Projectile/Relative Motion

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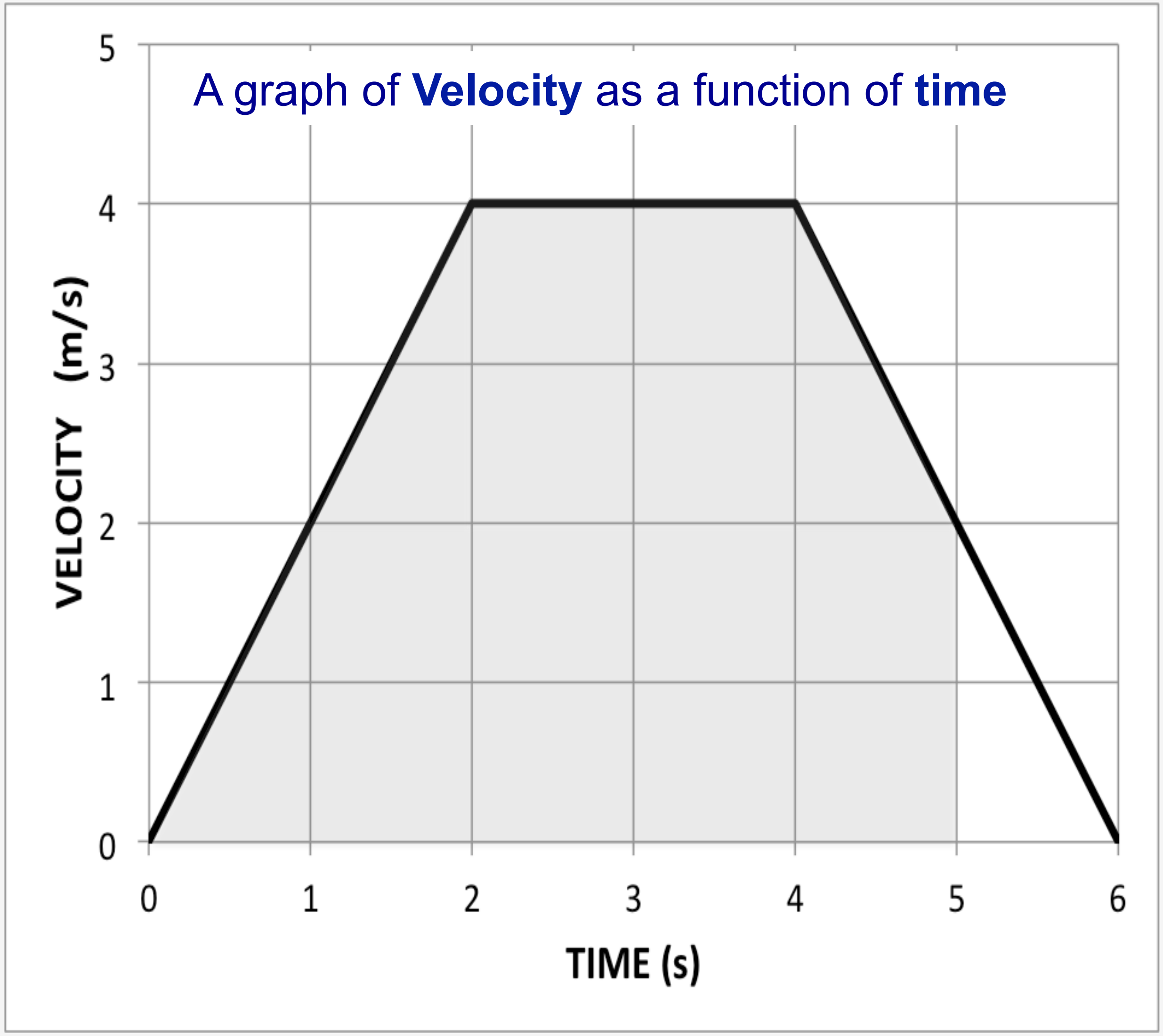
Phys 211, Lectures 6/7

Sep 08/10, 2015

Clicker Question 1 (30 s)



The particle starts at the origin at t = 0 and is moves along the X-axis.

