Find the Laplace transforms of:

1. \( f(t) = 3t^4 - 2t^{3/2} + 6 \)

2. \( f(t) = \sin t \cos t \)

Find the inverse Laplace transforms of

3. \( f(s) = \frac{3}{s^2 - 5s + 6} \) using a partial fraction expansion

4. \( f(s) = \frac{3}{s^2 - 5s + 6} \) using the Bromwich integral

5. \( f(s) = \frac{k^2}{s(s^2 + k^2)} \) using a partial fraction expansion

6. \( f(s) = \frac{k^2}{s(s^2 + k^2)} \) using the Bromwich integral

7. Use the Laplace transform to solve the initial value problem:

\[ \frac{d^2 y}{dt^2} + y = f(t) \]

with \( y(0) = 0, \ y'(0) = 0 \) and \( f(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \).

8. Use the Laplace transform to solve the initial value problem:

\[ \frac{d^2 y}{dt^2} + \beta^2 y = A \sin \omega t \]

with \( y(0) = 1, \ y'(0) = 0 \).

9. Consider two masses, \( m_1 \) and \( m_2 \), oscillating under the influence of ideal springs, spring constant \( k \), and coupled with a coupling strength \( a \). The relevant equations are

\[ m_1 \frac{d^2 X(t)}{dt^2} + kX(t) + a(X(t) - Y(t)) = 0 \]

\[ m_1 \frac{d^2 Y(t)}{dt^2} + kY(t) + a(Y(t) - X(t)) = 0 \]

Find \( X(t), Y(t) \) using the Laplace transform assuming \( X(0) = X_0, \ X'(0) = 0, Y(0) = Y_0, \ Y'(0) = 0 \).