Vortex instability in molybdenum-germanium superconducting films

Manlai Liang and Milind N. Kunchur*
Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208, USA
(Received 21 August 2010; published 18 October 2010)

We studied the high driving force regime of the current-voltage transport response in the mixed state of amorphous molybdenum-germanium superconducting films to the point where the flux flow becomes unstable. The observed nonlinear response conforms with the classic Larkin-Ovchinnikov picture with a quasiparticle-energy-relaxation rate dominated by the quasiparticle recombination process. The measured energy relaxation rate was found to have a magnitude and temperature dependence in agreement with theory.

DOI: 10.1103/PhysRevB.82.144517 PACS number: 74.40.Gh, 74.25.Uv, 72.15.Lh, 73.50.Gr

I. INTRODUCTION AND THEORY

In a type-II superconductor, magnetizing fields between the lower critical field \(H_{c1}\) and upper critical field \(H_{c2}\) introduce flux vortices containing a quantum of flux \(\Phi_0 = h/2e\). A transport electric current density \(j\) perpendicular to the vortices and to exerts a “Lorentz” driving force \(F_\ell = j\Phi_0\). The consequent vortex motion generates an electric field \(E = vB\) and is opposed by a viscous drag \(\eta v\) (where \(\eta\) is the coefficient of viscosity and \(v\) the vortex velocity) so that in steady state \(j\Phi_0 = \eta v \equiv E\) and the response is Ohmic as long as the flow is not hindered by pinning. This regime of flux motion corresponds to free flux flow (FFF). While superficially the physics appears simple, and bears resemblance to a hydrodynamic system, this resemblance and apparent simplicity are deceiving. First of all the so-called Lorentz force in a superconductor actually has the opposite direction to the usual electromagnetic Lorentz force and while it has the right magnitude \(jE\), it is short coming to a nonmonotonic response to be dictated by the LO mechanism, which is in-
ducting transition temperature), $c$ is an unknown constant of order unity and $E^*$ is given by
\[ E^* = \frac{D}{14\zeta(3)(1-t)}B^2, \]
where $D$ is the diffusion constant, $\zeta(x)$ is the Riemann zeta function, and $\tau_e$ is the energy relaxation time.

In our previous work\cite{13} we found that the expressions for $\sigma_f(0)$ in Ref. 13 (their Eqs. 22 and 30) did not fit the data well over a significant range. Instead the following expression:\cite{33}
\[ \sigma_f = \sigma_n + \sigma_n \left( 1 - \frac{b}{vb} \right) \]

based on the mean-field result of time-dependent Ginzburg-Landau (TDGL) theory,\cite{34-39} more accurately represented the behavior over an extended range; here $b = B/\mu_0H_c^2$ is the reduced magnetic field ($H_c^2$ is the upper critical magnetizing field) and $v \approx 0.3$ is a dimensionless constant. The right-hand side of Eq. (3) represents a two-fluid-model sum of the normal conductivity $\sigma_n$ and the flux-flow conductivity contribution. The first term is negligible compared to the second term for the range of conditions where we study the LO nonlinear behavior of $\sigma_f$ and is not affected by $E$ and $B$ fields. Thus combining Eq. (1) with the second term of Eq. (3) gives the following nonlinear $j(E)$ relationship:
\[ j = E \left[ \sigma_n \left( 1 - \frac{b}{vb} \right) \right] \left[ \frac{1}{1 + (E/E^*)^2} \right]. \]

Note that this nonlinear function is only valid until the vortex stops shrinking, which occurs at a field\cite{13} $E_c \sim E^*/(1-t)^{1/4}$. At very high $E$ the system eventually enters the normal state and then $\sigma = \sigma_n$.

II. EXPERIMENTAL TECHNIQUES

The samples A, B, and C used in this experiment are exactly the same as the samples A, B, and C used in our prior work on free flux flow.\cite{33} The samples consist of MoGe films of thickness 50 nm sputtered onto silicon substrates with 200-nm-thick oxide layers using an alloy target of atomic composition Mo$_{0.79}$Ge$_{0.21}$. The deposition system had a base pressure of 2×10$^{-7}$ Torr and the argon-gas working pressure was maintained at 3 mTorr during the sputtering. The growth rate was 0.15 nm/s. The samples were patterned into bridges of length $l = 102$ μm and width $w = 6$ μm using photolithography and argon ion milling. Some parameters of the samples are as follows: Sample A: $T_c = 5.56$ K, $R_n = 555 \Omega$, $\mu_0H_c^2 = 3.13$ T/K, and $D = 0.35$ cm$^2$/s. Sample B: $T_c = 5.41$ K, $R_n = 555 \Omega$, $\mu_0H_c^2 = 3.13$ T/K, and $D = 0.35$ cm$^2$/s. Sample C: $T_c = 5.01$ K, $R_n = 630 \Omega$, $\mu_0H_c^2 = 3.0$ T/K, and $D = 0.37$ cm$^2$/s. Here, $R_n$ is the normal-state resistance, $H_c^2 = \frac{\mu_0}{\mu_0}H_c^2$ is the upper critical-field slope, and the diffusion coefficient $D$ was calculated from\cite{40} $D = -8k_B/2\pi\epsilon\mu_0H_c^2$.