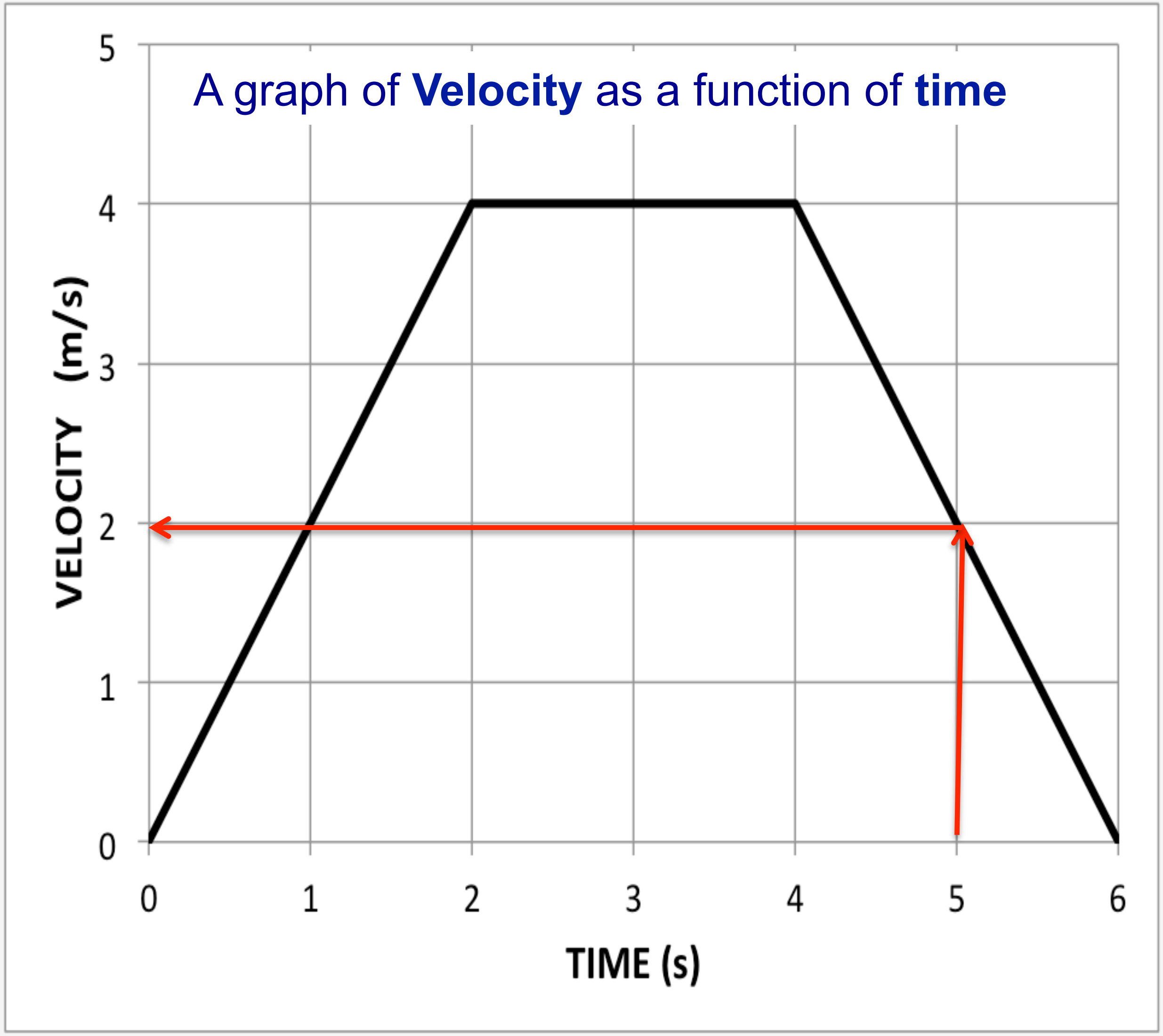
What is the **X co-ordinate** of the particle at **t = 5 s**?

$$x - x_0 = \int_{t=0}^{t} v dt$$

Find area under the curve between t = 0 and t = 5. x_0 is 0. Area of each square is 1 and shaded area is 15.

Answer: 15 m

Clicker Question 2 (30 s)



