Kinetic Energy And Work

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Phys 211, Lecture 11

Oct 13, 2015

Clicker Question 1 (30 s)

- A rope which weighs 2 lb/ft is overhanging from a building 100 ft tall.
- The fully extended length of the rope is 50 ft.
- Consider the work done against gravity in pulling the rope up from the top of the building and answer the following

A. More work is done in pulling the top half of the rope compared to the bottom half
B. Equal work is done in pulling both halves
C. More work is done in pulling the bottom half of the rope compared to the top half
D. Cannot be determined ‘

Work: Gravitational Force

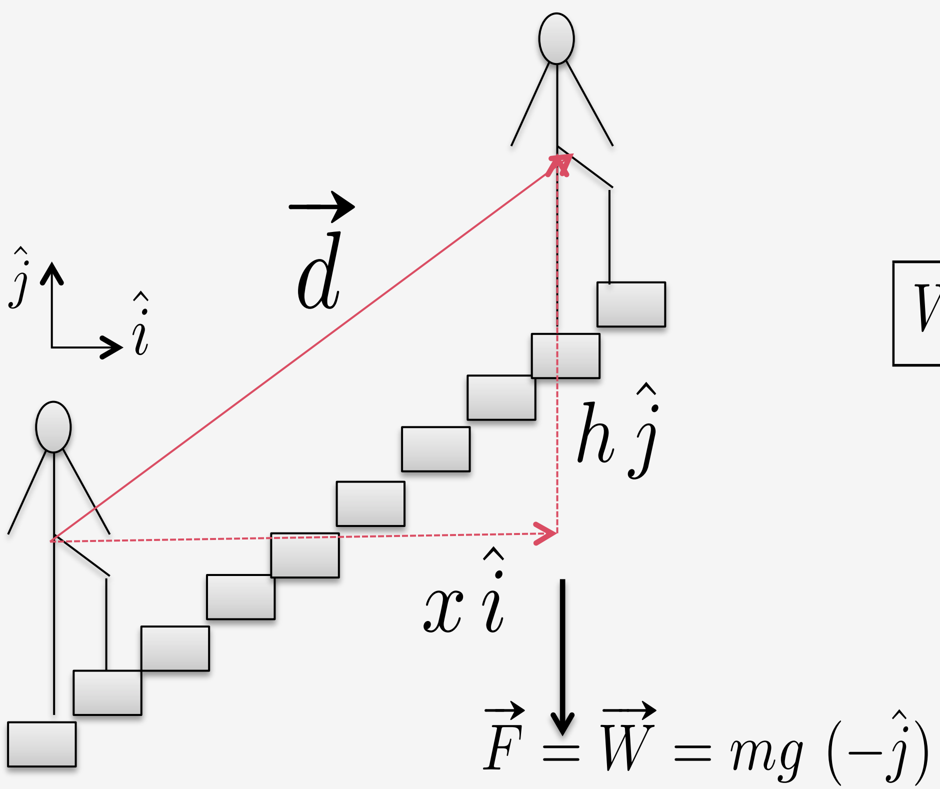
- Example: Gravitational force (weight) acting on an object can do work.

$$W = \vec{F} \cdot \vec{d}$$

$$= -(mg\,\hat{j}) \cdot (x\,\hat{i} + h\,\hat{j})$$

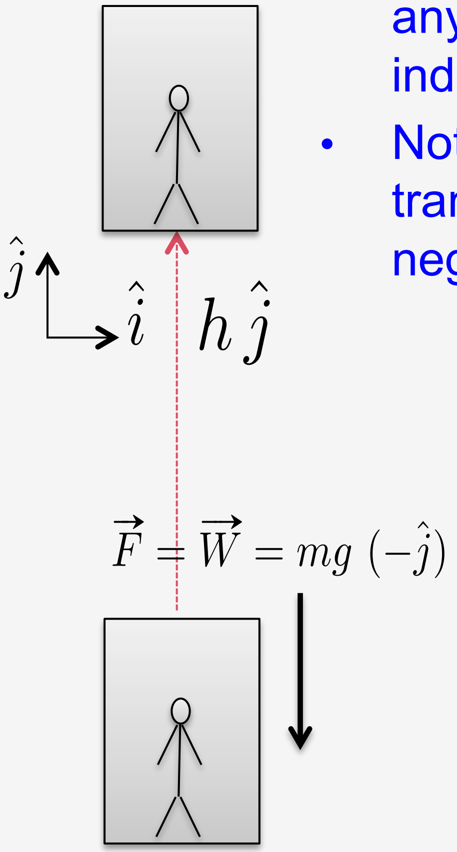
$$= -mgx(\hat{j} \cdot \hat{i}) - mgh(\hat{j} \cdot \hat{j})$$

