CONTRIBUTED PAPER

Circuit Elements With Memory: Memristors, Memcapacitors, and Meminductors

Nanoscale devices, that store information without need for a power source, can be used for non-volatile memory, and promise to allow simulation of learning, adaptation and spontaneous behavior.

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ABSTRACT | We extend the notion of memristive systems to capacitive and inductive elements, namely, capacitors and inductors whose properties depend on the state and history of the system. All these elements typically show pinched hysteretic loops in the two constitutive variables that define them: current-voltage for the *memristor,* charge-voltage for the *memcapacitor,* and current-flux for the *meminductor*. We argue that these devices are common at the nanoscale, where the dynamical properties of electrons and ions are likely to depend on the history of the system, at least within certain time scales. These elements and their combination in circuits open up new functionalities in electronics and are likely to find applications in neuromorphic devices to simulate learning, adaptive, and spontaneous behavior.

KEYWORDS | Capacitance; dynamic response; hysteresis; inductance; memory; resistance

I. INTRODUCTION

Circuit elements that store information without the need of a power source would represent a paradigm change in electronics, allowing for low-power computation and storage. In addition, if that information spans a continuous range of values, analog computation may replace the present digital one. Such a concept is also likely to be at the origin of the workings of the human brain and possibly of many other mechanisms in living organisms so that such circuit elements may help us understand adaptive and spontaneous behavior, or even learning.

One such circuit element is the memory-resistor (memristor for short), which was postulated by Chua in 1971 by analyzing mathematical relations between pairs of fundamental circuit variables [1]. The memristor is characterized by a relation between the charge and the flux, defined mathematically as the time integral of the voltage, which need not have a magnetic flux interpretation. This relation can be generalized to include any class of two-terminal devices (which are called memristive systems) whose resistance depends on the internal state of the system [2].

Many systems belong to this class, including the thermistor [3] (whose internal state depends on the temperature), molecules whose resistance changes according to their atomic configuration [4], or spintronic devices whose resistance varies according to their spin polarization [5], [6], [23]. Recently, memristive behavior and memory storage have been reported in solid-state TiO_2 thin films [7], [8], where the change in resistance is realized by the ionic motion of oxygen vacancies activated by current flow. Additionally, memristive behavior has been demonstrated in VO_2 thin films, where the memory mechanism is related to the insulator-to-metal transition in these structures [9]. Lastly, memristive behavior has been identified by Di Ventra and Pershin as a possible mechanism in the adaptive behavior of unicellular organisms such as amoebas [10].

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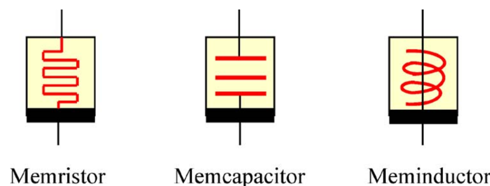

Fig. 1. *Symbols of the three different devices defined in this work: memristor, memcapacitor, and meminductor. In general, these devices are asymmetric, and we use the following convention when possible: when a positive voltage is applied to the second (upper) terminal with respect to the terminal denoted by the black thick line, the memory device goes into a state of high resistance, capacitance, or inductance, respectively.*

All these examples show the ubiquitous nature of memristive systems. In fact, it should not come as a surprise that many of the above examples refer to nanoscale systems, whose resistance is likely to depend on their state and dynamical history, at least within (possibly very short) time scales dictated by the fundamental state variables that control their operation [5], [11].

