Critical fields and critical currents of superconducting disks in transverse magnetic fields

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The large nonuniform and field-dependent demagnetizing factors of superconducting disks in transverse magnetic fields complicates the determination of the lower critical field and critical current from magnetization. Correcting the applied field with a constant ellipsiodally approximated demagnetization correction $D'$ can result in significant errors. In this study of the magnetization characteristics of lead and Nb-Ti disks with various aspect ratios $a$, we find an empirical relation $D'(a)$ that describes the scaling of the applied-field value $H'_{c1}$ ($H'_{c2}$), at which flux penetration occurs, with respect to the intrinsic (lower) critical field $H_{c1}$ ($H_{c2}$). A model is described for determining $H_{c1}$ and $J_c$ for such a geometry. The results have important implications for various magnetic measurements in high-$T_c$ superconductors. The errors that can result in the measured values of $H_{c1}$ and $J_c$ in the inferred penetration depths, and in the effective-mass anisotropies, are discussed.

Much information about the fundamental parameters of high-$T_c$ superconductors has been obtained through various types of magnetization measurements on single crystals. A potential complication in any measurement of magnetization is the presence of a demagnetizing field. This is especially serious for fields transverse to the Cu-O planes ($H^2$) because many of the crystals are in the shape of thin platelets with the Cu-O planes parallel to the large faces. In a measurement such as that of the upper critical field $H_{c2}$ (involving the reversible magnetization near $T_c$), the average susceptibility is small, making the demagnetization correction negligible. However, in measurements of the lower critical field $H_{c1}$ and the critical current density $J_c$, there is a large, spatially nonuniform, and field-dependent demagnetizing field. The overall effect is that the sample experiences an average field $H$, which is enhanced over the applied field $H_0$ by a factor $D'$. As a result, the apparent magnetization $V_{app} = -4\pi dm/dH_0$ (the initial magnetization slope) is larger than the geometric volume $V$, by a factor $D_{vol}'$. In addition, the applied field at which flux first penetrates ($H_{c1}'$ or $H_{c2}'$) is suppressed with respect to $H_{c}$ (type I) or $H_{c1}$ (type II) by a factor $(D'H_0)^{-1}$.

Ellipsoids are the only shapes that become homogeneously magnetized and for which the demagnetizing factor $D$ can be readily calculated. In this case the net field experienced by the body is $H = H_0/(1 + D4\pi\chi)$, where $\chi$ is the volume susceptibility. In the Meissner state of a superconductor $\chi = -1/4\pi$ so that $H/H_0 = 1/(1 - D)$, where $D_{el}$ is the usual demagnetizing factor for an ellipsoid (for which formulas and tables are available). Henceforth all demagnetizing corrections will be described in terms of the inverted demagnetizing factors defined by $D' = 1/(1 - D)$.

In the literature on high-$T_c$ superconductors, the common practice appears to be to substitute $D'H_0$ with either $D_{vol}'$ or $D_{el}'$. It is not obvious that $D_{vol}'$ and $D_{el}'$ should be the same, and it is unclear how well either of these is approximated by $D_{el}'$. Furthermore, even for an actual ellipsoidal shape, for large eccentricities it has been shown by Saif that nucleation of flux lines occurs at an applied field larger than that of the corresponding magnetization-corrected (i.e., divided by $D_{el}'$) $H_{c1}$. In this study of the magnetization characteristics of type-I (lead) and type-II (Nb-Ti) superconducting disks, we find that $D'H_0$ and $D_{vol}'$ are material-independent functions of the diameter-to-thickness aspect ratio $a$. For all aspect ratios measured $D_{el}' > D_{vol}' > D'H_0$ (where the inequalities increase with $a$ and become negligible for $a \sim 2$). Note that this error in the demagnetization correction will be present in the measured $H_{c1}$ for all methods that rely on an onset of flux penetration at a given temperature and applied field such as $m$ versus $H_0$ isotherms (the most common method), zero-field-cooled dc magnetization versus temperature curves, $\mu_0 H_{c1}$ or $H_{c2}$ (of magnetic relaxation, and field dependence of the rf penetration depth. Furthermore, most of these methods are also susceptible to a rounding of the onset region due to premature flux penetration (of weakly superconducting regions and sharp points) and pinning (which makes gradual the deviation from initial linearity of $m$ versus $H_0$). The solution to the last problem is to fit the data to some sort of model (as has been done, for example, for the second and third methods) rather than arbitrarily picking some point as the onset of flux penetration. In this work we describe a method for fitting the $m$ versus $H_0$ data to unambiguously obtain $H_{c1}$; additionally, the fitting also provides a good estimate of the low-field $J_c$, overcoming the complications caused by the low-field field to be described later.

