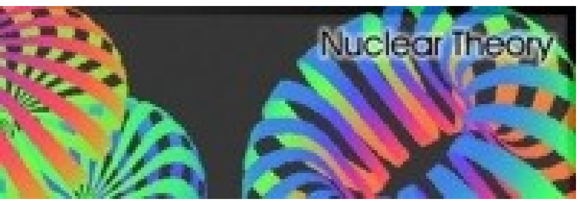
Argonne NATIONAL LABORATORY

Emergence of DSEs in Real-World QCD

Craig Roberts

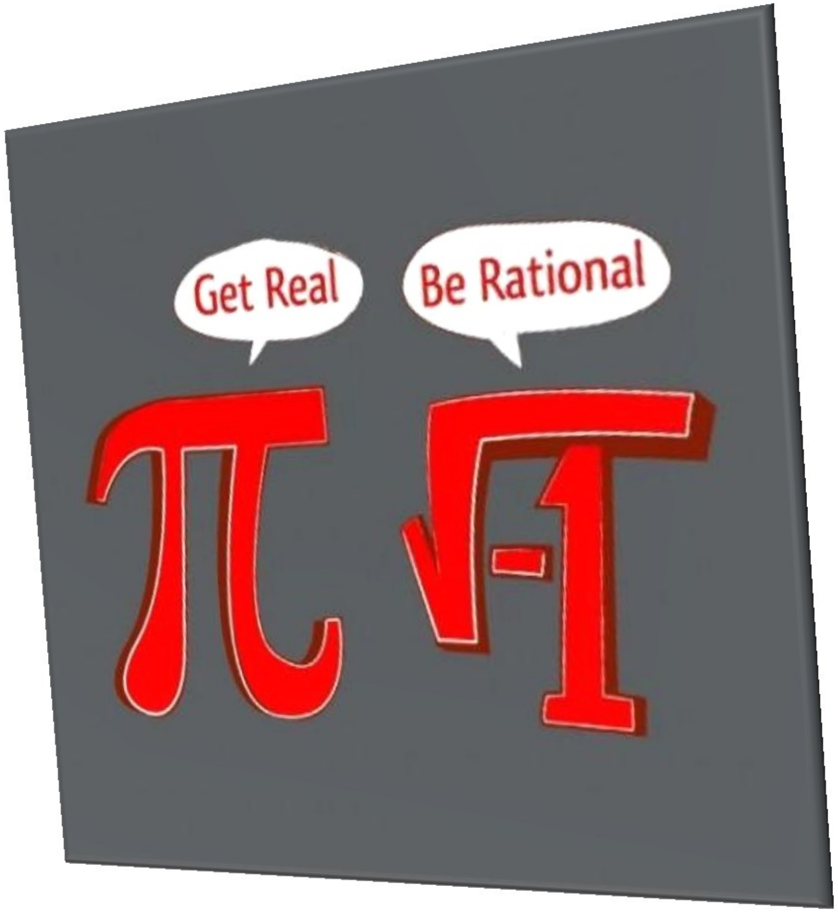


Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

National Science Foundation
WHERE DISCOVERIES BEGIN

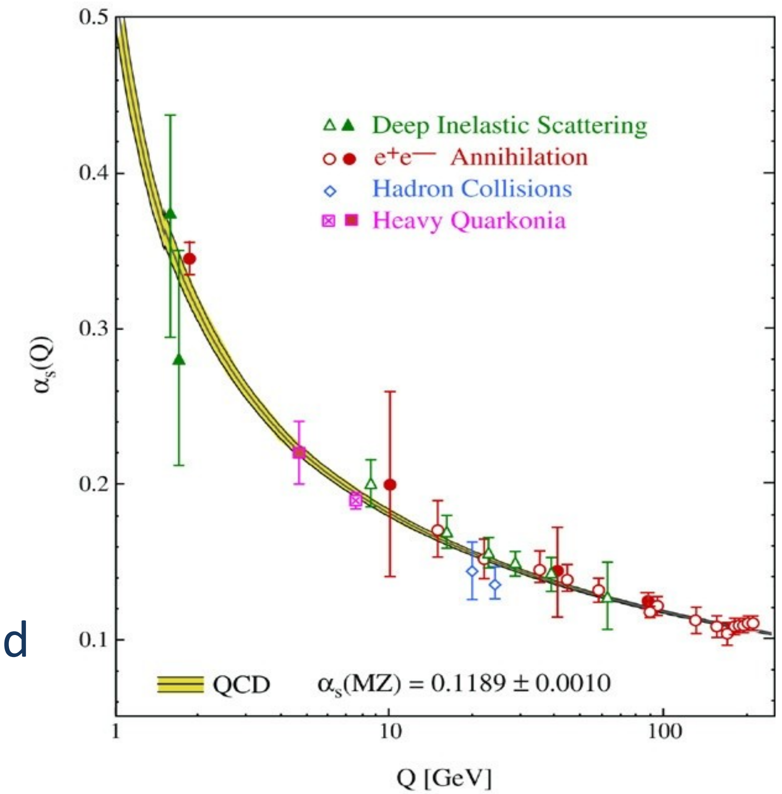
U.S. DEPARTMENT OF ENERGY



Getting real

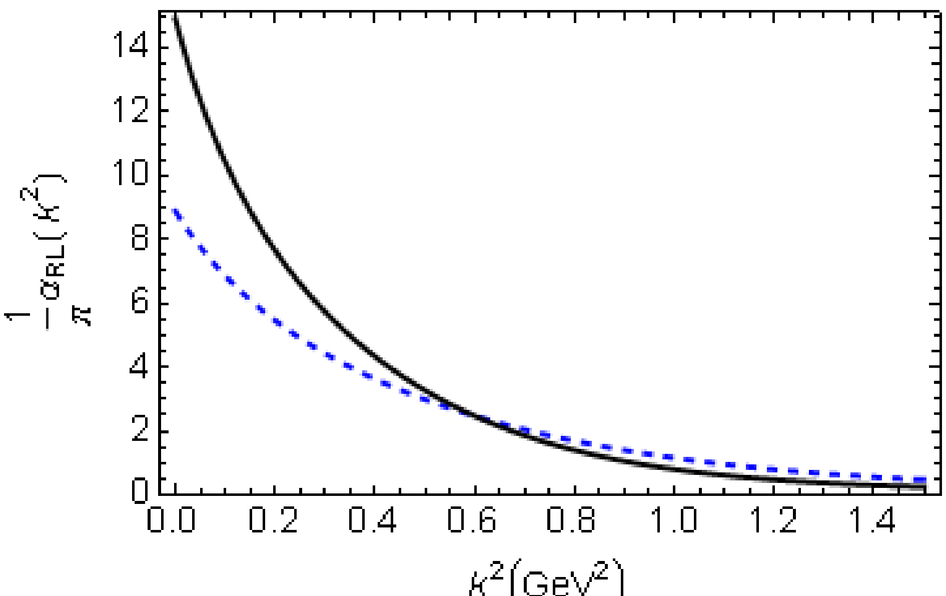
Charting the Interaction

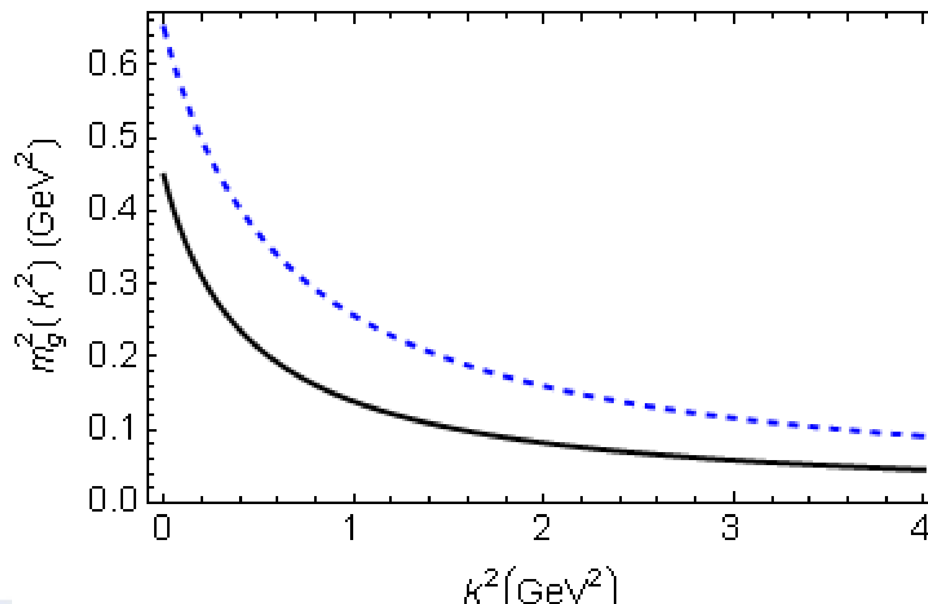
- Interaction in QCD is not momentum-independent
  - Behaviour for Q^2>2GeV^2 is well know; namely, renormalisation-group-improved one-gluon exchange
  - Computable in perturbation theory
- Known = there is a "freezing" of the interaction below a scale of roughly 0.4GeV, which is why momentum-independent interaction works
- Unknown
  - Infrared behavior of the interaction, which is responsible for
    - Confinement
    - DCSB
  - How is the transition to pQCD made and is it possible to define a transition boundary?