In this paper, we show that the above concept of memory device is not necessarily limited to resistances but can in fact be generalized to capacitative and inductive systems. Quite generally, if x denotes a set of n state variables describing the internal state of the system, $u(t)$ and $y(t)$ are any two complementary constitutive variables [12] (i.e., current, charge, voltage, or flux) denoting input and output of the system, and g is a generalized response, we can define a general class of nth-order u-controlled memory devices as those described by the following relations:

$$y(t) = g(x, u, t)u(t) \quad (1)$$
$$\dot{x} = f(x, u, t) \quad (2)$$

where f is a continuous n-dimensional vector function and we assume on physical grounds that, given an initial state $u(t = t_0)$ at time t_0, (2) admits a unique solution.[1]

Memcapacitive and *meminductive* systems are special cases of (1) and (2), where the two constitutive variables that define them are charge and voltage for the memcapacitance and current and flux for the meminductance (see Fig. 1 for the symbols we use for these devices). In this paper, we will discuss several properties of these systems that make them unique compared to their "standard" definitions and will provide a few examples. Combined with the general class of memristive systems and/or standard circuit elements, these devices may lead to new functionalities in electronics.

[1]Hypothetical choice of some nongeneric nonlinear function $f(x, u, t)$ in (2) could give rise to nonunique solutions (see, e.g., [13, p. 1063]).

II. MEMRISTIVE SYSTEMS

For completeness, let us introduce first the notion of memory-resistive systems [2]. We also provide a simple analytical example of memristive behavior that has been employed in describing "learning circuits" [10]. For a broad range of memristive system models, see, e.g., [14].

Definition: From (1) and (2), an nth-order current-controlled memristive system is described by

$$V_M(t) = R(x, I, t)I(t) \quad (3)$$
$$\dot{x} = f(x, I, t) \quad (4)$$

where x is a vector representing n internal *state variables*, $V_M(t)$ and $I(t)$ denote the voltage and current across the device, and R is a scalar, called the *memristance* (for memory resistance) with the physical units of ohm. The equation for a charge-controlled memristor is a particular case of (3) and (4) when R depends only on charge, namely

$$V_M = R(q(t))I \quad (5)$$

with the charge related to the current via time derivative $I = dq/dt$. Note that the analytical equation describing the TiO_2 device derived in the work by the Hewlett-Packard group [7] has precisely this form, and it therefore represents an *ideal* memristor.

We can also define an nth-order *voltage*-controlled memristive system from the relations

$$I(t) = G(x, V_M, t)V_M(t) \quad (6)$$
$$\dot{x} = f(x, V_M, t) \quad (7)$$

where we call G the *memductance* (for memory conductance).

Properties: Several properties of memristive systems were identified by Chua and Kang in their seminal paper [2]. Here, we highlight just a few of them. Considering devices with $R(x, I, t) > 0$ in (3), it has been proven that these devices are passive [2]. Another important feature, following from the above inequality, is the "no energy discharge property," which is related to the fact that a memristive system cannot store energy, like a capacitor or an inductor. As a manifestation of these characteristics, we notice from (3) [or (6)] that $V_M = 0$ whenever $I = 0$ (and vice versa). In addition, for a periodic current input, the memristive system shows a "pinched hysteretic loop" [2].

Moreover, a memristive system behaves as a linear resistor in the limit of infinite frequency and as a nonlinear resistor in the limit of zero frequency provided that $\dot{x} = f(x, I) = 0$ in (4) [respectively, $\dot{x} = f(x, V_M) = 0$ in (7)]

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has a steady-state solution. These two properties are easy to understand on physical grounds. Irrespective of the underlying physical mechanisms that define the state of the system, at very low frequencies, the system has enough time to adjust its value of resistance to a momentary value of the control parameter (either current or voltage), so that the device behaves as a nonlinear resistor [from (3) or (6)]. On the other hand, at very high frequency, there is not enough time for any kind of resistance change during a period of oscillations of the control parameter, so that the device operates as a usual (linear) resistor.

Lastly, it is worth pointing out that, when a current flows in a resistor, at the microscopic level local resistivity dipoles form due to scattering of carriers at interfaces [15]. Namely, charges accumulate on one side of the resistor with consequent depletion on the other side, with possible accumulation and depletion over the whole spatial extent of the resistor [11]. These dipoles develop *dynamically* over length scales on the order of the screening length [16], namely, within length scales over which electrons can effectively screen the electric field of a local charge. In other words, the self-consistent formation of local resistivity dipoles takes some time and is generally accompanied by some energy storage related to the capacitance of the system. We thus expect that in nanoscale systems, memristive behavior is always accompanied to some extent by a (possibly very small) capacitative behavior and as such the hysteresis loop may not exactly cross the origin at frequencies comparable to the inverse characteristic time of charge equilibration processes. This is, for instance, found in the transverse voltage response of the spin Hall effect in inhomogeneous semiconductor systems [17].

