2.38) The numbers of molecules of each type N_A and N_B remain the same, but each molecule can now be in one of its original possible locations or in one of the other types' locations.

Thus Ω goes from being $(N_A!)(N_B!)$ to $(N_A + N_B)!$ The multiplicity goes up by a factor

$$\frac{(N_A + N_B)!}{N_A!\, N_B!} = \binom{N}{N_A}$$

$$\approx \frac{(N_A + N_B)^{(N_A + N_B)}}{N_A^{N_A}\, N_B^{N_B}}$$

Since $\frac{N_A}{N_A + N_B} = 1 - x$ and $\frac{N_B}{N_A + N_B} = x$

we can write the increase factor (Ω_{factor}) as

$$\frac{(N_A + N_B)^{N_A}}{N_A^{N_A}} \cdot \frac{(N_A + N_B)^{N_B}}{N_B^{N_B}} = \frac{1}{(1-x)^{N_A}} \cdot \frac{1}{x^{N_B}}.$$

Thus,

$$\Delta S = \Delta(k \ln \Omega) = k \ln \Omega_{factor}$$

$$= -k\left\{ N_A \ln(1-x) + N_B \ln x \right\}$$

$$= -Nk\left\{ (1-x)\ln(1-x) + x \ln x \right\}$$