

2.17)

If $q \ll N$,

$$\begin{aligned}\Omega(N, q) &= \binom{N+q-1}{q} \approx \binom{N+q}{q} = \binom{N+q}{N} \\ &= \frac{(N+q)^{(N+q)}}{q^q N^N}\end{aligned}$$

$$\ln \Omega = (N+q) \ln(N+q) - q \ln(q) - N \ln(N)$$

Now we approximate $\ln(N+q)$ by

$$\ln(N+q) \approx \ln N \left(1 + \frac{q}{N}\right) \cong \ln N + \frac{q}{N}$$

$$\text{Thus, } \ln \Omega \approx (N+q) \left(\ln N + \frac{q}{N}\right) - q \ln(q) - N \ln(N)$$

$$\cong q \ln N - q \ln q + q$$

where we neglected $\frac{q^2}{N} = q \left(\frac{q}{N}\right)$ relative to q and the even larger $q \ln(N/q)$ in the last step.

$$\text{Thus, } \Omega(N, q) \approx \exp\left\{q \ln\left(\frac{N}{q}\right) + q \ln e\right\} = \left(\frac{Ne}{q}\right)^q$$