Linear Motion

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Clicker Question 1 (30 s)

$$\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$|\vec{A} \cdot \vec{B}| = \boxed{1}$$

Clicker Question 2 (30 s)

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{B} = -\hat{i} + 2\hat{j} - 3\hat{k}$$

$$|\vec{A} \times \vec{B}| = 0$$

Clicker Question 3 (30 s)

$$\hat{k} \cdot (\hat{i} \times \hat{j}) = 1$$

Properties

- A body that is moving is assumed rigid (ie, all points in that body move identically)
  - This rigid body is also assumed not to rotate along any axis
  - This rigid body can be replaced with a single point-particle to describe the motion
- Motion is in one-dimension and along a straight line
- We look at the motion only
  - do not care what caused it to move.
  - do not consider motion that is very fast (say close to the speed of light)
- We consider only inertial reference frame
  - The coordinate system is either at rest or moving with constant speed

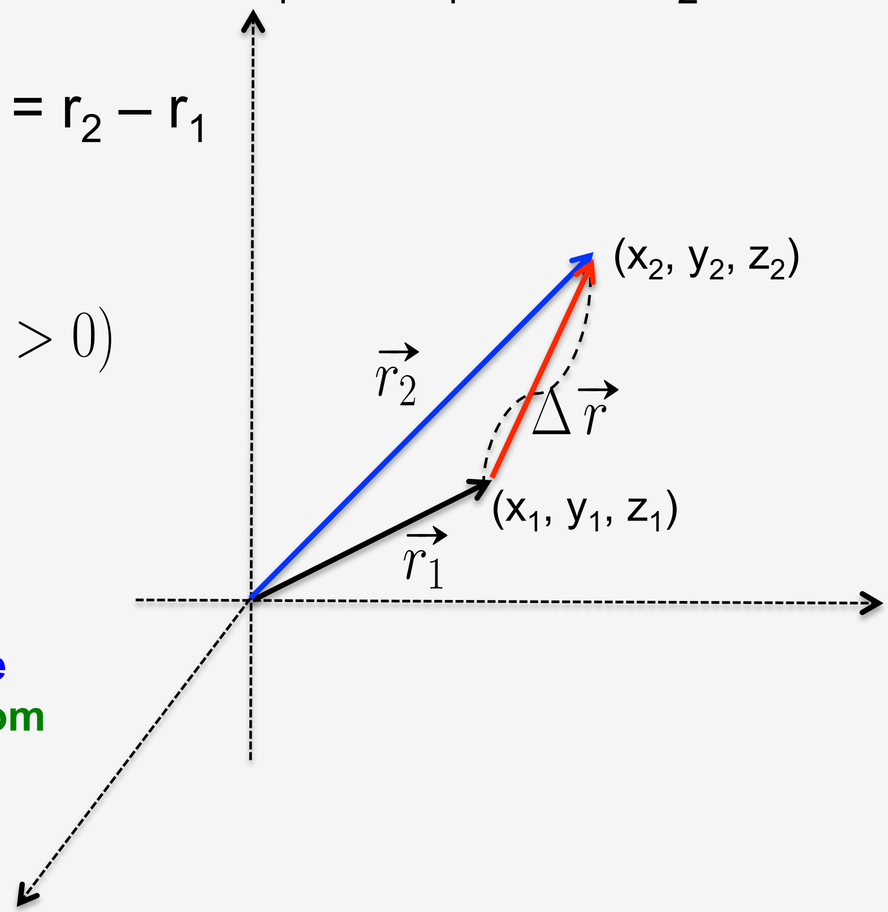
Displacement

- Displacement is the change in position of a particle
  - Say the particle's position vector was r_1 at $t=t_1$ and at r_2 at $t=t_2$
  - Displacement is given by $\Delta r = r_2 - r_1$

$$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{i} \qquad at\ t = 0$$

$$\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{i} \qquad at\ t = t_1\,(t_1 > 0)$$

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$



- Notes
  - **The actual distance traveled during the time (dotted curve) may be different from displacement (vector shown in red)**
  - **Displacement is a vector. Distance is a scalar**

Velocity

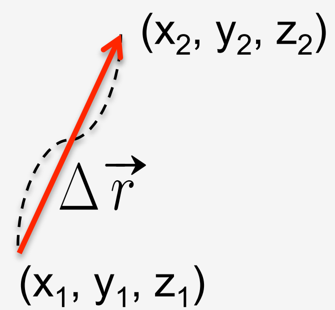
- Velocity is the rate of change of displacement
  - Say the particle's position vector was r1 at t=t0 and at r2 at t=t1
  - Displacement is given by Δr = r2 – r1
- Average vs Instantaneous velocity
  - Average velocity is given by $v_{avg} = \Delta r / \Delta t$ (SI UNIT: m/s)
  - Instantaneous velocity is given by lim (Δt → 0) $V_{avg} = dr/dt$