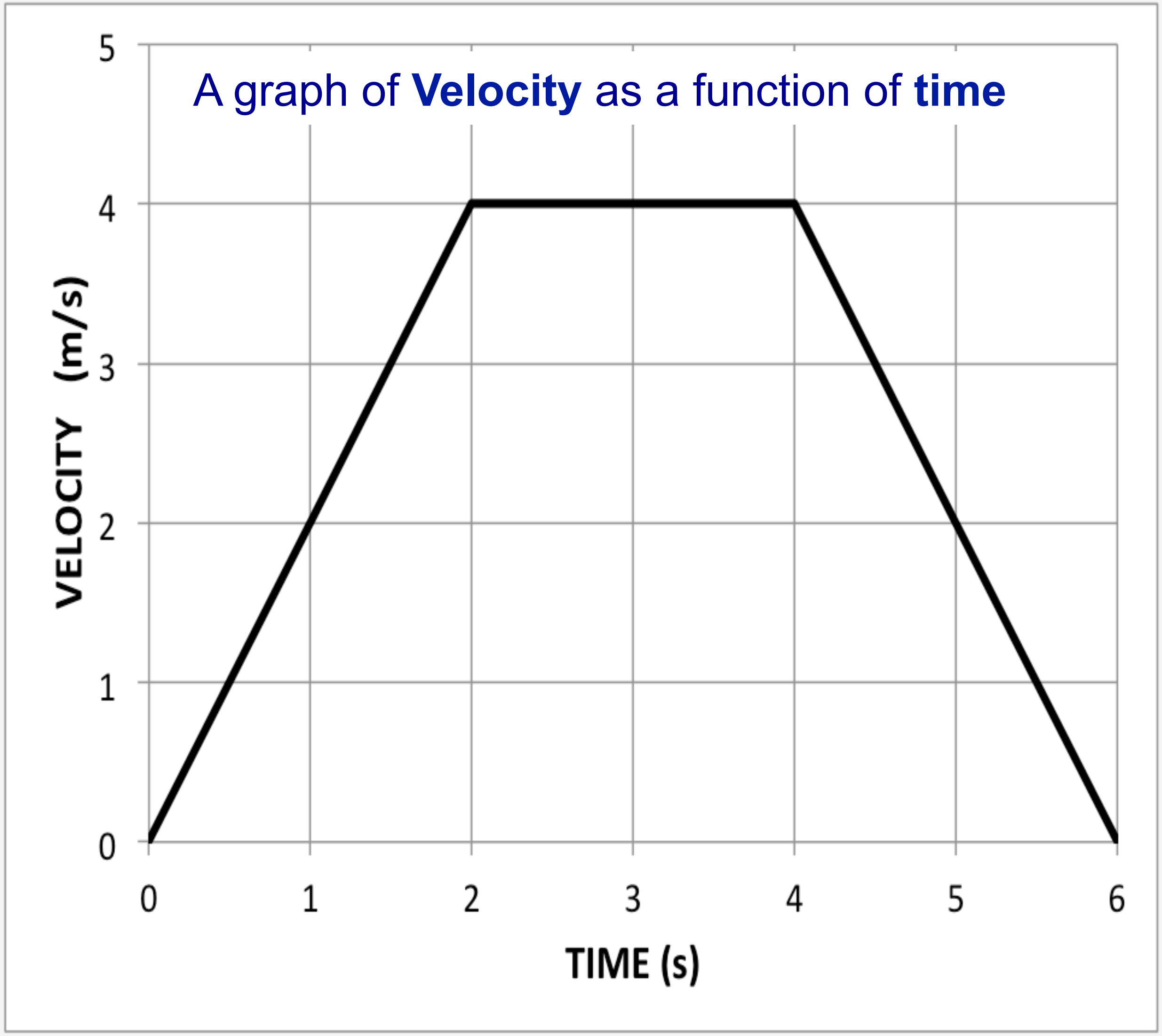
The particle starts at the origin at t = 0 and is moves along the X-axis.

What is the **velocity** of the particle at **t = 5 s** ?

Read the value on the y-axis for t = 5, which is 2

Ans: 2 m/s

Clicker Question 3 (30 s)



The particle starts at the origin at t = 0 and is moves along the X-axis.