Examples: Several systems have been found to satisfy the above properties [1]–[3], [5], [7]–[9], and many more are likely to be found that fit this classification. In particular, memristive behavior is a property of thermistors [3], molecular systems [4], spintronic devices [5], and thin-film nanostructures [7]–[9], as well as numerous examples presented in [14].

As an example, let us consider a model of a voltage-controlled memristive system used in [10]. In this model, the rate of system resistance change (R changes between two limiting values R_1 and R_2) is characterized by a constant α when $|V_M| \leq V_T$ and β when $|V_M| > V_T$, where V_M is the voltage applied to the memristor and V_T is a threshold voltage.[2] Mathematically, the response of this memristive system is described by (6) and (7) with

$$G = x^{-1} \quad (8)$$

$$\dot{x} = (\beta V_M + 0.5(\alpha - \beta)[|V_M + V_T| - |V_M - V_T|]) \times \theta(x - R_1)\theta(R_2 - x) \quad (9)$$

[2]This model was inspired by an activation-type resistance state found in nanostructure memristors [8].

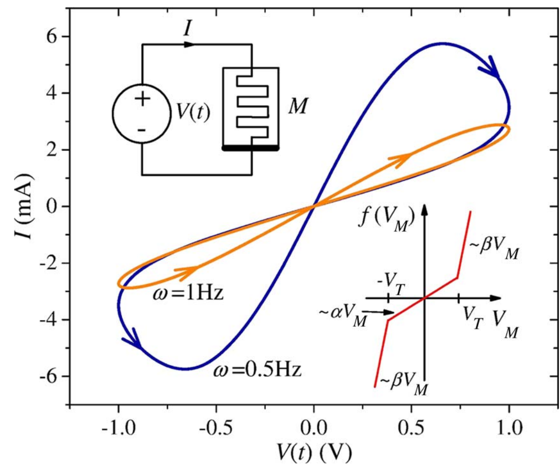

Fig. 2. ***Simulation of a voltage-controlled memristive system. This plot was obtained by a numerical solution of (6)–(9) for the case of a memristive system M connected directly to an alternating (ac) voltage source $V(t) = V_0 \sin(2\pi\omega t)$ as shown in the inset using the following parameter values: $R_1 = 20\ \Omega$, $R_2 = 500\ \Omega$, $V_T = 0.5$ V, $\alpha = 500\ \Omega/(\mathrm{V}\cdot \mathrm{s})$, $\beta = 2\alpha$, $V_0 = 1$ V. In our model, the rate of resistance change is defined by the function $f(V_M) = (\beta V_M + 0.5(\alpha - \beta)[|V_M + V_T| - |V_M - V_T|])$ entering (9) and schematically shown in the inset for positive values of α and β. At $|V_M| < V_T$, the slope of $f(V_M)$ is determined by α and at $|V_M| > V_T$ by β.***

where $\theta(\cdot)$ is the step function. The simulation results[3] depicted in Fig. 2 clearly show a pinched hysteresis loop and a hysteresis collapse with increasing frequency ω of the alternating (ac) voltage source $V(t) = V_0 \sin(2\pi\omega t)$.

We can now extend the above definitions to capacitances and inductances. The memory devices that result share many characteristics of memristive systems but with a fundamental difference: they store energy.