Notes:

Speed = (distance / time interval) it's a scalar

velocity = (displacement / time interval) and is a vector

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$



$$\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Acceleration

- Acceleration is the rate of change of velocity
  - Say the particle's velocity vector was v_1 at $t=t_1$ and at v_2 at $t=t_2$
  - Change in velocity is given by $\Delta v = v_2 - v_1$
- Average vs Instantaneous acceleration
  - Average acceleration is given by $a_{avg} = \Delta v / \Delta t$ (SI UNIT: (m/s)/s = m/s^2)
  - Instantaneous acceleration is given by lim ($\Delta t \rightarrow 0$) a_{avg} = dv/dt

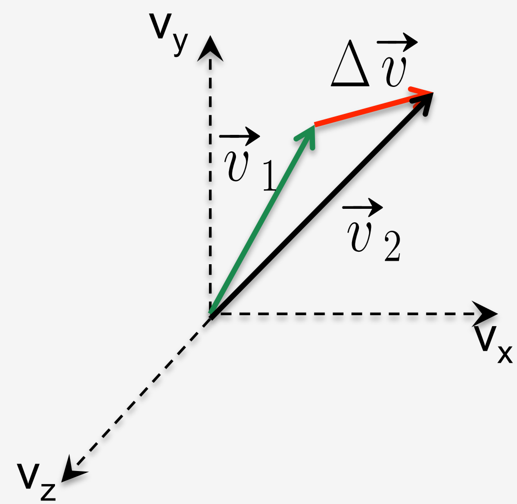
Notes:

Deceleration is negative acceleration

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}\left\{\frac{d\vec{x}}{dt}\right\} = \frac{d^2\vec{x}}{dt^2}$$



Summary

quantity	1D	2D	3D
position $\vec{r}$	$r_x\,\hat{i}$	$r_x\,\hat{i} + r_y\,\hat{j}$	$r_x\,\hat{i} + r_y\,\hat{j} + r_z\,\hat{k}$
avg. velocity $\frac{\Delta\vec{r}}{\Delta t}$	$\frac{\Delta r_x}{\Delta t}\,\hat{i}$	$\frac{\Delta r_x}{\Delta t}\,\hat{i} + \frac{\Delta r_y}{\Delta t}\,\hat{j}$	$\frac{\Delta r_x}{\Delta t}\,\hat{i} + \frac{\Delta r_y}{\Delta t}\,\hat{j} + \frac{\Delta r_z}{\Delta t}\,\hat{k}$
inst. velocity $\frac{d\vec{r}}{dt}$	$\frac{dr_x}{dt}\,\hat{i}$	$\frac{dr_x}{dt}\,\hat{i} + \frac{dr_y}{dt}\,\hat{j}$	$\frac{dr_x}{dt}\,\hat{i} + \frac{dr_y}{dt}\,\hat{j} + \frac{dr_z}{dt}\,\hat{k}$
avg. acceleration $\frac{\Delta\vec{v}}{\Delta t}$ inst. acceleration $\frac{d\vec{v}}{dt}$ inst. acceleration $\frac{d^2\vec{r}}{dt^2}$	$\frac{\Delta v_x}{\Delta t}\,\hat{i}$ $\frac{dv_x}{dt}\,\hat{i}$ $\frac{d^2 r_x}{dt^2}\,\hat{i}$	$\frac{\Delta v_x}{\Delta t}\,\hat{i} + \frac{\Delta v_y}{\Delta t}\,\hat{j}$ $\frac{dv_x}{dt}\,\hat{i} + \frac{dv_y}{dt}\,\hat{j}$ $\frac{d^2 r_x}{dt^2}\,\hat{i} + \frac{d^2 r_y}{dt^2}\,\hat{j}$	$\frac{\Delta v_x}{\Delta t}\,\hat{i} + \frac{\Delta v_y}{\Delta t}\,\hat{j} + \frac{\Delta v_z}{\Delta t}\,\hat{k}$ $\frac{dv_x}{dt}\,\hat{i} + \frac{dv_y}{dt}\,\hat{j} + \frac{dv_z}{dt}\,\hat{k}$ $\frac{d^2 r_x}{dt^2}\,\hat{i} + \frac{d^2 r_y}{dt^2}\,\hat{j} + \frac{dr_z}{dt^2}\,\hat{k}$