Flux flow in a two-band superconductor with delocalized electric fields

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In conventional flux flow, vortex dissipation is localized in the vicinity of the vortex core leading to a viscous coefficient η that is independent of flux density B and a flux-flow conductance $G_f \propto 1/B$. An anomalous behavior arises when a substantial quasiparticle population exists away from the cores and when the electric field and dissipation extend into those regions—a scenario that is realized in a disordered two-band superconductor with slow branch-imbalance relaxation. In this case, η rises linearly with B and G_f becomes independent of B, as observed in disordered magnesium diboride.

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I. INTRODUCTION

In the mixed state of a type-II superconductor, an applied flux density B has two effects on transport characteristics such as conductance-versus-temperature $G(T)$ or voltage-versus-current $V(I)$ curves: (1) suppression of superconductivity is manifested by a reduction in the effective critical temperature or critical current, and (2) the transition becomes increasingly broadened due to flux motion. For example, near T_c, the shift in transition temperature is approximately linear in B, and the $G(T)$ curve becomes increasingly shallow[1,2] with a $1/B$ field dependence that persists even in the case of highly driven nonlinear and unstable flux flow.[3,4] This observed B dependence arises because dissipation during flux motion is localized in the vicinity of the vortex core, so that each vortex has a viscous drag coefficient η that is independent of the other vortices and, hence, independent of B.

A fundamentally different behavior can result when the vortex dissipation becomes delocalized and spreads throughout the volume of the sample. Two conditions need to be met for this to happen: (1) the electric-field penetration depth ζ needs to be long compared to the coherence length ξ so that E extends long distances and fills the region between the vortex cores, and (2) there needs to be a sufficient concentration of quasiparticles everywhere so that not only is there an electric field everywhere but dissipation as well. The first condition is satisfied when ζ is unusually long, because of slow branch-imbalance relaxation as a consequence slow electron-phonon scattering, and when elastic scattering from disorder shrinks ξ without affecting ζ. The second condition is satisfied at high temperatures when the gap is reduced or in a two-band superconductor in which one band is significantly quenched by a magnetic field.

This type of anomalous mixed-state response has been observed in magnesium diboride[5–8] and one example (data from Ref. 8) is shown in Fig. 1: panels (a) and (b) show that clean MgB_2 conforms to the conventional $V \propto B$ response, whereas panel (c) shows that the presence of impurity scattering transforms the response so that $V(I)$ curve shapes become anomalously B independent.

We show here that $\eta \propto B$ is expected when the electric-field penetration depth exceeds the coherence length and intervortex spacing, and when a sea of quasiparticles from a nearly normal π band causes vortex dissipation to spread throughout the entire volume instead of being localized near the core of the vortices.

II. ELECTRIC-FIELD PENETRATION DEPTH

The electric-field penetration depth ζ has been used to describe the injection of charge carriers from a normal conductor where an electric field is present into a superconductor where the field decays spatially.[9,10] The same concept serves to describe the penetration of an electric field from the core of a moving vortex into the surrounding superconduct-

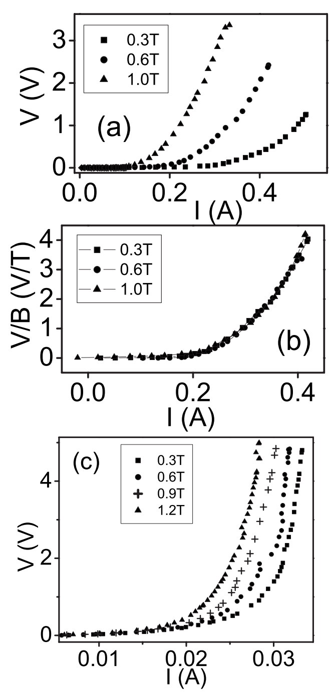

FIG. 1. (a) $V(I)$ curves for a clean hybrid physical-chemical vapor deposition (HPCVD) *in situ* epitaxial MgB_2 film showing conventional B-dependent ($V \propto B$) broadening due to flux motion. $T=10$ K. (b) Above V values divided by B and then shifted horizontally and made to collapse. (c) $V(I)$ curves for a disordered carbon-doped HPCVD *in situ* MgB_2 film. $T=6$ K; the curve shapes are approximately independent of B. (Data from Ref. 8.)

