2.37 The "entropy of mixing" is the increase in entropy of the two gases when mixed, i.e.,

$$\Delta S_{mixing} = S_{A+B} - S_A - S_B$$

Using the Sackur-Tetrode equation (2.49) we get, (writing $U/N = \frac{3}{2}kT$ for each gas before and after mix.)

$$\Delta S_{mixing} = N(1-x)\,k\left[\ln\left(\frac{V(2m\pi kT/h^2)^{3/2}}{N(1-x)}\right) + \frac{5}{2}\right]$$

$$+ Nx\,k\left[\ln\left(\frac{V(2m\pi kT/h^2)^{3/2}}{Nx}\right) + \frac{5}{2}\right]$$

$$- N(1-x)\,k\left[\ln\left(\frac{V(1-x)}{N(1-x)}\left(\frac{2m\pi kT}{h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$- Nx\,k\left[\ln\left(\frac{Vx}{Nx}\left(\frac{2m\pi kT}{h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= -Nk\,(1-x)\ln(1-x) - Nk\,x\ln x$$

$$= -Nk\left[x\ln x + (1-x)\ln(1-x)\right]$$

Note that we used the fact that the energy per molecule (U/N) is the same after mixing, but the volumes $V(1-x)$ and $V(x)$ were smaller before mixing.

Check: For $x = 1/2$ $\quad \Delta S_{mix} = -Nk\left[\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2}\ln\frac{1}{2}\right]$

$$= -Nk\ln\frac{1}{2} = Nk\ln 2.$$