CAPA 02: Solutions

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Useful Identities

$$\hat{i}\cdot\hat{i} = 1 \quad \hat{j}\cdot\hat{j} = 1 \quad \hat{k}\cdot\hat{k} = 1$$
$$\hat{i}\times\hat{j} = \hat{k} \quad \hat{j}\times\hat{k} = \hat{i} \quad \hat{k}\times\hat{i} = \hat{j}$$

••23 If $\vec{B}$ is added to $\vec{C} = 3.0\hat{i} + 4.0\hat{j}$, the result is a vector in the positive direction of the y axis, with a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$?

<u>**Solution**</u> Let $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{B} + \vec{C} = \vec{D} = d\hat{j} \tag{1}$$
$$\Rightarrow \; (x+3)\hat{i} + (y+4)\hat{j} + z\hat{k} = d\hat{j}$$
$$\Rightarrow \; (x+3) = 0 \quad (y+4) = d \quad z = 0$$
$$|\vec{D}| = d = (y+4) = |\vec{C}| = \sqrt{3^2 + 4^2} = 5 \tag{2}$$
$$\Rightarrow \; y = 1$$
$$\vec{B} = -3\hat{i} + \hat{j} \quad \Rightarrow \; |\vec{B}| = \sqrt{(-3)^2 + 1^2} = 3.2$$

••39 Vector $\vec{A}$ has a magnitude of 6.00 units, vector $\vec{B}$ has a magnitude of 7.00 units, and $\vec{A}\cdot\vec{B}$ has a value of 14.0. What is the angle between the directions of $\vec{A}$ and $\vec{B}$?

<u>**Solution**</u>

$$|\vec{A}\cdot\vec{B}| = |\vec{A}|\;|\vec{B}|\;\cos\theta \quad \Rightarrow \; \cos\theta = \frac{|\vec{A}\cdot\vec{B}|}{|\vec{A}|\;|\vec{B}|} = \frac{14}{6\times 7} \quad \Rightarrow \; \theta = 54.14^\circ$$

••44 In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$,

$$\vec{v} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k} \quad \text{and} \quad \vec{F} = 4.0\hat{i} - 20\hat{j} + 12\hat{k}.$$

What then is $\vec{B}$ in unit-vector notation if $B_x = B_y$?

Solution Let $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$. Given $B_x = B_y$. The equation gives the force on a charged particle moving in a magnetic field. We are asked to find the magnetic field vector $\vec{B}$, given the velocity $\vec{v}$ of a charged particle with charge q units experiencing (ie moving under the influence of) a force $\vec{F}$.

$$\begin{aligned}
\vec{F} &= q\vec{v} \times \vec{B} \\
\Rightarrow \; 4.0\hat{\imath} - 20\hat{\jmath} + 12\hat{k} &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ qv_x & qv_y & qv_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4.0 & 8.0 & 12.0 \\ B_x & B_x & B_z \end{vmatrix} \\
\Rightarrow \; 4.0\hat{\imath} - 20\hat{\jmath} + 12\hat{k} &= (8B_z - 12B_x)\hat{\imath} - (4B_z - 12B_x)\hat{\jmath} + (4B_x - 8B_x)\hat{k}
\end{aligned}$$

$$\Rightarrow \; 4.0 = (8B_z - 12B_x)$$

$$\Rightarrow \; -20.0 = -(4B_z - 12B_x)$$

$$\Rightarrow \; 12.0 = (4B_x - 8B_x)$$

Solving, we get $\vec{B} = -3.0\hat{\imath} - 3.0\hat{\jmath} - 4.0\hat{k}$