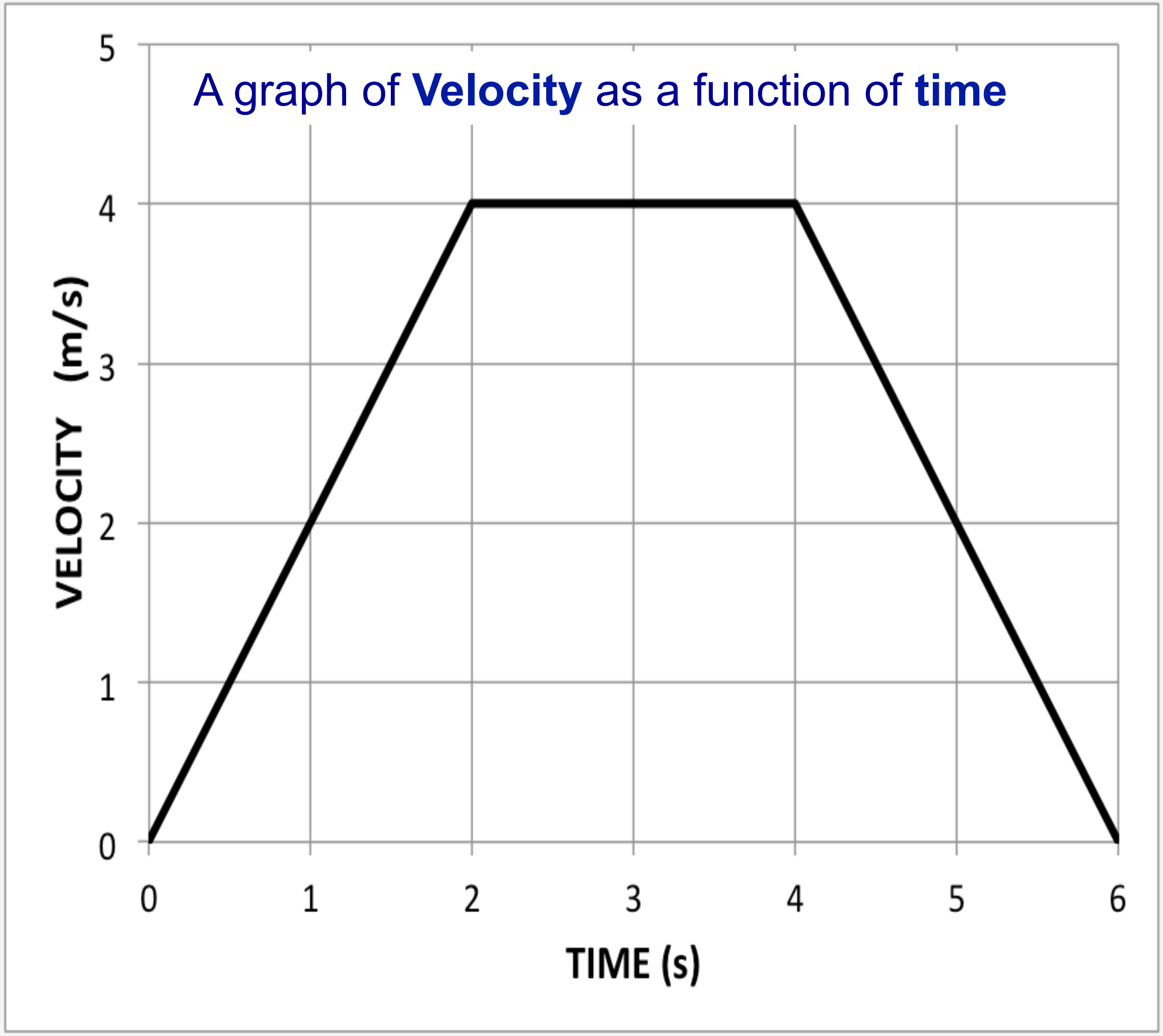
What is the **acceleration** of the particle at **t = 5 s** ?

Find the slope of the line at t = 5 s.

Slope = (0-4)/(6-4) = -2

Ans: -2 m/s^2

Clicker Question 4 (30 s)



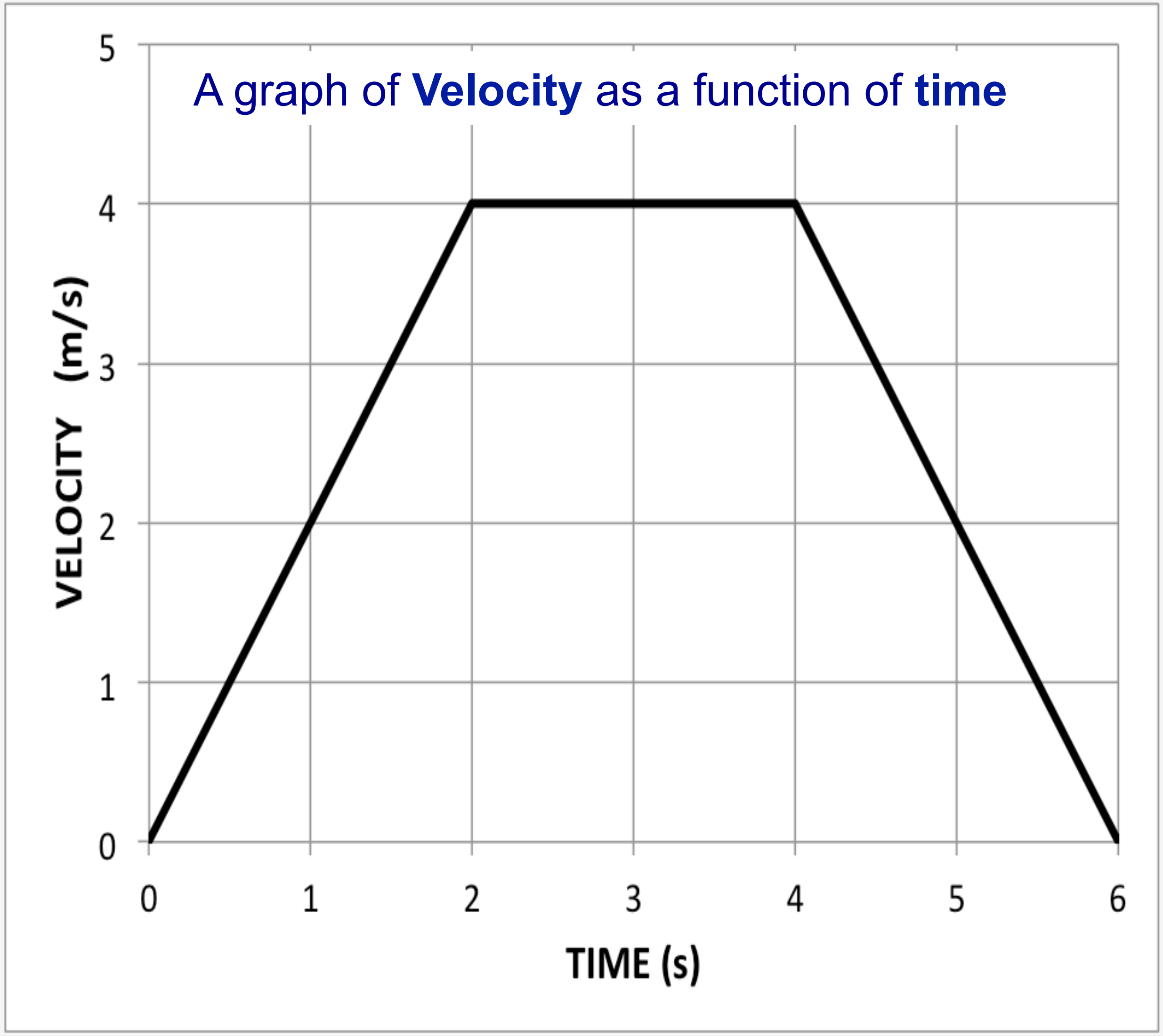
The particle starts at the origin at t = 0 and is moves along the X-axis.