$$\boxed{W = -mgh}$$



Work: Gravitational Force

- Note: In the previous example, if we took the elevator straight up (by a height "h") the amount of work done by gravity would be the same as taking a flight of stairs of height "h" and length "x". Or any other path for that matter. The work done by gravity is independent of the horizontal displacement.
- Note also as the object goes "up" against gravity, the energy is transferred from the object into the system, therefore work is negative



$$W = \vec{F} \cdot \vec{d}$$

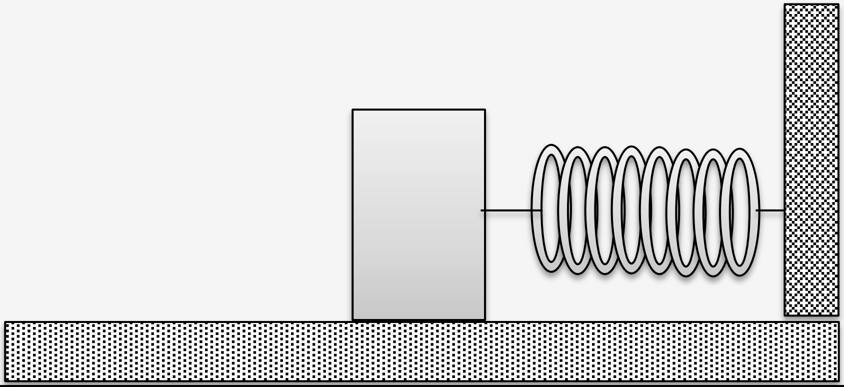
$$= -(mg\,\hat{j}) \cdot (h\,\hat{j})$$

$$= -mgh(\hat{j} \cdot \hat{j})$$

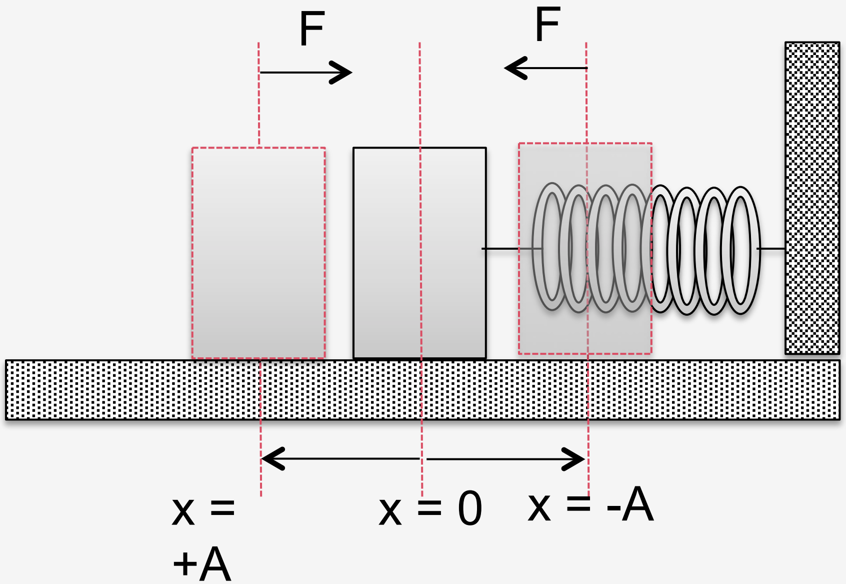
$$\boxed{W = -mgh}$$

Work: Spring Force

- Consider mass-spring system shown. Ignore friction.
- Force exerted by spring (to bring it back to relaxed position) is called the "restoring" force.
- Force exerted on the object (that stretches or compresses the spring away from it's relaxed position) is called the "applied" force
- Applied force and restoring force are equal in magnitude and opposite in (horizontal) direction