Figure 1 shows some examples of nonlinear $j(E)$ curves. $j$ (and hence the viscous drag force) has a local maximum value of $j^*$ at the instability field $E^*$. In a current biased circuit, where the source resistance is larger than the sample resistance as is the case here, $E$ jumps (indicated by arrows) upon increasing $j$ to the vicinity of the intrinsic $j^*$. In a voltage biased measurement, where the source resistance is lower than the sample’s, there will not be a jump in $E$ and instead $j$ will be seen to decrease. The macroscopically averaged behavior will have a negative $dj/dE$ and the flux matter fragments into compressional, shear\cite{44} or other types of elastic domains such that any given domain is moving in a response region with $dj/dE > 0$ locally. The macroscopic...
curve will then not follow the primitive curve [e.g., Eq. (4)] but will show steps in the region where \( \frac{dj}{dE} < 0 \). In the present experiment we are only concerned with the region of the transport response up to \( E \) where \( \frac{dj}{dE} > 0 \) macroscopically.

The solid lines in Fig. 1 are fits to Eq. (4) and are seen to follow the trends of the data. The parameters \( \nu \) and \( E^* \) (location of peak) were adjusted to improve the fits, but have fit values of the expected magnitudes: \( \nu \sim 0.3 \) and \( E^* \) from the peaks is slightly higher than the position of the actual jumps, which is to be expected and has been observed by others (e.g., Ref. 21). For subsequent analysis, we take \( E^* \) to be the actual measured value of \( E \) at the threshold of the jump.

Figure 2 plots \( E^2 \), obtained from the \( j(E) \) curves as discussed above, against \( B^2 \). In agreement with Eq. (2), the two quantities are directly proportional to each other (i.e., the critical vortex velocity \( v^* = \frac{E^*}{B} \) is independent of \( B \)). From the measured slope and Eq. (2), we obtain the energy relaxation time \( \tau_v \). Figure 3 plots the corresponding relaxation rate \( \tau_v^{-1} \) against the reduced temperature for each of the three samples.

As discussed in the introduction, the LO effect occurs when \( \tau_{ee} \) is long compared with \( \tau_{ep} \), resulting in a nonthermal shape of the quasiparticle distribution function. The extent of the distribution function distortion is controlled by the rate of energy relaxation from quasiparticles to phonons, which occurs through two processes: one process is the inelastic scattering between a quasiparticle and a phonon and the other is the recombination of two quasiparticles to form a Cooper pair with the emission of a phonon. As discussed by Kaplan et al., the energy relaxation is mainly dominated by the latter recombination process which has a rate that can be written as

\[
\tau_v^{-1} = T \left( \frac{k_B T}{\Delta} \right)^{1/2} \exp \left( - \frac{\Delta}{k_B T} \right),
\]
where $T$ is a temperature-independent characteristic time constant (in the terminology of Kaplan et al., $T \approx \tau_0/55$) and $\Delta$ is the temperature-dependent superconducting energy gap. Taking the Bardeen-Cooper-Schrieffer (BCS) temperature dependence for $\Delta$ we are able to fit the data in Fig. 3 with Eq. (5) (solid lines) with $T \approx 3.5 \times 10^{-11}$ s. As can be seen, the measured functional form of $\tau_0^2(t)$ is in agreement with Eq. (5). While there is insufficient information in the literature to theoretically compute the magnitude of $\tau_0$, the temperature-dependent superconducting energy gap $\Delta$ has a value of $T_c(4.5 \text{ K})$ and $T_D(240 \text{ K})$ as $\text{MoGe}$ (for which $T_c \approx 5.3 \text{ K}$ and $T_D \approx 260 \text{ K}$) has a value of $T(3.3 \times 10^{-11}$ s) that is comparable to the one we obtained for $\text{MoGe}(3.5 \times 10^{-11}$ s). ($T_c$ and $T_D$ are parameters that are indicative of the electron-phonon coupling and phonon density of states, respectively.) If quasiparticle-phonon scattering is the dominant relaxation process, rather than quasiparticle recombination, then the rate is given by a function of the form $\tau_0^2 \approx \tau_0^{7/2}$ instead of Eq. (5). This power-law function (taken with an adjustable constant of proportionality $a$) is plotted as dashed red lines on Fig. 3 and is clearly at odds with the data. Thus our study of the vortex instability is able to distinguish the two routes of energy decay and provides a confirmation of the recombination rate expression of Eq. (5).

**IV. CONCLUSIONS**

In conclusion, we have studied the high driving force regime of vortex dynamics in one of the simplest and nearly model superconductors (unpinned, isotropic, low-temperature, weak-coupling BCS, etc.). In recent work\(^{33}\) we found that for the FFF regime these MoGe films provided a detailed confirmation of the TDGL mean-field prediction for $\sigma_d(0)$, while the LO expressions for the same regime showed very limited applicability. On the other hand in the present work we find that the LO expression [Eq. (1)] for the non-linear modulation factor for $\sigma_d(0)$ and the LO result [Eq. (2)] for the relationship between the instability electric field $E$ and the energy relaxation time $\tau_0$ are well obeyed in this system. Furthermore we were able to distinguish between the two principal quasiparticle-phonon energy relaxation processes and confirm the predicted temperature dependence [Eq. (5)] for the recombination process. Both regimes of instability, the hot electron as well as the distribution-function type, thus provide a valuable tool for investigating key scattering processes through the response of the mixed state.

**ACKNOWLEDGMENTS**

The authors gratefully acknowledge Jiong Hua, Zhili Xiao, James M. Knight, and Boris I. Ivel. This work was supported by the U. S. Department of Energy under Grant No. DE-FG02-99ER45763.