What is the
average velocity
of the particle
between **t = 1 s and t = 5 s** ?

$$\left|v_{avg}\right| = \Delta x/\Delta t$$
$$= \frac{x(5) - x(1)}{t(5) - t(1)}$$
$$= (15 - 1)/(5 - 1) = 3.5$$

Ans: 3.5 m/s

Clicker Question 5 (30 s)



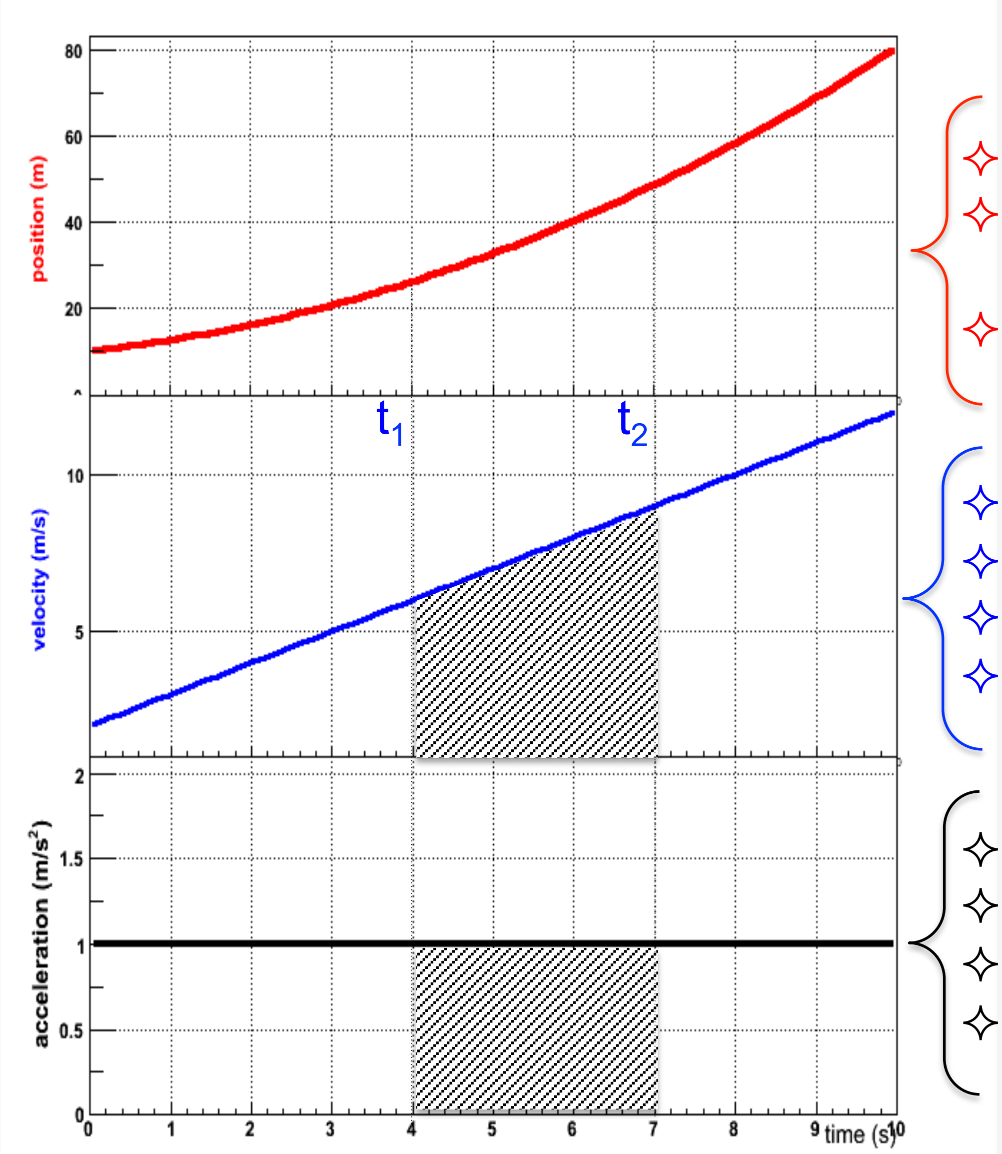
The particle starts at the origin at t = 0 and is moves along the X-axis.

