

6.42)

$$(a) F = N(-kT \ln Z) = +NkT \ln(1 - e^{-\beta \epsilon})$$

$$(b) S = -\partial_T F = -\partial_\beta F \left( \frac{d\beta}{dT} \right) = \frac{1}{kT^2} \partial_\beta F$$

$$= \frac{N}{kT^2} \partial_\beta \left( \frac{\ln(1 - e^{-\beta \epsilon})}{\beta} \right) = \frac{N}{kT^2} \left\{ \frac{-1}{\beta^2} \ln(1 - e^{-\beta \epsilon}) + \frac{\epsilon e^{-\beta \epsilon}}{\beta(1 - e^{-\beta \epsilon})} \right\}$$

$$= Nk \left\{ \frac{\beta \epsilon e^{-\beta \epsilon}}{(1 - e^{-\beta \epsilon})} - \ln(1 - e^{-\beta \epsilon}) \right\}$$