Vectors

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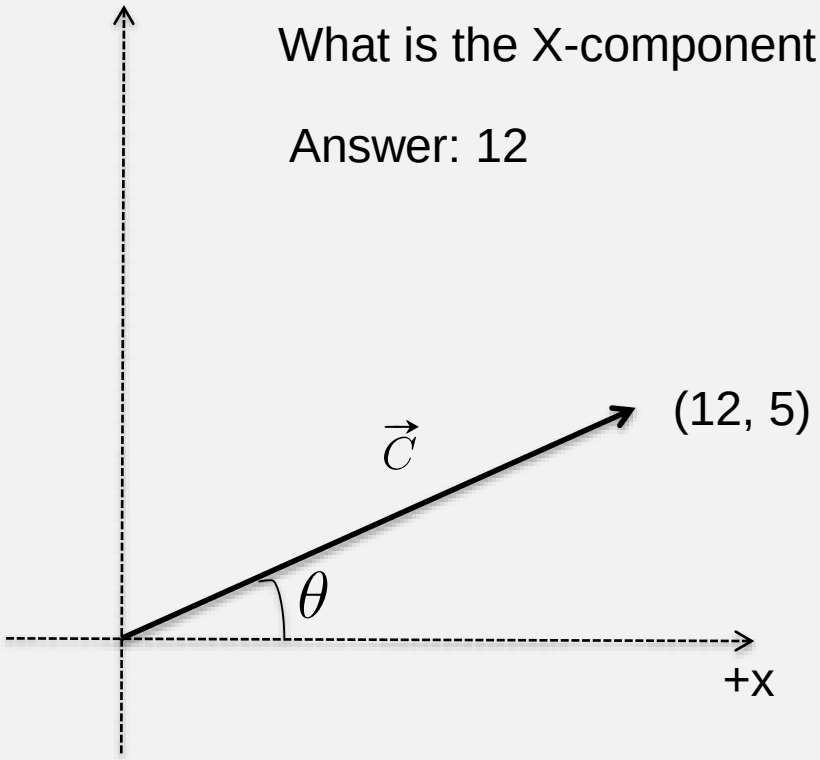
Phys 211, Lecture 3, Aug 27, 2015

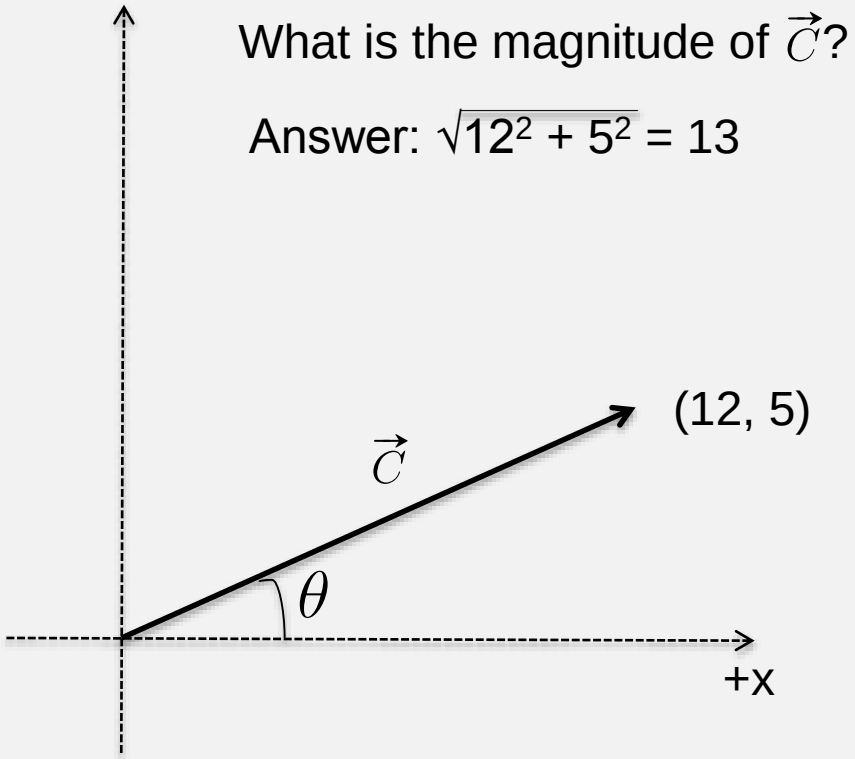
Clicker Question 1 (30 s)