What is the **average acceleration** of the particle between **t = 1 s and t = 5 s** ?

$$\left|\vec{a}_{avg}\right| = \Delta v/\Delta t = \frac{v(5)-v(1)}{t(5)-t(1)} = (2-2)/(5-1) = 0$$

Ans: 0 m/s^2

Graphical Solution



Motion with constant acceleration

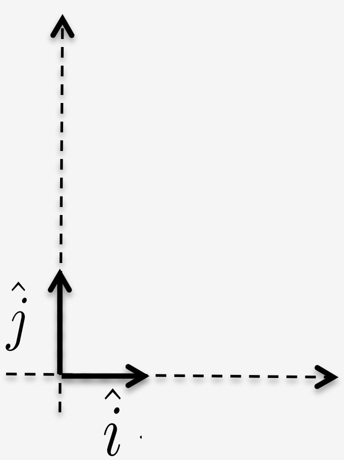
- ✧ graph of x(t)
- ✧ slope of the curve at a time t is the instantaneous velocity at t
- ✧ slope changes with time

- ✧ graph of v(t)
- ✧ slope of the curve is acceleration
- ✧ slope is constant = acceleration
- ✧ area under the curve gives $x(t_2) - x(t_1)$

- ✧ graph of a(t)
- ✧ slope is zero
- ✧ y-value at any time t is = acceleration
- ✧ area under the curve gives $v(t_2) - v(t_1)$

Projectile Motion

- Motion with constant acceleration
  - 2D motion, $a = g = 9.8\ m/s^2$ along y-axis and $a = 0$ along x-axis
  - Assumptions: We ignore air resistance
  - Initial velocity (v_0), angle of projection (θ) are given
  - To describe the motion, we choose the X, Y axes intersecting at the origin as our reference frame and write down the components of vectors
  - At any instant t ($t > 0$) , let the position vector be r. Position vector at $t = 0$ is 0



$$\vec{r} = x\,\hat{i} + y\,\hat{j}\,, \quad \vec{r}_0 = 0$$

$$\vec{v}_0 = v_0 \cos\theta\,\hat{i} + v_0 \sin\theta\,\hat{j}$$

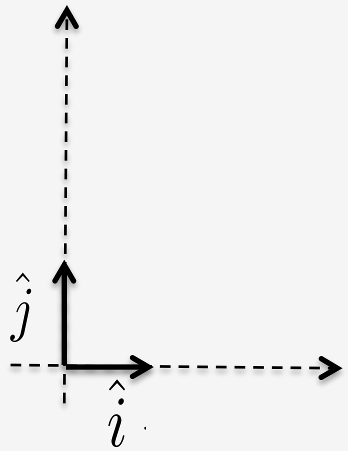
$$\vec{a} = -g\,\hat{j}$$

Projectile Motion

- Equations of motion at any instant

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2}\vec{a}t^2$$

$$\Rightarrow x\,\hat{i} + y\,\hat{j} = v_0 t\cos\theta\,\hat{i} + v_0 t\sin\theta\,\hat{j} - \frac{1}{2}gt^2\,\hat{j}$$