Two types of samples were used for these experiments: disks of Nb-Ti, and disks and squares of lead. The Nb-Ti disks were cut from a rod of the material with a composition of 53 at. % Nb and a $T_c$ of 9 K. The disks were
etched in a solution of HF (49%), HNO₃ (70.9%) and water (3:7:15) to remove the surface layer. The lead disks and squares were cut out of sheets pressed from 99.999%-pure pellets. These were then washed in mineral acid and solvents to clean the surface. Magnetization measurements were made in a commercial superconducting-quantum-interference-device spectrometer (SHE model 905). Temperatures were stable within 0.01 K during measurement of the isotherms. During zero-field cooling, residual fields of up to 1.5 Oe were present. The maximum error in the field at all values was of a similar magnitude.

Seven disks of lead were measured (at T = 5.5 K) with aspect ratios ranging from 2 to 80. Figure 1 shows M versus H₀ for the disks with a = 1.93, 5.98, 22.2 and 80.1; intermediate measured values of a have been omitted from the figure for clarity. Initially flux is completely excluded and the behavior is linear in H₀. From the slope we obtain the apparent-volume inverted demagnetizing factor D'ᵥₒ₁ = Vₐₚₓ/V = -4πa(∂M/∂H₀)ᵯ, shown in the third column of Table I. At applied fields H₀ = H'ᵯ, indicated by arrows in Fig. 1, the field at the edge of the disks exceeds the critical field; the sample enters the intermediate state of type-I superconductors, parts of it (initially a thin peripheral layer) becoming normal. The deviation from initial linearity that marks entry into the intermediate state also corresponds roughly to the peak in the M-H₀ curve — unlike the hard type-II case, where H = H'ᵯ is signaled by a gradual deviation in the M-H₀ curve and the peak at the higher field corresponds to complete flux penetration (H*). As H₀ → H'ᵯ (marked by the last arrow in Fig. 1), M decreases linearly to zero. As expected, this H'ᵯ, which of course is an intrinsic property independent of shape, is about the same for all samples. The ratio H'ᵯ/H'ᵯ = D'ᵥₒ₁, the inverted demagnetizing factor describing the scaling of the (lower) critical field, and is shown in column 2 of Table I. The fourth column shows for comparison the usual D'ᵥₒ₁ for an ellipsoid of revolution with major-to-minor axes ratio equal to a; note that both an inscribed as well as escribed ellipsoid (with minimum volume difference) have the same a as the corresponding disk. The formula used for calculating D'ᵥₒ₁ is

\[
D'ᵥₒ₁ = \frac{D'_ᵥₒ₁ - 1}{D'_ᵥₒ₁} = \frac{a^2}{(a^2-1)} \left( 1 - \frac{\arcsin[(a^2-1)/a]}{(a^2-1)^{1/2}}\right).
\]

We find D'ᵥₒ₁ > D'ᵥₒ₁ > D'ᵥₒ₁ throughout the measured range of a, the disparity increasing with a. These features can be seen more clearly in Fig. 2, which shows the three D's as functions of a. Also included in Table I (and shown in Fig. 2) are data for three lead squares. For the squares a was taken as √4/π(l/1). As can be seen, the inverted demagnetizing factors of the squares behave just like those of the disks—an equivalence useful for high-Tc applications. An interpolation of the empirically found values of D'ᵥₒ₁ in Table I can now be used to find the H'ᵯ and Jₚ of a hard type-II superconducting disk in transverse field—an application that is demonstrated below. It should be mentioned that the inequality found earlier between D'ᵥₒ₁, D'ᵥₒ₁ and D'ᵥₒ₁ might be expected to be inverted for the Hᵯ₁ orientation (H₀ in the plane of the disk or plateletlike crystal). Some measurements made in this orientation indeed seem to support that expectation; however, this point will not be elaborated further since the magnitude of the discrepancy, and hence of its consequences, is much smaller for this field orientation.

For thin superconducting disks, Bean⁸ and other critical-state models,⁹,¹⁰ which describe the magnetization of a long cylinder in terms of the bulk Jₚ, are inaccurate unless H₀ ≫ M. Experiments in conventional low-

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**TABLE I.** Inverted demagnetizing factors (Hᵯ₁ orientation) from apparent Hᵯ₁(D'ᵥₒ₁), apparent volume (D'ᵥₒ₁), and ellipsoidal approximation (D'ᵥₒ₁). a is the aspect ratio.