What is the X-component of $\vec{C}$

Answer: 12

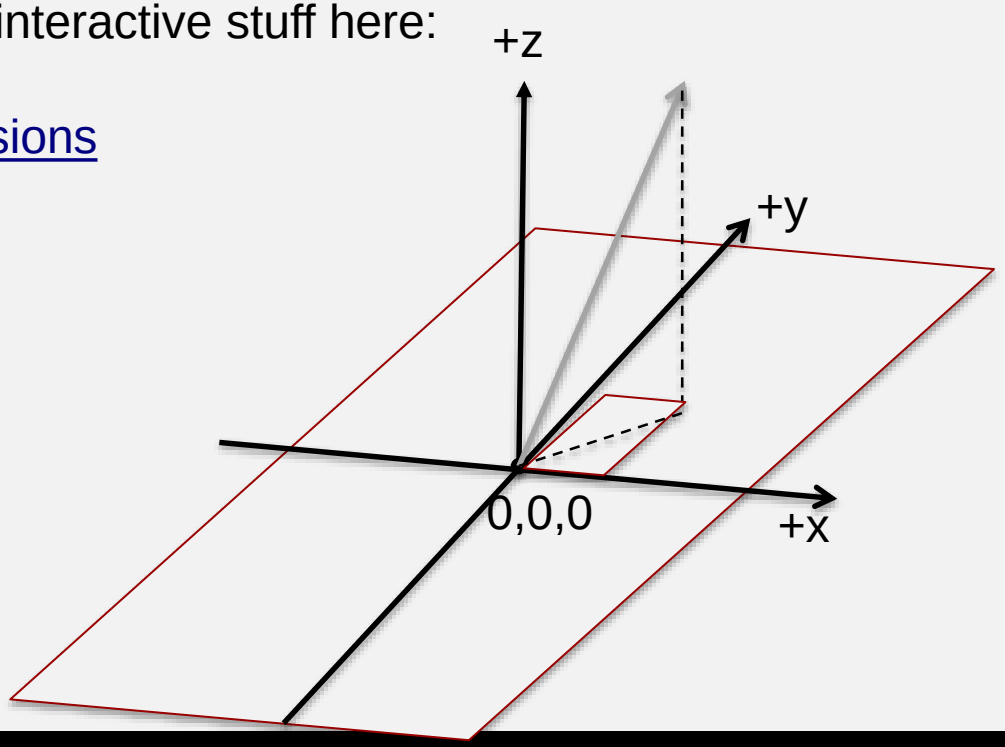
Clicker Question 2 (30 s)



Cartesian Coordinate System

- Used to uniquely determine the position of a point in a regular 3D space (Euclidian space)
- Also called the right-handed system

