7.28)

(a) $g(n)dn = \frac{1}{4} \cdot 2\pi n\, dn = \frac{\pi}{2} n\, dn$.

Since $E = \frac{n^2 h^2}{2mL^2}$, $dE = \frac{h^2}{mL^2} \cdot n\, dn$.

Thus, using $g(E)dE = g(n)\,dn$ we get

$$g(E) = \frac{m\pi L^2}{2h^2}.$$

The total number of particles, N, $= \int_0^{E_F} g(E)\,dE = \frac{m\pi L^2 E_F}{2h^2}$

and $E_F = \frac{2h^2}{m\pi}\left(\frac{N}{A}\right)$ where the area $A = L^2$.

$$\langle E \rangle = \frac{\int_0^{E_F} E\, g(E)\, dE}{\int_0^{E_F} g(E)\, dE} = \frac{E_F^2/2}{E_F} = \frac{E_F}{2}.$$

(b) We showed above that $g(E) = \frac{m\pi L^2}{2h^2}$, a constant.

Let $g_0 = m\pi L^2/2h^2$.

(c) Knowing that $\mu \equiv \left(\frac{\partial U}{\partial N}\right)_{S,V}$ we find U first (and N) as a function of T.

$$N = \int_0^\infty g_0\, \bar{n}(E)\, dE = g_0 \int_0^\infty \frac{dE}{e^{(E-\mu)\beta} + 1}.$$

Defining $x \equiv \frac{(E-\mu)}{k_B T} = \beta(E-\mu)$ we get

$$N = g_0 \int_{-\beta\mu}^{\infty} \frac{dx/\beta}{(e^x + 1)} = \frac{g_0}{\beta}\int_{-\beta\mu}^{\infty} \frac{dx}{e^x + 1} = \frac{g_0}{\beta}\int_{-\beta\mu}^{\infty} \frac{e^{-x}dx}{1 + e^{-x}}.$$

Let $y = e^{-x}$, then $dy = -e^{-x}dx$ and

$$N = \frac{g_0}{\beta}\int_0^{e^{\beta\mu}} \frac{dy}{1+y} = \frac{g_0}{\beta} \ln(1 + e^{\beta\mu}).$$

Thus, $1 + e^{\beta\mu} = e^{N\beta/g_0}$

$$\mu = \frac{\ln\left(e^{N\beta/g_0} - 1\right)}{\beta} = k_B T \ln\left(e^{N\beta/g_0} - 1\right).$$

Since we showed above that $N = g_0 E_F$, we get

$$\mu = k_B T \ln\left(e^{\beta E_F} - 1\right)$$

Thus, at high temperatures $(\beta \to 0)$ $\mu \approx k_B T \ln\left(\frac{E_F}{kT}\right)$.

For low temperatures $\mu \approx E_F$ (as expected).

[Note: at high T $\mu \approx -k_B T \ln(k_B T) + k_B T \ln(E_F)$ and so, like an ideal gas, decreases with increasing T and is negative.]

(d), (e) See above.