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ing medium.[11] The mechanism governing the process in superconductors with nonzero gap is the relaxation of unequal populations in the electronlike and holelike excitations (branch imbalance). Tinkham[9] showed that this relaxation is mediated by inelastic scattering in a superconductor with an isotropic gap, but that it can be accomplished by elastic scattering if there is anisotropy. The electric-field penetration depth is given by $\zeta=\sqrt{D\tau_Q}$ in terms of the diffusion constant D and the branch-imbalance relaxation time τ_Q. The part of τ_Q due to phonons is related to the electron-phonon scattering time $\tau_{\rm ph}$. There is reason to expect that the inelastic scattering by phonons is exceptionally small in MgB_2 and, therefore, to the extent that other mechanisms for branch relaxation can be neglected, that the electric-field penetration depth is exceptionally large. The evidence comes from the high Debye temperature ($\approx$1000 K) in MgB_2, indicating that few phonons are present at superconducting temperatures, and from the fact that there is little evidence of temperature dependent resistivity near superconducting temperatures in the samples used in the experiment.

In the work of Hu and Thompson[11] on the structure of moving vortices in single-band superconductors, an arbitrary electric-field penetration depth was used, even though they were considering the case of gapless superconductors where ζ is $1/\sqrt{12}$ times the coherence length ξ. Numerical work based on their formulas for $\zeta \gg \xi$ shows penetration of both charge and electric field into the region outside the vortex core, and shows a generally decreased local electric field for a given vortex velocity. The decrease in the local field occurs because spreading the field over a larger region must result in a decrease because the spatially averaged field must still be equal to the macroscopic electric field E determined by the vortex velocity and the external B field, $E=vB$. This is the field measured by the voltage across the sample.

In adapting the single-band results qualitatively to the two-band case of MgB_2 in the range of temperatures and fields where the anomalous behavior is observed, we must take note of the fact that the vortices are sustained by the σ-band electron pairs and that the π-band electrons form a normal conducting background. The π electrons will react to the electric field of the moving vortex both inside and outside of the core, adding to the dissipation of the σ electrons, which occurs primarily near the core.

In a single-band superconductor, τ_Q can be expressed in terms of the electron-phonon scattering time $\tau_{\rm ph}$ when elastic scattering and electron-electron scattering do not contribute. The expression is[12]

$$\tau_Q = \frac{2\hbar T}{\pi|\Delta|^2}\sqrt{1+\frac{4\|\Delta|^2\tau_{\rm ph}^2}{\hbar^2}}. \quad (1)$$

The temperature dependence of ζ arises from this factor, both from the explicit factor $T=tT_c$ and from the temperature dependence of the gap Δ and of the electron-phonon scattering time $\tau_{\rm ph}$. In the numerical estimates below, we take the standard empirical temperature dependence $\Delta=\Delta_0\sqrt{1-t}$ for the gap and assume that $\tau_{\rm ph}^{-1}$ is proportional to the number of phonons present, i.e., proportional to t^3. With these assumptions, Eq. (1) indicates that ζ approaches infinity as $1/\sqrt{1-t}$ at the transition temperature, the same temperature dependence as the coherence length. At $T=0$, ζ approaches infinity as $1/T$ because of the infinite electron-phonon scattering time in Eq. (1).

In calculating the reduced temperature t, we must account for the decrease of T_c with B. This can be obtained with sufficient accuracy by making a linear fit to the experimental data near $B=0$. The part of ζ determined by elastic scattering in the presence of gap anisotropy and the part due to scattering by π-band electrons are assumed to be temperature independent.

If the value of ζ exceeds the vortex spacing $l_\phi \approx \sqrt{\Phi_0/B}$, the electric field will extend throughout the sample. In that case, the π electrons will contribute fully to the dissipation, while the σ-band dissipation will still be restricted to the region near the vortex core.

In the next section, these ideas are taken as the basis for a simple model for the viscous coefficient of a moving vortex in dirty samples of MgB_2. In the absence of a full treatment of vortex dissipation in a two-band system, we expect that this model can give an indication of the dependence of this coefficient on the temperature and on the magnetic field. A full treatment would be necessary if we are to understand the system quantitatively and to deal with such questions as the effects of π-band screening on the electric-field penetration.