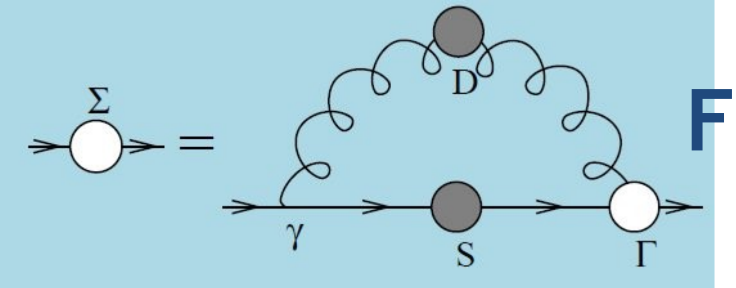


DSE Studies - Phenomenology of gluon

- Wide-ranging study of π & ρ properties
- Effective coupling
  - Agrees with pQCD in ultraviolet
  - Saturates in infrared
    - $\alpha(0)/\pi$ = 8-15
    - $\alpha(m_G^2)/\pi$ = 2-4
- Running gluon mass
  - Gluon is massless in ultraviolet in agreement with pQCD
  - Massive in infrared
    - $m_G(0)$ = 0.67-0.81 GeV
    - $m_G(m_G^2)$ = 0.53-0.64 GeV