<table>
<thead>
<tr>
<th>a</th>
<th>D'ᵥₒ₁</th>
<th>D'ᵥₒ₁</th>
<th>D'ᵥₒ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.93</td>
<td>1.80</td>
<td>2.04</td>
<td>2.07</td>
</tr>
<tr>
<td>2.86</td>
<td>1.97</td>
<td>2.29</td>
<td>2.65</td>
</tr>
<tr>
<td>5.98</td>
<td>2.77</td>
<td>3.90</td>
<td>4.63</td>
</tr>
<tr>
<td>9.13</td>
<td>3.14</td>
<td>5.00</td>
<td>6.63</td>
</tr>
<tr>
<td>22.2</td>
<td>5.95</td>
<td>11.9</td>
<td>14.9</td>
</tr>
<tr>
<td>46.0</td>
<td>9.65</td>
<td>19.2</td>
<td>30.1</td>
</tr>
<tr>
<td>80.1</td>
<td>12.2</td>
<td>33.1</td>
<td>51.8</td>
</tr>
</tbody>
</table>

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FIG. 1. Transverse-field magnetization at T = 5.5 K of four lead disks (a's indicated in key). The arrows indicate H'ᵯ, the field at which a sample enters the intermediate state (deviation from the initial linear behavior), and the critical field H'ᵯ (at which the magnetization extrapolates linearly to zero). The lines connect consecutive points.
$T_c$ (Ref. 11) as well as high-$T_c$ (Ref. 12) materials indicate that the flux front that penetrates a disk-shaped superconductor is initially cylindrical, but deviates from this geometry as it progresses into the sample. The critical state is finally reached through the faces (i.e., along the thickness) so that the field that is shielded or trapped in the center is of order $J_c r$ rather than $J_c r$, where $t$ and $r$ are the thickness and radius. As the flux penetrates into the disk the demagnetizing factor decreases because of the decreasing average susceptibility. Matters are further complicated by the field dependence of $J_c(B)$, which is especially severe in high-$T_c$ superconductors (see Ref. 10, and references therein). To overcome these difficulties we develop a model that describes the field dependence of the moment $m(H_0)$ for fields just above $H_{c1}$ where only a small portion of the sample has been penetrated by the in-migrating flux front. Because most of the sample volume is still flux free in this evolving incomplete critical state, the average susceptibility (and hence $D_{He}$) is essentially constant over the field range of interest; the field at the periphery of the disk can then be taken to be $D_{He} \times H_0$. As a function of the depth $x$, from the surface of a long cylinder, for $x \leq c(H_0 - H_{c1})/(4\pi J_c)$, the flux density typically varies as

$$B(x) = H_0 - H_{c1} - 4\pi J_c x/c.$$  

This expression for $B(x)$ includes the effect of $H_{c1}$ (equilibrium magnetization), but neglects the small sharp rise in the equilibrium magnetic induction at $H_{c1}$. Because $B$ is small over the entire flux-density profile for the field range considered ($B \leq H_0 - H_{c1}$; $B \rightarrow 0$ as $H_0 \rightarrow H_{c1}$), the field dependence of $J_c$ does not enter Eq. (1): the quantity that goes in is the low-field $J_c$. The moment is obtained straightforwardly by integrating $(B - H_0)$ over the sample volume. To include the effect of demagnetization in that expression we make the substitutions $H_0 \rightarrow H = D_{He}^* H_0$ and $V \rightarrow V_{app} = D_{vol} V$ to arrive at

$$-\frac{4\pi m}{H_0} = V_{app} = D_{vol}^* V, \quad \frac{H_{c1}}{D_{He}^*} = H_{c1}^*$$

$$-\frac{-4\pi m}{H_0} = \frac{D_{vol}^* V}{D_{He}^*} \left[ \frac{3H_{c1} + G_1}{3H_0} \right]$$

$$-\frac{(H_{c1} + G_1 - D_{He}^* H_0^3)}{3H_0 G_1^3}$$

for $H_0 > \frac{H_{c1}}{D_{He}^*}$, (2)

where $G_1 = 4\pi J_c r/c$. For a thin disk in parallel field the demagnetizing factor is small and, with its neglect, the corresponding expressions are

$$-\frac{4\pi m}{H_0} = V_{app} = V, \quad H_0 \leq H_{c1},$$

$$-\frac{4\pi m}{H_0} = V - V \left[ \frac{(H_0 - H_{c1})^2}{H_0 G_1} \right]$$

for $H_0 > H_{c1}$, (3)

where $G_1 = 4\pi J_c r/c$.