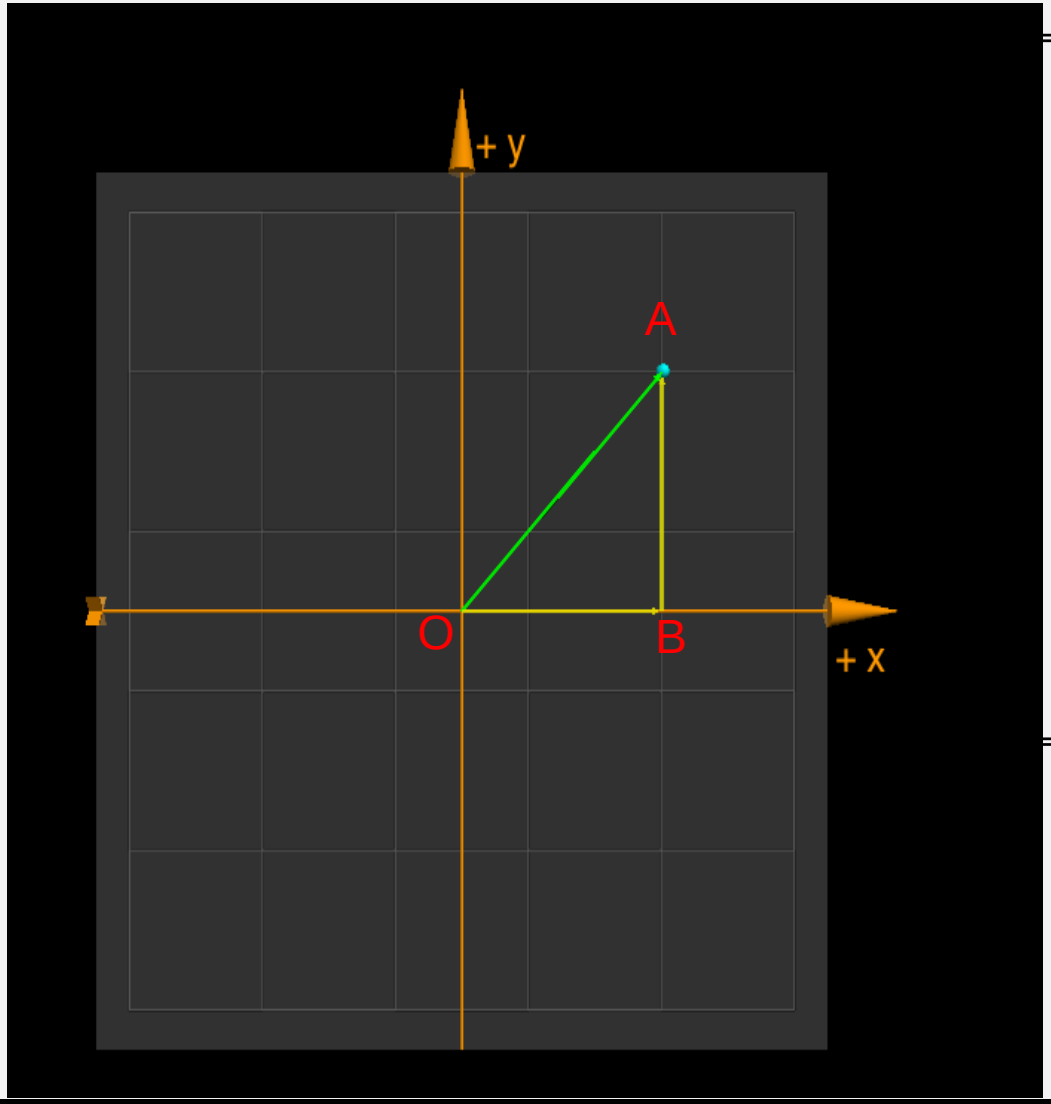
Check out some cool 3D interactive stuff here:
Cartesian Coordinates
Vectors in 2 and 3 dimensions



Definitions

- Some physical quantities cannot be completely described by a single number (and its unit)
  - Examples: position, velocity, acceleration etc.
  - These physical quantities are called vectors
- Others can be completely described by a single number (and its unit)
  - Examples: Speed, Density, Temperature etc.
  - Such physical quantities are called scalars

Components



$\overrightarrow{OB}$ is the x-component of $\overrightarrow{OA}$ and $\overrightarrow{BA}$ is the y-component of $\overrightarrow{OA}$