III. VISCOUS DRAG COEFFICIENT

The average electric field E_v near a vortex can be estimated by setting its spatial average equal to the average of the macroscopic field E:

$$E_v \pi \zeta^2 \approx E l_\phi^2 \approx E\frac{\Phi_0}{B}. \quad (2)$$

The viscous drag coefficient η can then be obtained by equating the total energy dissipated per unit time per unit vortex length to the drag force ηv per unit length times the velocity:

$$\eta v^2 = \sigma_\sigma \pi \xi^2 E_v^2 + \sigma_\pi \pi \zeta^2 E_v^2. \quad (3)$$

Note that the σ band contributes to the dissipation in a cylinder of radius ξ, while the π band contributes within a radius ζ. The drag coefficient then follows using the relation $v=E/B$ and Eq. (2):

$$\eta = \frac{\Phi_0^2}{\pi\zeta^2}\left(\sigma_\sigma\frac{\xi^2}{\zeta^2}+\sigma_\pi\right). \quad (4)$$

The last equation holds as long as $\zeta < l_\phi$. If that is not the case, the electric field spreads throughout the material, and the π band will produce almost as much dissipation as in the normal state. Then ζ in Eq. (4) can be replaced by the vortex spacing and the dissipation due to the σ band can be neglected when the coherence length is much less than the vortex spacing. This gives the estimate

$$\eta \approx \frac{\sigma_\pi \Phi_0 B}{\pi}, \quad (5)$$

which is in contrast to the B-independent Bardeen-Stephen result

$$\eta \approx \frac{\sigma_n \Phi_0 H_{c2}}{\pi}. \tag{6}$$

Thus, the flux-flow conductivity $\sigma_{ff} = \eta / B\Phi_0$, which is proportional to B^{-1} in the conventional case, becomes independent of B for the drag coefficient in Eq. (5). This change in field dependence could account for the experimentally observed anomalies if certain conditions on the conductivities, discussed below, are satisfied.

IV. PARAMETERS

Equation (1) shows that ζ becomes very large near T_c, so that field-independent $I(V)$ and $G(T)$ curves would be expected there. To assess whether this behavior is likely to be realized at temperatures far from the transition temperature, we estimate the value of ζ in MgB_2 in that temperature range. The estimate requires knowledge of the diffusion constant $D = v_F^2 \tau$ and the branch-imbalance relaxation time τ_Q. Estimates of these quantities depend on the carrier density n. Measurements of the Hall coefficient[13] $R = 1/ne$ indicate a carrier density of about 150 holes nm^{-3}. However, the zero-temperature magnetic penetration depth $\lambda(0) = \sqrt{mc^2/4\pi ne^2}$, quoted as 152 nm in Ref. 14, gives a value of 1.22 nm^{-3}. Both of these seem extreme in comparison to the number of unit cells per nm^3, which is 34. In the estimates, we therefore take a presumed value of two carriers per unit cell or 70 carriers per nm^3. The transport relaxation time τ can now be obtained from the measured resistivity, $\rho = 14\ \mu\Omega$ cm, using the Drude formula $\rho = m/ne^2\tau$. The resulting transport relaxation time is 3.6×10^{-15} s. Fermi velocities $v_{F\sigma} = 4.4 \times 10^5$ m/s and $v_{F\pi} = 8.2 \times 10^5$ m/s are quoted in Ref. 15. Using an average value of 6.4×10^5 m/s, we find a diffusion constant $D = 1.5 \times 10^{-3}$ $m^2\ s^{-1}$ and a mean free path of 2.3 nm.

An estimate of the electric-field penetration depth using Eq. (1) requires an estimate of the electron-phonon scattering time τ_{ep}. A direct estimate of this time from the Bloch-Grüneisen formula,[16] using parameters appropriate to MgB_2 and neglecting details of the Fermi surface and phonon spectrum, gives $\tau_{ep} = 2.2 \times 10^{-10}$ s. This results in an electric-field penetration depth ζ of the order of 300 nm. The estimate is crude and very sensitive to the Debye temperature. An earlier estimate of ζ given by Tinkham requires extrapolating the high temperature value of τ_{ep}, where the resistivity is proportional to T, back to the Debye temperature. If we again use the Bloch-Grüneisen formula to do the extrapolation, we obtain a value about 3 orders of magnitude smaller, $\zeta \sim 0.3$ nm. Fortunately, the details of the anomalous regime do not depend sensitively on the value of ζ. All that is required for the anomalous flux dynamics is for ζ to exceed $l_\phi \approx 45/\sqrt{B}$ nm, where B is in Tesla. This condition is satisfied in MgB_2 for zeta values in the higher part of the estimated range, especially for large B.