III. MEMCAPACITATIVE SYSTEMS

Definition: We define an nth-order voltage-controlled memcapacitive system by

$$q(t) = C(x, V_C, t)V_C(t) \quad (10)$$

$$\dot{x} = f(x, V_C, t) \quad (11)$$

where $q(t)$ is the charge on the capacitor at time t, $V_C(t)$ is the corresponding voltage, and C is the *memcapacitance* (for memory capacitance), which depends on the state of

[3]All numerical calculations presented in this paper have been performed using a fourth-order Runge–Kutta integration method.

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the system. Similarly, we can define an nth-order *charge-*controlled memcapacitive system from

$$V_C(t) = C^{-1}(x, q, t)q(t) \tag{12}$$
$$\dot{x} = f(x, q, t) \tag{13}$$

where C^{-1} is an inverse memcapacitance.

In addition, we define a subclass of the above devices we call voltage-controlled *memcapacitors* when (10) and (11) reduce to

$$q(t) = C\left[\int_{t_0}^{t} V_C(\tau)\, d\tau\right] \cdot V_C(t) \tag{14}$$

and a charge-controlled memcapacitor when (12) and (13) can be written as

$$V_C(t) = C^{-1}\left[\int_{t_0}^{t} q(\tau)\, d\tau\right] \cdot q(t) \tag{15}$$

where, in the above two equations, the lower integration limit (initial moment of time) may be selected as $-\infty$ or zero if $\int_{-\infty}^{0} V_C(\tau)d\tau = 0$ and $\int_{-\infty}^{0} q(\tau)d\tau = 0$, respectively, and the multiplication symbol $\cdot$ is used for clarity.

Properties: It follows from (10) that the charge is zero whenever the voltage is zero. Note, however, that in this case, $I = 0$ does not imply $q = 0$, and thus this device can store energy. Therefore, the latter can be both added to and removed from a memcapacitive system.

From a microscopic point of view, a change in capacitance can occur in two ways: i) due to a geometrical change of the system (e.g., a variation in its structural shape as it takes place in some MEMS capacitors) or ii) in the quantum-mechanical properties of the carriers and bound charges of the materials composing the capacitor (manifested, e.g., in a history-dependent permittivity ε), or both.

In either case, inelastic (dissipative) effects may be involved in changing the capacitance of the system upon application of the external control parameter. These dissipative processes release energy in the form of heating of the materials composing the capacitor. However, this heat may not be simply accounted for as a (time-dependent) resistance in series with the capacitor.

Similarly, there may be situations in which energy not from the control parameter but from sources that control the equation of motion for the state variable (11) is needed to vary the capacitance of the system (e.g., in the form of elastic energy or via a power source that controls, say, the permittivity of the system via a polarization field). This energy can then be released in the circuit, thus amplifying the current. Therefore, (10) and (11) for memcapacitive systems postulated above could, in principle, describe both active and passive devices.

However, starting from a fully discharged state, the amount of removed energy from a *passive* memcapacitive system, by definition, cannot exceed the amount of previously added energy. Mathematically, this property can be written in the form

$$U_C = \int_{t_0}^{t} V_C(\tau)I(\tau)\, d\tau \geq 0 \tag{16}$$

under the condition that at $t = t_0$, no energy is stored in the system. Equation (16) should be valid for any form of the control parameter, such as the voltage $V_C(t)$ applied to the passive memcapacitive system. Moreover, in memcapacitive systems with dissipative processes (such as heating), the equality sign in (16) for $t > t_0$ is not realized (assuming $V_C(t) \neq 0$).

Fig. 3 shows a schematic memcapacitive system hysteresis loop passing through the origin. The shaded areas give the energy U_i added to/removed from the system. Note that this energy is associated with some internal degree of freedom, i.e., related to some physical elastic or inelastic processes that accompany the change in capacitance. The system is passive if $U_1 + U_2 = 0$, dissipative if $U_1 + U_2 > 0$, and active if $U_1 + U_2 < 0$.

