1. Solve the integral equation

\[ \phi(x) = 1 + 2x - \int_0^x (t - x) \phi(t) \, dt \]

converting it to a differential equation and solving the differential equation (plus initial conditions).

2. Convert the following integral equation to an equivalent initial value problem (derive ODE and initial conditions)

\[ u(x) = e^x - \int_0^x (x-t) u(t) \, dt \]

3. Find the extremals of the following functional

\[ \int_a^b \frac{(y')^2}{x^3} \, dx . \]

4. Write out the equation of motion corresponding to the Lagrangian

\[ L = m_0c^2 \left[ 1 - \sqrt{1 - \frac{v^2}{c^2}} \right] - V(x), \text{ where } v = \dot{x} \text{ is the velocity.} \]

5. Fermat’s principle of optics states that a light ray will follow the path \( y(x) \) for which

\[ \int_{s_1, s_2} n(y, x) \, ds \]

is a minimum when \( n \) is the index of refraction. For \( y_2 = y_1 = 1, -x_1 = x_2 = 1 \), find the ray path if \( n = e^y \) and \( n = a(y - y_0) \), \( y > y_0 \).

6. Find the curve minimizing the integral

\[ \int_0^1 \left( \frac{1}{2} y''^2 + yy' + y' + y \right) \, dx . \]

7. Find the ratio of \( R \) (radius) to \( H \) (height) that will minimize the total surface area of a right-circular cylinder of fixed volume.