Figure 3 shows $-m/H_0$ plotted against $H_0$ for a Nb-Ti disk ($a = 8.8$) in both field orientations at $T = 7.1$ K. The solid lines are fits to Eqs. (2) and (3) with $D_{vol}^*$, $J_c$, and $H_{c1}$ as the fitting parameters. $D_{vol}^*$ affects the "height" of the initial horizontal portion. Although it was varied to optimize the fit in that region, it can also be taken from the interpolation of the lead data. In fact, the values of $D_{vol}^*$ obtained from the fits fall right on the lead-data $D_{vol}^*$ curve in Fig. 2: $D_{vol}^*$ is therefore really not an independently indeterminable fitting parameter. $H_{c1}$ shifts the whole curve to the right or left, whereas $J_c$ controls the slope and curvature of the flux-entry portion—allowing these independently acting parameters to be

![Graph showing the relationship between $-m/H_0$ and $H_0$ for a Nb-Ti disk.](image)
TABLE II. \( J_c \) (10^3 A/cm^2) and \( H_{c1} \) (Oe) of Nb-Ti disks in perpendicular (\( H^\parallel \)) and parallel (\( H^\perp \)) field orientations obtained from fits of the data to Eqs. (2) and (3). \( D'_{Hc} \) and \( D'_{el} \) refer to the use of either lead-data-interpolated or ellipsoidally approximated inverted demagnetizing factors in Eq. (2).

<table>
<thead>
<tr>
<th>Aspect ratio</th>
<th>With ( D'_{Hc} )</th>
<th>With ( D'_{el} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8 (( H^\parallel ))</td>
<td>6</td>
<td>94</td>
</tr>
<tr>
<td>24 (( H^\parallel ))</td>
<td>7</td>
<td>90</td>
</tr>
<tr>
<td>8.8 (( H^\perp ))</td>
<td>6</td>
<td>103</td>
</tr>
<tr>
<td>26 (( H^\perp ))</td>
<td>6</td>
<td>128</td>
</tr>
<tr>
<td>61 (( H^\perp ))</td>
<td>7</td>
<td>120</td>
</tr>
</tbody>
</table>

The fit over an extended range of \( H_0 \) serves to reduce the ambiguity in \( H_{c1} \) due to the rounded transition region. Table II shows the results for several Nb-Ti disks (all at \( T = 7.1 \) K). Because of its negligible demagnetization and fewer unknowns, it is well known that the \( H^\parallel \) case is described reasonably well by critical-state models (agreement between the magnetic and transport \( J_c \)'s is typically within 15%). We will therefore use the parameters determined for that orientation as "reference values." Table II shows \( J_c \) and \( H_{c1} \) (columns 2 and 3) determined directly by inserting in Eq. (2) \( D'_{Hc} \) interpolated from the lead measurements (Table I). Also shown (columns 4 and 5) are a set of \( J_c \) and \( H_{c1} \) values obtained by replacing \( D'_{Hc} \) in Eq. (2) by \( D'_{el} \). As can be seen, the ellipsoidal approximation results in large errors in \( J_c \) and \( H_{c1} \), which increase with \( a \). On the other hand, with \( D'_{Hc} \) the parameters found from fits have roughly the same values for all \( a \). The scatter in \( J_c \) and \( H_{c1} \) may be partly due to the uncertainty in \( D'_{Hc} \) values interpolated from Table I. In addition, the values of \( H_{c1} \) will include any error in the applied field, which can be up to \( \pm 1.5 \) Oe. Demagnetization will magnify this error up to approximately 20 Oe. In view of these uncertainties, it is evident from Table II that Eqs. (2) and (3) provide a consistent and adequate description of the data. Note that replacing \( D'_{Hc} \) with \( D'_{vol} \) in Eq. (2) will give results that are intermediate to \( D'_{el} \) and \( D'_{Hc} \), but still largely self-consistent. Figure 4 compares the value of \( H_{c1} \) measured here with published zero-temperature values (\( H_{c10} \)) for different compositions of the alloy. \( H_{c10} \) was obtained from the 7.1 K value using \( H_{c1} = (H_{c10})^2 / (k^2 2) \) and \( k(t) = k(1.25 - 0.30t^2 + 0.05t^4) \), and assuming a parabolic temperature dependence for \( H_c \). No published values were available for comparison with the \( J_c \)'s obtained here. For both field orientations, \( J_c \) flows perpendicularly to the axis of the original rod from which disks were cut — although the field is along the rod axis for the \( H^\parallel \) case and perpendicular to the rod axis for the \( H^\perp \) orientation. Because of limitations on sample dimensions and resultant heating effects, the transport \( J_c \) could not be determined directly by either continuous-dc or pulsed methods.