$$\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}$$

$$|\overrightarrow{OB}| = 6.0\,,\ |\overrightarrow{BA}| = 6.0$$

$$|\overrightarrow{OA}|^2 = |\overrightarrow{OB}|^2 + |\overrightarrow{BA}|^2$$

$$\Rightarrow |\overrightarrow{OA}| = \sqrt{6^2 + 6^2} \approx 8.5$$

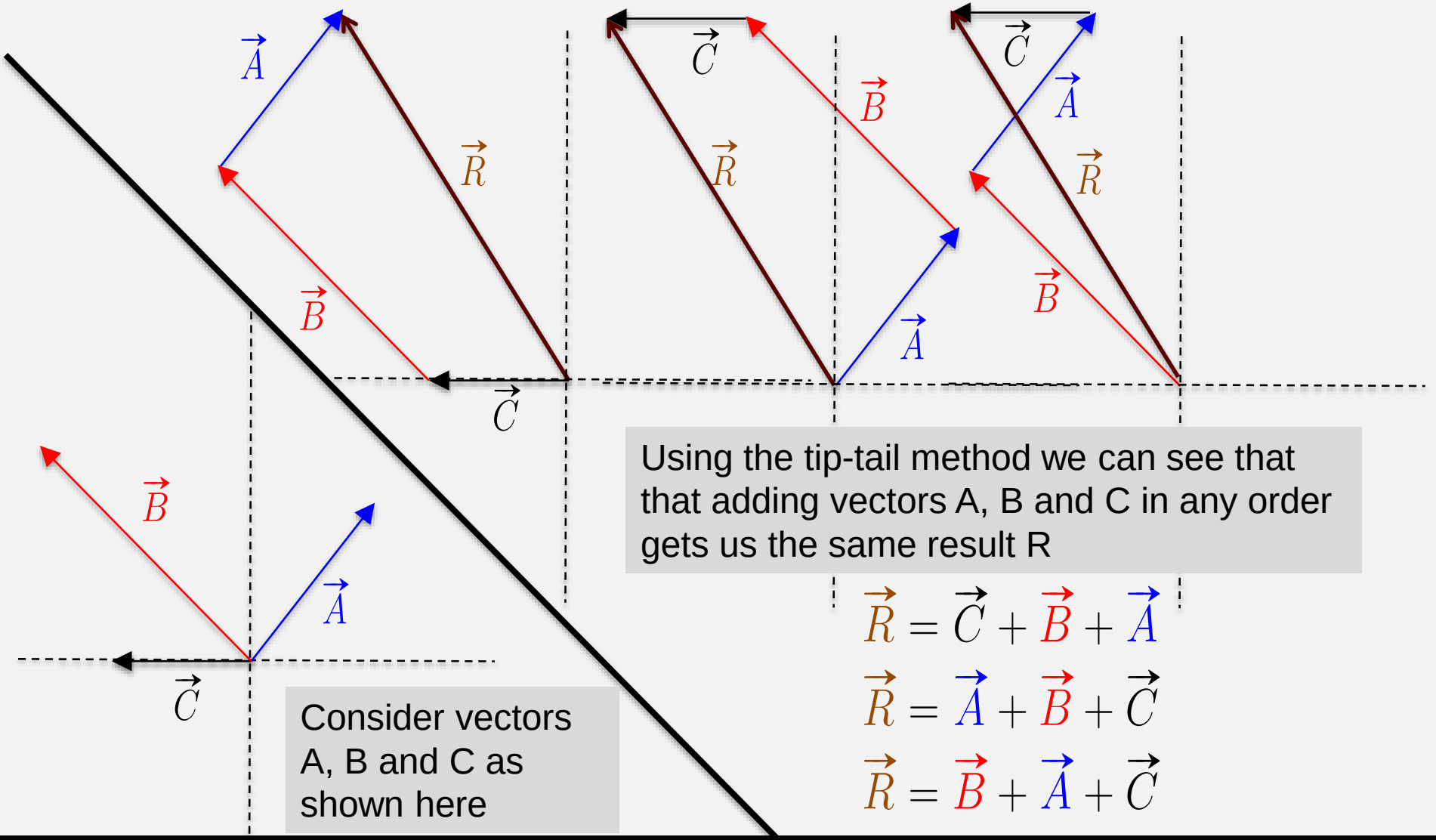
$$\tan\theta = |\overrightarrow{BA}|/|\overrightarrow{OB}| = 6.0/6.0 = 1$$

$$\Rightarrow \theta = 45^\circ$$

$$\overrightarrow{OA} + \overrightarrow{AO} = 0$$

$$\Rightarrow \overrightarrow{OA} = -\overrightarrow{AO}$$

Vector Addition



Using the tip-tail method we can see that that adding vectors A, B and C in any order gets us the same result R

$$\vec{R} = \vec{C} + \vec{B} + \vec{A}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