Finally, since the state (11) has (by assumption) only a unique solution at any given time $t \geq t_0$, then if $V_C(t)$ is periodic with frequency ω, namely, $V_C(t) = V_0 \cos(2\pi\omega t)$,

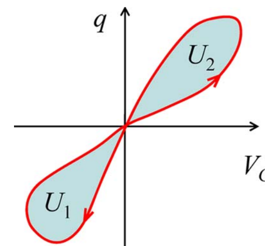

Fig. 3. ***Schematics of a pinched hysteresis loop of a memcapacitive system. The energy added to/removed from the system ($\int V_C(q)dq$) is the area between the curve and the q-axis. The areas of shaded regions U_1 and U_2 give the amount of added/removed energy in each half-period. The signs of U_1 and U_2 are determined by the direction on the loop. For the direction shown here, U_2 is positive and U_1 is negative.***

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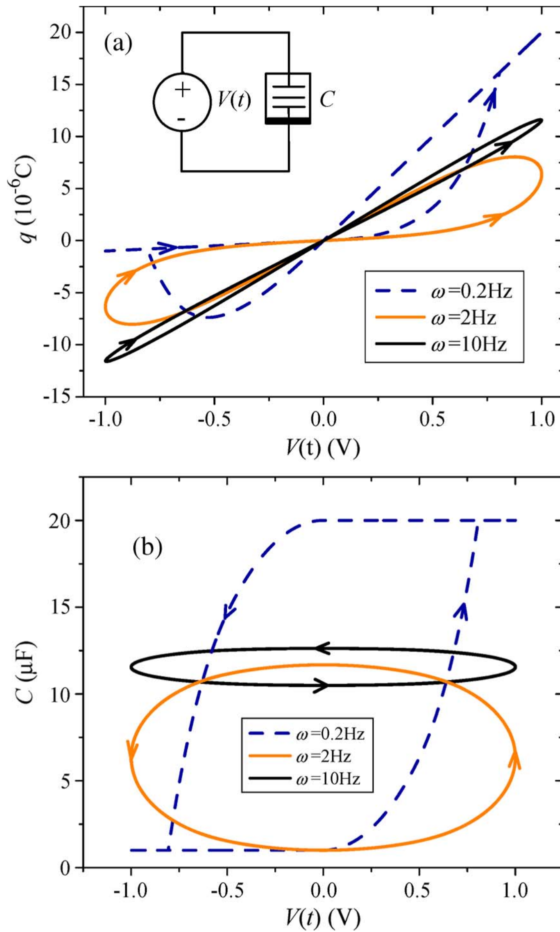

Fig. 4. ***Simulation of a circuit [shown in the inset of (a)] composed of a voltage-controlled memcapacitive system. The following parameter values were used in this calculation:*** $C_1 = 1\ \mu F$, $C_2 = 20\ \mu F$, $V_T = 0.5$ V, $\alpha = 50\ \mu F/(V \cdot s)$, $\beta = 2\alpha$, $V_0 = 1$ V. ***The curves at 10 Hz (black curves) have been generated with*** $C(t = 0) = 10$ mF; ***the other curves with*** $C(t = 0) = 1$ mF.

then the q–V_C curve is a simple loop passing through the origin, namely, there may be at most two values of the charge q for a given voltage V_C if we consider a voltage-controlled device (see Fig. 4) or two values of the voltage V_C for a given charge q for a charge-controlled system. This loop is also antisymmetric with respect to the origin if, for the case of (10) and (11), $C(x, V_C, t) = C(x, -V_C, t)$ and $f(x, V_C, t) = f(x, -V_C, t)$. Note also that there may be cases in which, at certain instants of time, both the capacitance and charge are zero so that, from (12), the voltage on the memcapacitive system is finite. In this case, the hysteretic loop does not pass through the origin.

As in the case of memristive systems, a memcapacitive system behaves as a linear capacitor in the limit of infinite frequency and as a nonlinear capacitor in the limit of zero frequency, assuming (11) and (13) admit a steady-state solution. The origin of this behavior rests again on the system's ability to adjust to a slow change in bias (for low frequencies) and the reverse: its inability to respond to extremely high-frequency oscillations.