The results of these magnetization studies on lead and Nb-Ti disks have obvious implications for various magnetic measurements (in the \( H^\parallel \) orientation) on single crystals of high-\( T_c \) superconductors. First of all we saw that the shielding volume equals \( (D'_{vol})^{-1} \times 4\pi dndm/dH_0 \) with \( D'_{vol} < D'_{el} \). An incomplete "superconducting fraction" will therefore be inferred if \( D'_{el} \) is used for the demagnetization correction. Second, estimates of low-field \( J_c \) from magnetization can now be made more accurately using the model presented above. With a simple extension a similar approach can also be used for calculating the low-field magnetic relaxation rate, \( \gamma \) memory effects, \( 18 \) and other phenomena that result from flux creep. Thus it will be possible to determine the low-field flux-pinning energy \( U_{p}^{2}(B \approx 0) \) with greater accuracy — a quantity used as a parameter in the comparison of different materials.\(^{20,29}\) Finally, when the usual incorrect inverted-demagnetizing factors \( D'_{el} \) or \( D'_{vol} \) are used in determining \( H_{c1} \), the reported values will be overestimates of the intrinsic values. An error of similar magnitude will be carried over into the deduced penetration depths as well as the effective-mass anisotropy factor \( \gamma = |m_e / m_{ab}| \) found, for example, from an approximate London formulation.\(^{4,21,22}\) Knowing the anisotropy factor accurately is of importance in understanding the nature of the superconducting state in high-\( T_c \) materials, since different theoretical models\(^{31,32}\) make different predictions about the relationship between the anisotropies in the lower and upper critical field. In Y-Ba-Cu-O, torque magne-
tometry, 24 measurements of $H_{c2}$ 25 and those of fluctuation conductivity 26 produce $\gamma \approx 5$. The values of $H_{c1}$ and $H_{c2}$, and of the penetration depths $\lambda_1$ and $\lambda_2$, show a considerable amount of scatter. Reported anisotropies in these parameters cover a range 3–10. 4,27 A recent — and perhaps the most reliable — measurement of $H_{c1}$ by Krusin-Elbaum et al. 6 leads to a $\gamma$ of 3.5. If their results are recalculated with the proper demagnetization correction $D_I^H$, $\gamma$ is reduced to about 1.7—significantly lower than the value obtained from $H_{c2}$. With some of the other high-$T_c$ superconductors, notably Bi-Sr-Ca-Cu-O, the crystals have even larger aspect ratios and use of the proper $D^*$ is of correspondingly greater importance. In Bi-Sr-Ca-Cu-O a rather large range of $H_{c1}$ and deduced $\gamma$ values have been reported. 28–30 Our own measurements 29 originally reported an implied $\gamma$ of about 35. With the new $D^*$ this would have to be revised to approximately 20. Resistive measurements of $H_{c2}$ on single crystals 31 yield a $\gamma$ of 25–60, whereas similar measurements on thin films 32 yield $\gamma \sim 15$. At present no measurements on $H_{c2}$ from reversible dc magnetization are available. Farrell et al. 33 obtain $\gamma = 3000$ from torque magnetometry. Thus the scatter in the different values of $\gamma$ make it difficult to draw any definite conclusions; however, as in Y-Ba-Cu-O it seems that the anisotropy factor found from $H_{c1}$ is smaller than that obtained from $H_{c2}$.

It will be interesting to see if this is a general property of high-$T_c$ superconductors once additional independent and reliable measurements of the various compounds become available.

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14For figures of such profiles see, for example, P. G. de Gennes, Superconductivity of Metals and Alloys (Benjamin, New York, 1966); or H.Ullmaier, Irreversible Properties of Type-II Superconductors, Vol. 76 of Springer Tracts in Modern Physics (Springer-Verlag, Berlin, 1975).