Work: Spring Force

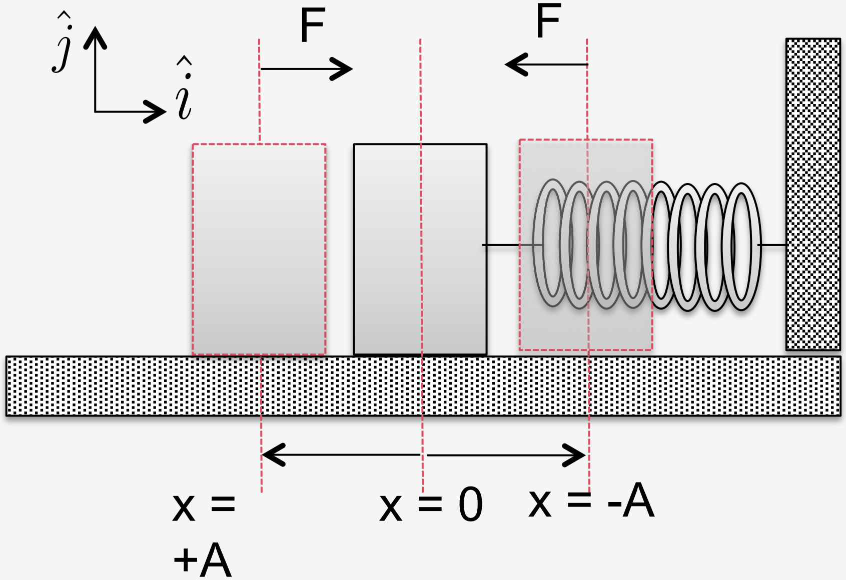


$$\vec{F} \propto \vec{x}$$

$$\vec{F} = -k\,\vec{x}$$

- Restoring force vector and displacement vector are opposite to each other.
- For small displacements the restoring force is proportional to displacement (linear spring).
- The constant "k" is the spring constant and is a measure of the "stiffness" of the spring. Larger the value of "k", stiffer it is. It's value depends on material, temperature and other factors. Units: N/m

Work: Spring Force



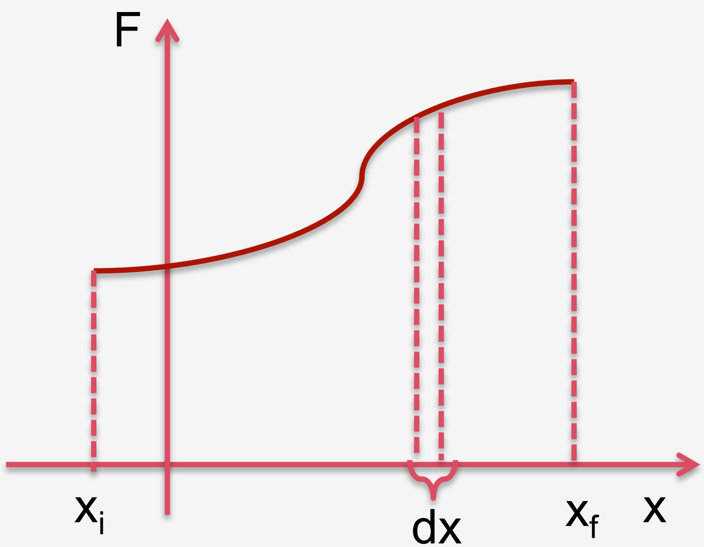
$$dW = \vec{F} \cdot d\vec{x}$$
$$= -k\,x\,\hat{i} \cdot dx\,\hat{i}$$
$$W = -k\int_{x_i}^{x_f} x\,dx$$
$$= \frac{1}{2}k(x_i^2 - x_f^2)$$

- Note: Work done by spring force is zero for any displacement that ends where it began ($x_i = x_f$)
- Work done by spring for a displacement of x_f from relaxed position ($x_i = 0$) is:

$$W = -\frac{1}{2}kx_f^2$$

Work: Variable Force

- In general the same work formula applies to a variable force.
- The work done can be graphically calculated by finding the area under the curve from a Force vs. displacement graph



$$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$$

Power

- Power is the rate of doing work
- If a force does work of W in a time Δt, average power is:

$$P_{avg} = \frac{W}{\Delta t}$$

- The instantaneous power is:

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

- Units: J/s or watts (W)