We have made these estimates without differentiating the contributions of the two bands to the diffusion constant or the carrier density. There is, however, evidence that in the superconducting state, the π band is considerably cleaner than the σ band. Measurements of the critical field H_{c2} have been made on the samples that show lack of broadening and lack of field dependence in conductance and voltage curves. Figure 2 shows such data (symbols). They can be fitted (solid line) to the theory of Gurevich,[17] which is obtained by adapting the Eilenberger and Usadel equations to the case of a two-band superconductor. It turns out that the shape of the H_{c2} vs T curve is very sensitive to the ratio of the diffusion constants of the two bands, which is roughly proportional to the ratio of scattering times. The fit between experimental data and theory in Fig. 2 was made with a ratio D_π/D_σ of 30. In obtaining this fit, we found it necessary to increase the interband electron-phonon coupling constants slightly compared to the commonly accepted ones.[17–19] The coupling constants giving the best fit are $\lambda_{\sigma\sigma} = 0.81$, $\lambda_{\pi\pi} = 0.285$, $\lambda_{\sigma\pi} = 0.150$, and $\lambda_{\pi\sigma} = 0.115$. The last two are greater than the ones used by Gurevich by a factor of 1.25.

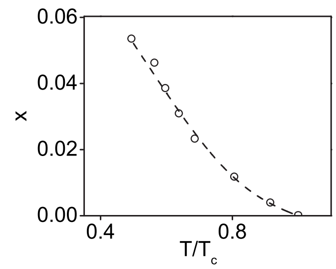

FIG. 2. Fit of experimental data for H_{c2} vs T to the theory of Gurevich cited in the text. The abscissa is the reduced temperature T/T_c and the ordinate is the dimensionless critical field $x = H_{c2}D_\sigma/2\Phi_0 T_c$. Parameters are the electron-phonon coupling constants $\lambda_{\sigma\sigma} = 0.81$, $\lambda_{\pi\pi} = 0.285$, $\lambda_{\sigma\pi} = 0.150$, and $\lambda_{\pi\sigma} = 0.115$, and the ratio of diffusion constants $D_\pi/D_\sigma = 30$.

Regarding the aspect of a quenched π band providing a sea of delocalized quasiparticles, it is known that a magnetic field affects the two bands to different extents: The π-band superconductivity is suppressed at low fields[20–22] and $H_{c2}(T \to 0)$ is mainly controlled by the σ band.[23] As suggested in Ref. 24, the total superfluid density can be represented by

$$n_s = n\{(1 - w_\pi) + [w_\pi \exp(-B/H^*)]\}, \tag{7}$$

where the weight w_π of the π band diminishes on the scale of $H^* \approx 3000$ G (sometimes called the "virtual H_{c2}"). Their value for H^* obtained from the neutron-scattering form factor[24] is consistent with the result from tunneling.[21]

V. DISCUSSION

Experimentally, it has been shown[5,8] that disordered MgB_2 shows $V(I)$ and $G(T)$ curves that have strangely B-independent shapes, which merely shift as a function of B. In this paper, we have considered a different regime of flux dynamics that can be expected in a two-band superconductor with delocalized vortex electric fields. In a conventional single-band mixed state, vortex dissipation is localized to a region $R \sim \xi$ because usually $\zeta \sim \xi$ and because far from the upper-critical phase boundary, the quasiparticle density diminishes rapidly for $r > \xi$ and, hence, there is a natural cutoff. In this conventional case, the viscous coefficient is independent of B, leading to a flux-flow conductance that is inversely proportional to B. In the delocalized scenario, dissipation extends outside the core region and fills the whole volume because of the availability of the quasiparticles of the quenched (π) band and because $\zeta > l_\phi > \xi$. In this latter delocalized case, the viscous coefficient is proportional to B, leading to a flux-flow conductance that is independent of B as observed experimentally.

This model naturally gives rise to a crossover field that must be exceeded for the anomalous behavior to occur. Two conditions must be met for the delocalized scenario: (1) $\zeta > l_\phi$ in order to have delocalized electric fields, and (2) the π band's participation in the superconductivity must be quenched (i.e., the virtual H_{c2} must be exceeded) so as to provide a sea of quasiparticles that will extend the dissipation into the region between the vortex cores. Interestingly, both conditions give a crossover field of $\sim$3000 G, which agrees with the experimental observation[8] that there is a conventional B dependence of $R(I)$ curves below a few thousand Gauss.

A further requirement for the transport characteristics to have a dramatic indifference to B is that the π quasiparticle conductivity be much higher than the σ quasiparticle conductivity. The observed shape of the $H_{c2}(T)$ indeed confirms that. At present, it is not known what causes the π band to have such a high conductivity in the superconducting state.

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