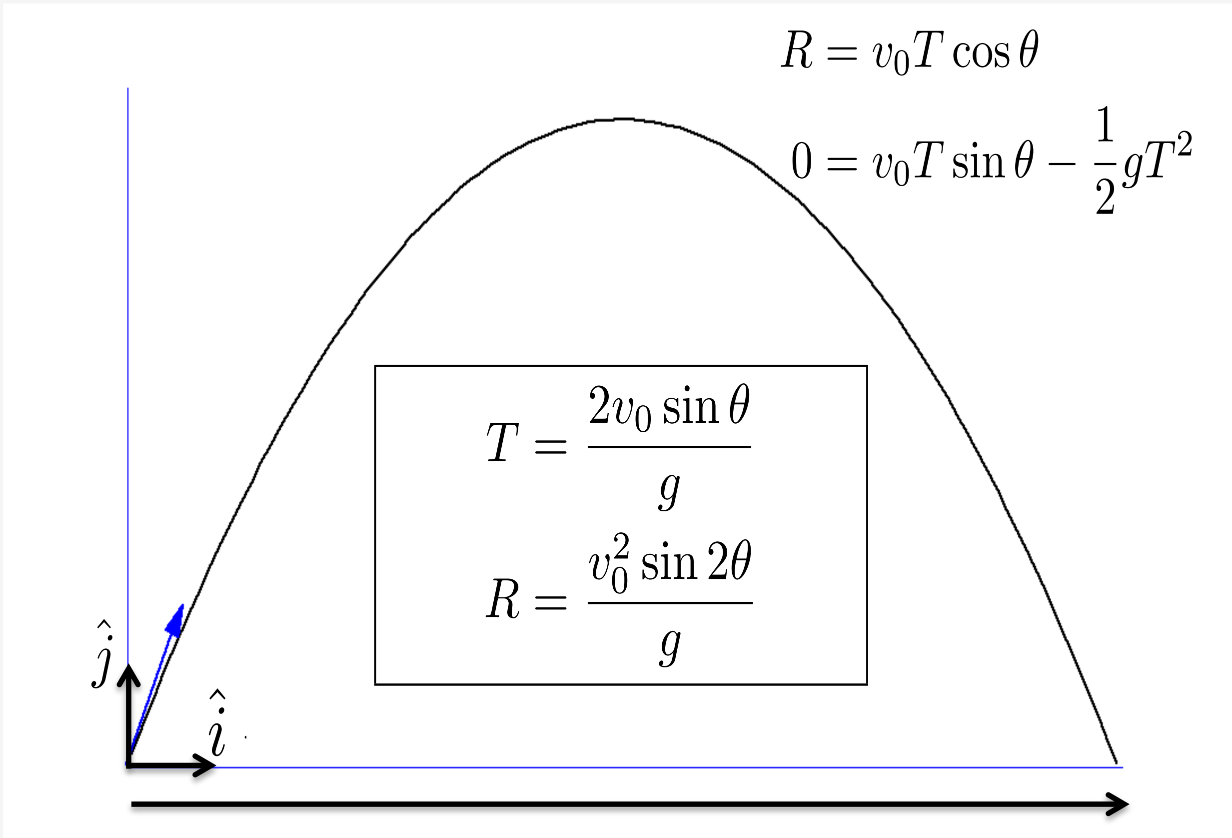


$$x = v_0 t\cos\theta$$

$$y = v_0 t\sin\theta - \frac{1}{2}gt^2$$

Projectile Motion

- Range, Time of flight
  - When t = T, x = R and y = 0, where T is the time of flight and R is the range



