

Careful calculation of

$$\mu_{JT} = \frac{T \left(\frac{\partial V}{\partial T} \right)_P - V}{C_p}, \text{ when } \left[P + a \left(\frac{n}{V} \right)^2 \right] [V - nb] = nRT.$$

$$PV - Pnb + \frac{an^2}{V} - anb \left(\frac{n}{V} \right)^2 = nRT$$

Differentiate w.r.t. T at constant pressure:

$$P \left(\frac{\partial V}{\partial T} \right)_P - \frac{an^2}{V^2} \left(\frac{\partial V}{\partial T} \right)_P + \frac{2anb n^2}{V^3} \left(\frac{\partial V}{\partial T} \right)_P = nR.$$

$$\text{So, } \left(\frac{\partial V}{\partial T} \right)_P = \frac{nR}{P - \frac{an^2}{V^2} + \frac{2anb n^2}{V^3}}$$

So,

$$\begin{aligned} T \left(\frac{\partial V}{\partial T} \right)_P &= \frac{nRT}{P - \frac{an^2}{V^2} + \frac{2anb n^2}{V^3}} \\ &= \frac{(P + a \left(\frac{n}{V} \right)^2)(V - nb)}{P - \frac{an^2}{V^2} + \frac{2anb n^2}{V^3}} \\ &\approx \frac{PV \left(1 + \frac{a}{P} \left(\frac{n}{V} \right)^2 - \frac{nb}{V} \right)}{P - \frac{an^2}{V^2} + \frac{2anb n^2}{V^3}} = \frac{V + \frac{a}{P} \frac{n^2}{V} - nb}{1 - \frac{a n^2}{P V^2} + \frac{2anb n^2}{V^3}} \end{aligned}$$

$$\begin{aligned} \text{Thus, } T \left(\frac{\partial V}{\partial T} \right)_P - V &\approx \left(V + \frac{a \cdot n}{RT} - nb \right) \left(1 + \frac{an^2}{P V^2} \right) - V \\ &\approx V \left(\frac{na}{RTV} - \frac{nb}{V} + \frac{an^2}{P V^2} \right) = V \left(\frac{2na}{RTV} - \frac{nb}{V} \right) \\ &= n \left(\frac{2a}{RT} - b \right) \end{aligned}$$

$$\text{Thus, } \mu_{JT} = \frac{\frac{2a}{RT} - b}{C_p}.$$

The inversion temperature $T = \frac{2a}{Rb}$ can be used, along with $b = 4 \cdot \frac{4\pi R^3}{3}$ to obtain a .