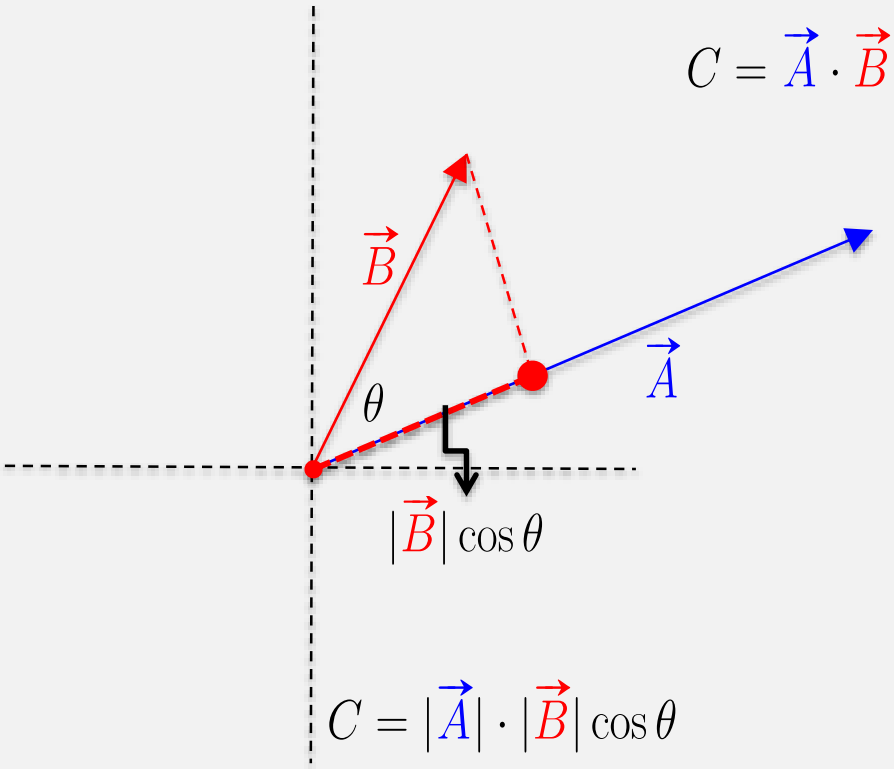
$$\vec{R} = \vec{B} + \vec{A} + \vec{C}$$