$$R = v_0 T \cos\theta$$

$$0 = v_0 T \sin\theta - \frac{1}{2} g T^2$$

$$T = \frac{2 v_0 \sin\theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

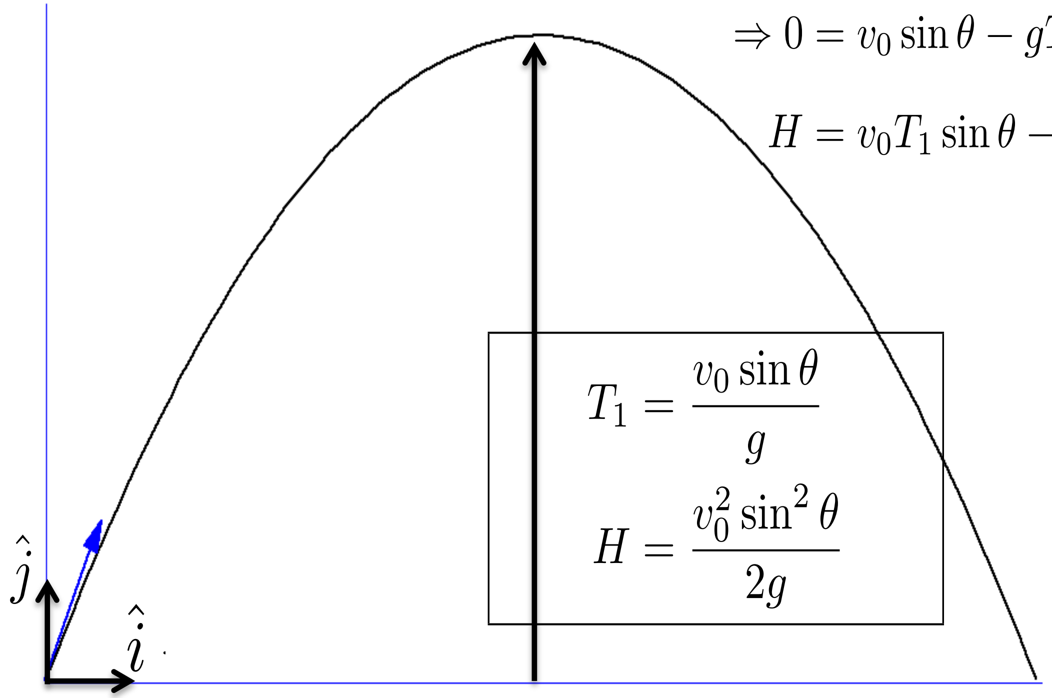
Projectile Motion

- Maximum Height, Time taken to reach it
  - When t = T1, v_y = 0, y= H where T1 is the time to reach max height and and H is the maximum height. Note that the vertical component of velocity is zero here.

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\Rightarrow 0 = v_0 \sin\theta - gT_1$$

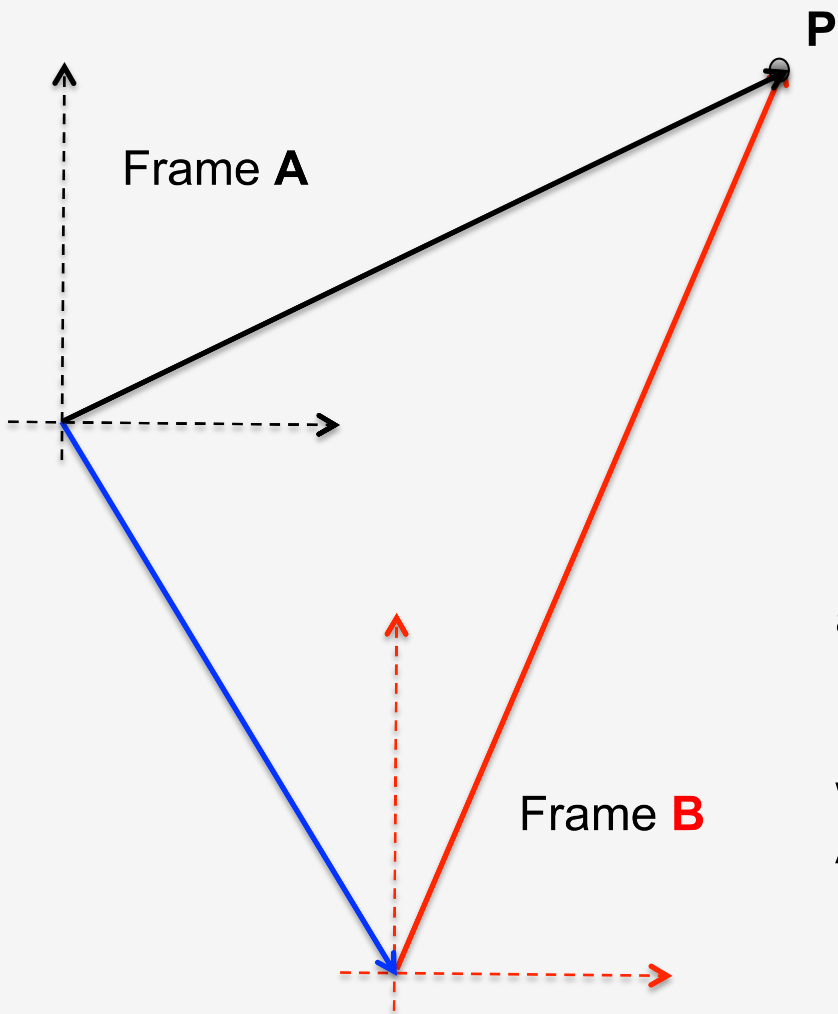
$$H = v_0 T_1 \sin\theta - \frac{1}{2} g T_1^2$$



$$T_1 = \frac{v_0 \sin\theta}{g}$$

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

Relative Motion



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\vec{a}_{PA} = \vec{a}_{PB} \; iff \;\; \vec{a}_{BA} = 0$$

If both frames measure the same acceleration at point P, they both have to move at constant velocity relative to each other. In other words, acceleration of B relative to A is zero