrt{|\vec{v}_{rt}|^2 + |\vec{v}_{tg}|^2} \\
&= \sqrt{30.0^2 + 10.9^2} \\
|\vec{v}_{rg}| &\approx 32.0\ m/s
\end{aligned}
$$

••71 A suspicious-looking man runs as fast as he can along a moving sidewalk from one end to the other, taking 2.50 s. Then security agents appear, and the man runs as fast as he can back along the sidewalk to his starting point, taking 10.0 s. What is the ratio of the man's running speed to the sidewalk's speed?

Solution Let the distance the man runs be "d". The velocity of the man relative to the sidewalk is $v_m + v_w$ when he is moving in the same direction and $v_m - v_w$ when he is moving opposite to it (like running on a treadmill). The same distance is traversed in both cases but it takes longer for the return trip.

$$
\begin{aligned}
d &= 2.5 \cdot (v_m + v_w) = 10 \cdot (v_m - v_w) \\
\Rightarrow\ v_m + v_w &= 4v_m - 4v_w \\
\Rightarrow\ \frac{v_m}{v_w} &= \frac{5}{3} = 1.67
\end{aligned}
$$