Consider vectors A, B and C as shown here

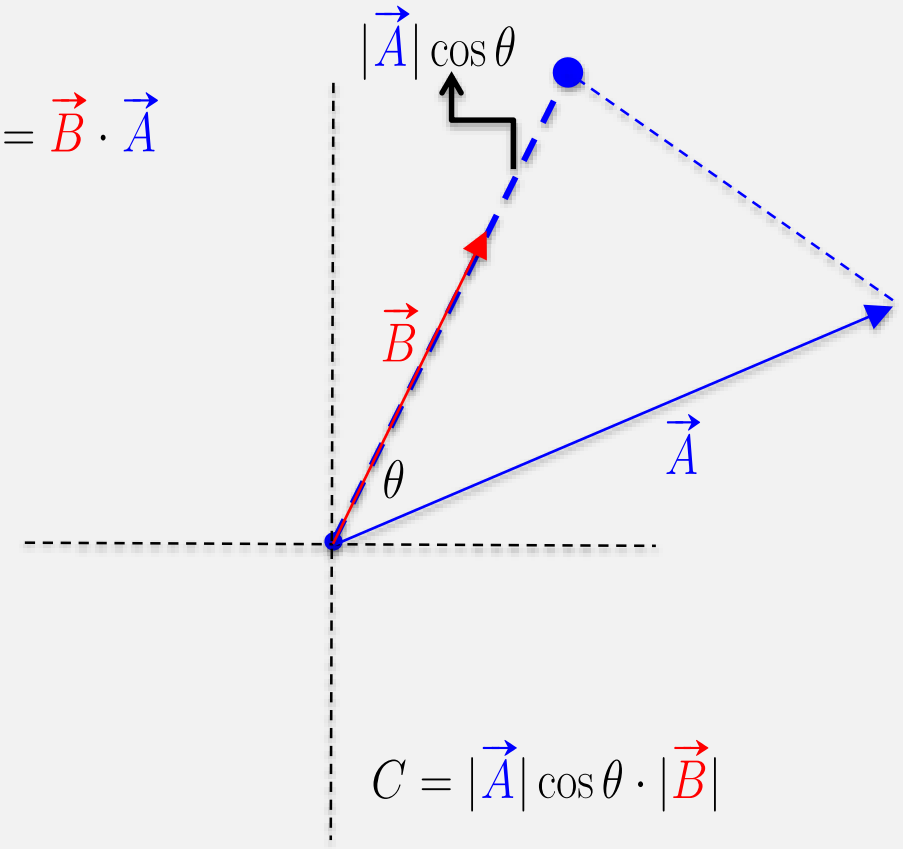
Multiplication: Dot Product

Note: The result of a dot product of two vectors is a scalar. It's value is the product of the magnitude of two vectors and the cosine of the angle between them.

$$C = \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$



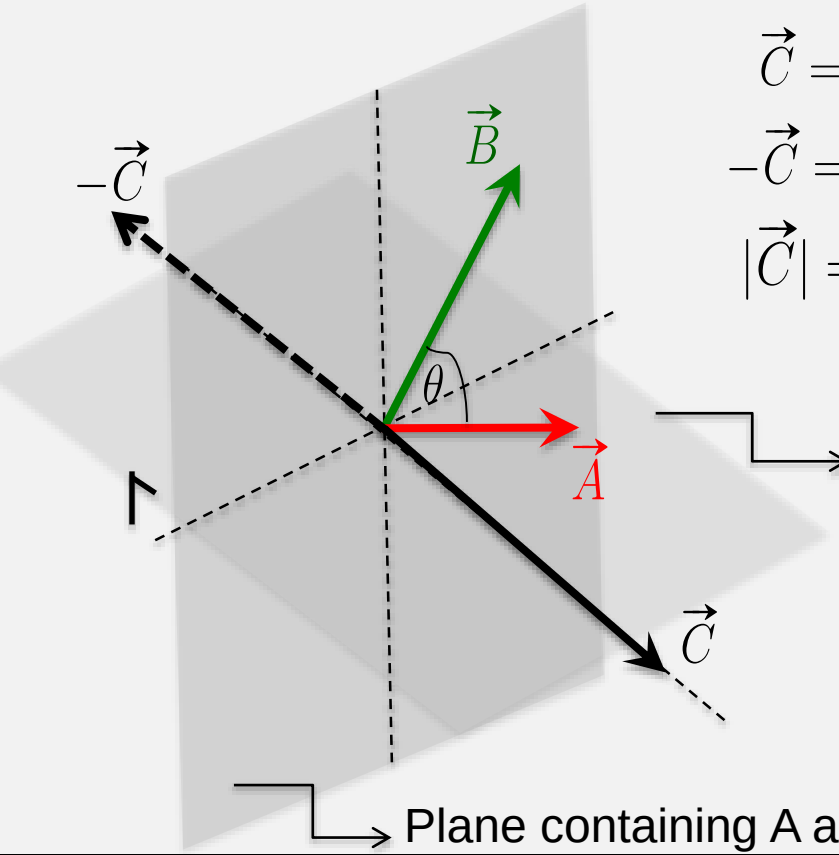
$$C = |\vec{A}| \cdot |\vec{B}| \cos\theta$$



$$C = |\vec{A}| \cos\theta \cdot |\vec{B}|$$

Multiplication: Cross Product

The result of a cross product of two vectors is a vector. It's value is the product of the magnitude of two vectors and the sine of the angle between them. Its direction is in a plane perpendicular to the plane of the vectors and is given by the right hand rule.



$$\vec{C} = \vec{A} \times \vec{B}$$

$$-\vec{C} = \vec{B} \times \vec{A}$$

$$|\vec{C}| = |\vec{A}| \cdot |\vec{B}| \sin\theta$$

$$\vec{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$