Examples: There are instances in which the capacitance C has been found to have a hysteresis-type dependence on the applied voltage [18]–[20]. These cases are related to nanoscale capacitors in which interface traps [18] or embedded nanocrystals [19], [20] are responsible for memory effects. Even though characteristic hysteresis properties of these systems were not discussed in the framework of the memcapacitive systems suggested in this paper, it is clear that they belong to the present classification.

Below, we provide a model example of a memcapacitive system that could be used to simulate experimental realizations of memcapacitors. To be specific, let us consider a voltage-controlled memcapacitive system according to (10) and (11). We focus on the electronic circuit shown in the inset of Fig. 4. This scheme consists of a memcapacitive system C connected to a source of ac-voltage $V(t) = V_0 \sin(2\pi\omega t)$. The capacitance C is chosen to vary in the range between its minimum and maximum values (C_1 to C_2) and changes according to

$$C = x \tag{17}$$

$$\dot{x} = (\beta V_C + 0.5(\alpha - \beta)[|V_C + V_T| - |V_C - V_T|]) \times \theta(x - C_1)\theta(C_2 - x). \tag{18}$$

Here, V_C is the voltage drop on the memcapacitive system, V_T is a threshold voltage, α is a variation rate for $|V_C| < V_T$, and β is a variation rate for $|V_C| > V_T$. The interpretation of the coefficients α and β is the same as in Fig. 2. The response of the circuit is simply described by the equation $V(t) = V_C = q/C$, where C can be found using (17) and (18). It is important to remember that in modeling of more complex circuits, the chain rule of differentiation should be used.[4]

Fig. 4 represents results of our numerical simulations. Typical features discussed above such as capacitance hysteresis, low-frequency and high-frequency behavior, and pinched hysteresis loop can be clearly distinguished. We also checked the energy conservation for the given circuit and found that for the present model, the memcapacitive system behaves as a passive device. However, for certain initial conditions, when combined in a circuit with resistors and/or inductors, the model in (17) and (18) may lead to an active behavior.

[4]In the case of a memory capacitor, since the capacitance C is time-dependent, the well-known relation $dq = CdV_C$ generalizes to $dq = CdV_C + V_C dC$.

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IV. MEMINDUCTIVE SYSTEMS

Let us now introduce the third class of memory devices. Let us define the "flux"

$$\phi(t) = \int_{-\infty}^{t} V_L(t')\,dt' \tag{19}$$

where $V_L(t)$ is the induced voltage on the inductor (equal to minus the electromotive force).

Definition: We call an nth-order current-controlled meminductive system one described by the equations

$$\phi(t) = L(x, I, t)I(t) \tag{20}$$
$$\dot{x} = f(x, I, t) \tag{21}$$

where L is the *meminductance* and an nth-order *flux-controlled* meminductive system the following:

$$I(t) = L^{-1}(x, \phi, t)\phi(t) \tag{22}$$
$$\dot{x} = f(x, \phi, t) \tag{23}$$

with L^{-1} the inverse meminductance.

We call current-controlled *meminductors* a subclass of the above devices when (20) and (21) reduce to

$$\phi(t) = L\left[\int_{t_0}^{t} I(\tau)\,d\tau\right] \cdot I(t) \tag{24}$$

and a flux-controlled meminductor when (22) and (23) can be written as

$$I(t) = L^{-1}\left[\int_{t_0}^{t} \phi(\tau)\,d\tau\right] \cdot \phi(t). \tag{25}$$

In the above two equations, the lower integration limit may be selected as $-\infty$ or zero if $\int_{-\infty}^{0} I(\tau)\,d\tau = 0$ and $\int_{-\infty}^{0} \phi(\tau)\,d\tau = 0$, respectively.

