CAPA 01: Solutions

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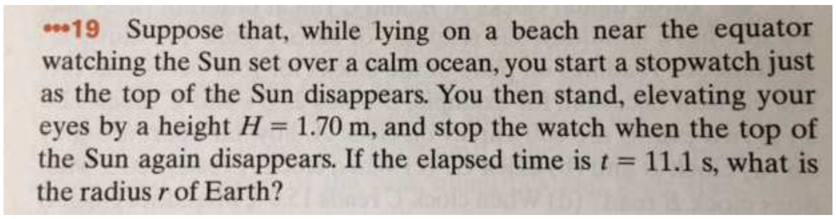
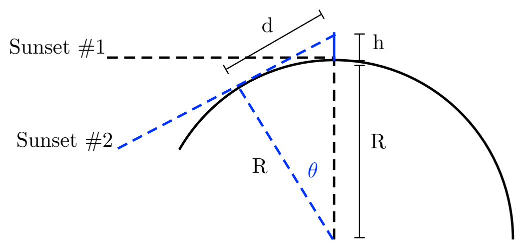

•••19 Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?



Solution Sunset #1 is the line of sight of the first sunset (lying down) and Sunset #2 is the second (standing up). The duration between these two events is t = 11.1 s. Your height is h = 1.7 m. Let radius of the earth be R. Earth would have rotated by angle θ between these two events. The lines of sight form tangents. Normals to those from points of intersection meet at the center of earth forming a right triangle with sides {R, R+h, d } as shown. Using trigonometry and the fact that your height h is miniscule compared to radius of the earth $h \ll R$, we can determine the approximate radius of the earth (ball park estimate). Earth takes 24 hrs (or 86400 s) for a full 360°rotation. In 11.1 s, it rotates by a small fraction of that given by $\theta = \frac{11.1}{86400} \cdot (360) = 4.625 \times 10^{-2}$ degrees.

$$\tan\theta = \frac{d}{R} \quad\Rightarrow\quad d = R\tan\theta \quad\Rightarrow\quad d^2 = R^2\tan^2\theta \tag{1}$$

$$d^2 + R^2 = (R+h)^2 = R^2 + h^2 + 2Rh \approx R^2 + 2Rh \quad\Rightarrow\quad d^2 = 2Rh \tag{2}$$

Using results (1) and (2) and eliminating d, we get

$$R = \frac{2h}{\tan^2\theta} = 2(1.7)/[\tan^2(0.04625°)] = \boxed{5218\,km}$$

Equatorial radius of the earth using more sophisticated (and detailed) measurements turns out to be 6371 km. Our answer is off (approximate), but we did well for a measurement using our stopwatch!

Solution Here's an excellent introduction to pulsars: http://www-outreach.phy.cam.ac.uk/camphy/pulsars/pulsars_index.htm The time for one complete rotation is called the time period (T, in units of

•16 Time standards are now based on atomic clocks. A promising second standard is based on *pulsars,* which are rotating neutron stars (highly compact stars consisting only of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR 1937 + 21 is an example; it rotates once every 1.557 806 448 872 75 ± 3 ms, where the trailing ±3 indicates the uncertainty in the last decimal place (it does *not* mean ±3 ms). (a) How many rotations does PSR 1937 + 21 make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?

seconds). The fraction of rotation completed in single unit of time is called the frequency (f, in units of hertz) which is by definition, the inverse of time period. For part (a) notice the number of significant figures is three (t = 7.00 days). Uncertainty per revolution is $\delta = \pm 3 \times 10^{-14} \times 10^{-3}\ s$

$$f = \frac{1}{1.55780644887275 \times 10^{-3}}\ Hz$$

$$(a)\ N = f \cdot t = \frac{(7.00)(24)(60)(60)s}{1.55780644887275 \times 10^{-3} s} = 388238218.4 = \boxed{3.88 \times 10^{8}\ rotations}$$

$$(b)\ N = f \cdot t\ \Rightarrow 1 \times 10^{6} = \frac{t}{1.55780644887275 \times 10^{-3} s}\ \Rightarrow t = \boxed{1557.80644887275\, s}$$

$$(c)\ \Delta t = \delta \cdot N = \pm(3 \times 10^{-17} s) \cdot (1 \times 10^{6}) = \boxed{\pm 3 \times 10^{-11}\ s}$$