Properties: For the sake of definiteness, we shall consider the current-controlled meminductive systems. Taking the time derivative of both sides of (20) yields

$$V_L = \frac{d\phi}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt}. \tag{26}$$

The second term on the right-hand side of the above equation is an additional contribution to the induced voltage due to a time-dependent L. The energy stored in the current-controlled meminductive system can be thus calculated as

$$U_L(t) = \int_{t_0}^{t} V_L(\tau)I(\tau)\,d\tau = \int_{t_0}^{t} \left[L\frac{dI}{dt} + I\frac{dL}{dt}\right] I(\tau)\,d\tau. \tag{27}$$

When L is constant, we readily obtain the well-known expression for the energy $U_L = LI^2/2$. Usually, this U_L is interpreted as the energy of the magnetic field generated by the current. Similarly to the passivity criterion of memcapacitive systems [see (16)], we can formulate, using $U_L(t)$ from (27), the passivity criterion of meminductive system: if at $t = t_0$ a meminductive system is in its minimal energy state, then $U_L(t) \geq 0$ at any subsequent moment of time.

Lastly, as in the case of memristive and memcapacitive systems, it is expected that when the external control parameter is a periodic function of time with frequency ω, namely, $I(t) = I_0 \cos(2\pi\omega t)$, the ϕ–I curve is a simple loop passing through the origin, namely, there may be at most two values of the flux ϕ for a given current I. This loop is antisymmetric with respect to the origin if, for the case of (20) and (21), $L(x, I, t) = L(x, -I, t)$ and $f(x, I, t) = f(x, -I, t)$. Note also that, as for memcapacitive systems, there may be cases in which, at certain instants of time, both the inductance and flux are zero so that, from (22), the current across the meminductive system is finite. In this case, the hysteretic loop does not pass through the origin.

Also, the magnetic permeability can adjust to slow periodic variations of the current but cannot at high frequencies, so that this device behaves as a nonlinear inductor at low frequencies [assuming (21) and (23) admit a steady-state solution] and a linear inductor at high frequencies.

Examples: In electronics, inductors are primarily of a solenoid type consisting of a coil of conducting material wrapped around a core. The inductance of a solenoid is proportional to the relative permeability μ_r of the material within the solenoid and also depends on the geometrical parameters of the system. The simplest way to introduce memory effects in such a system is to use the core material whose response to the applied magnetic field depends on its history. As an example, we can think about ferromagnetic materials' exhibiting a magnetic hysteresis such as iron. In fact, an electronic circuit with such inductor was recently analyzed [21]. In addition, memory effects in inductance (such as lagging) were discussed in the context of superconducting circuits [22]. Moreover,

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memory effects can be introduced by varying the inductor's shape.

A circuit model for a meminductive system can be formulated similarly to the models of memristive and memcapacitive systems discussed above. In particular, we can imagine a structure whose inductance increases up to a certain value when current flows in one direction and decreases down to another value if the current flow direction is reversed. This is a straightforward calculation that does not add much to the analysis of this problem. Therefore, we do not report it here.

V. CONCLUSION

We have extended the notion of memory devices to both capacitative and inductive systems. These devices have specific properties that typically appear most strikingly as a pinched hysteretic loop in the two constitutive variables that define them: current–voltage for memristive systems, charge–voltage for memcapacitive systems, and current–flux for the meminductive systems. Many systems belong to these classifications, especially those of nanoscale dimensions. Indeed, with advances in the miniaturization of devices, these concepts are likely to become more relevant since, at the nanoscale, the dynamical properties of electrons and ions may strongly depend on the history of the system, at least within certain time scales [11].

Apart from the obvious use of these devices in nonvolatile memories, several applications can be already envisioned, especially in neuromorphic devices to simulate learning, adaptive and spontaneous behavior. For instance, the identification of memristive behavior in primitive organisms such as amoebas [10] opens up the possibility to relate physiological processes that occur in cells with the theory of memory devices presented here. Along similar lines, one could envision simple models that identify memory mechanisms in neurons and use these memory devices to build such models in the laboratory.

Therefore, due to their versatility (including analog functionalities), the combined operation of these memory devices in electronic circuits is still largely unexplored, and we hope our work will motivate experimental and theoretical investigations in this direction. ■

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