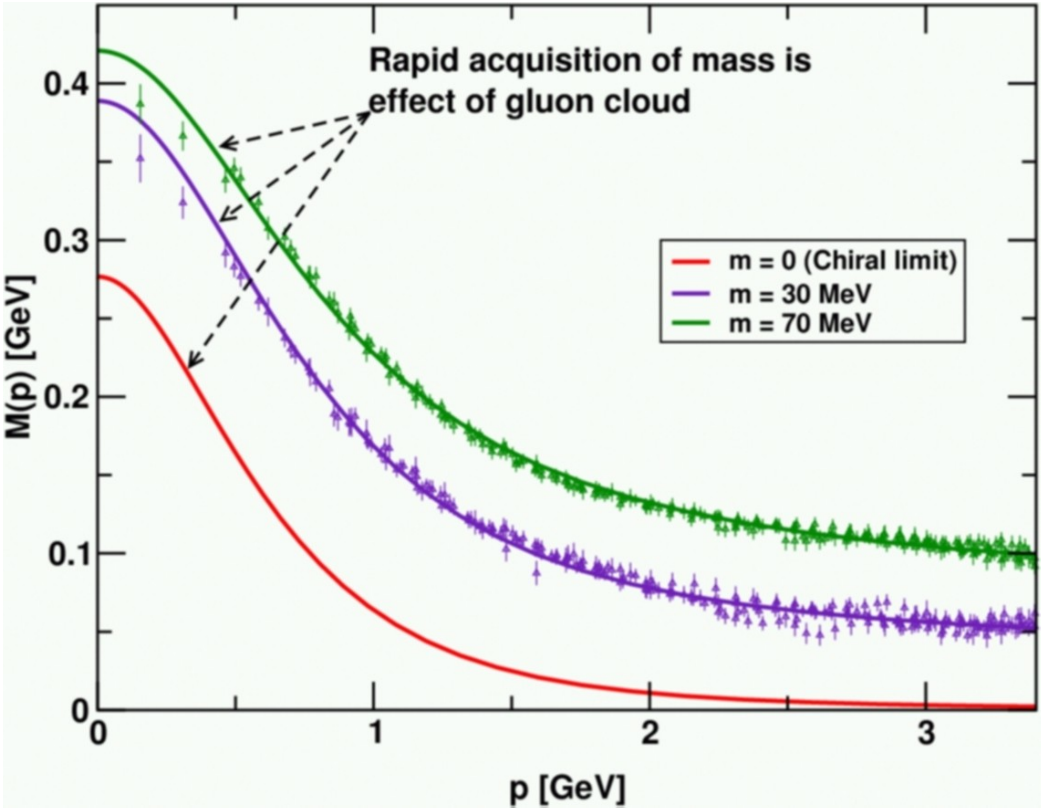




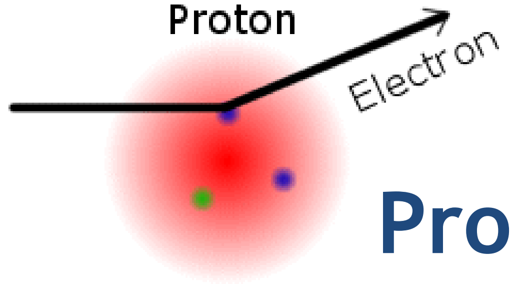
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. **Numerical simulations of lattice QCD** (data, at two different bare masses) **have confirmed model predictions** (solid curves) **that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates**. In this way, a quark that appears to be absolutely massless at high energies (m =0, **red curve**) acquires a large constituent mass at low energies.



C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50
M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227



Nucleon Structure
Probed in scattering experiments

- Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, $F_1=1$ & $F_2=0$, so that $G_E=G_M$ and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$\begin{aligned} J_\mu(P',P) &= ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,, \\ &= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P) \end{aligned}$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magntic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

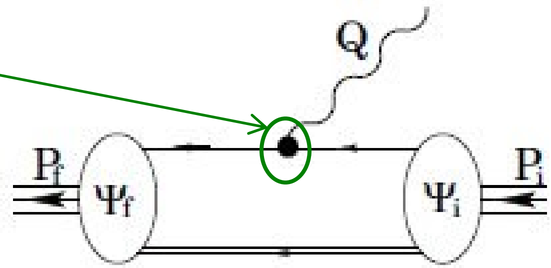
Nucleon form factors

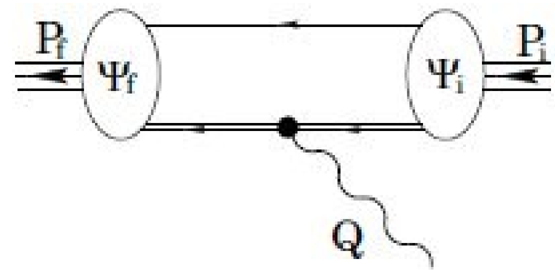
- For the nucleon & Δ-resonance, studies of the Faddeev equation exist that are based on the 1-loop renormalisation-group-improved interaction that was used efficaciously in the study of mesons
  - *Toward unifying the description of meson and baryon properties*
    G. Eichmann, I.C. Cloët, R. Alkofer, A. Krassnigg and C.D. Roberts
    arXiv:0810.1222 [nucl-th], Phys. Rev. C **79** (2009) 012202(R) (5 pages)
  - *Survey of nucleon electromagnetic form factors*
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
    arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36
  - *Nucleon electromagnetic form factors from the Faddeev equation*
    G. Eichmann, arXiv:1104.4505 [hep-ph]
- These studies retain the scalar and axial-vector diquark correlations, which we know to be necessary and sufficient for a reliable description
- In order to compute form factors, one needs a photon-nucleon current

L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. ***106*** *(2011) 072001*

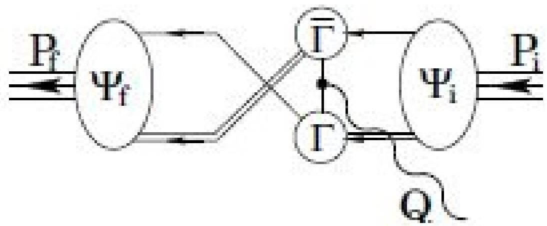
Photon-nucleon current

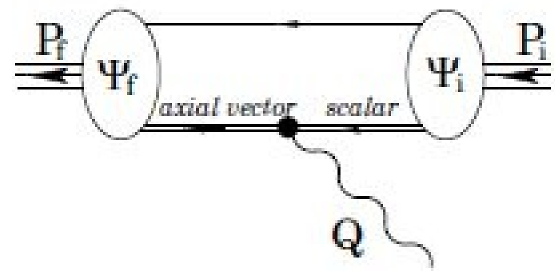
Vertex must contain the dressed-quark anomalous magnetic moment: Lecture 2B



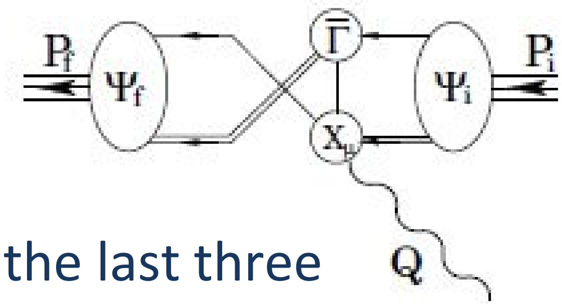


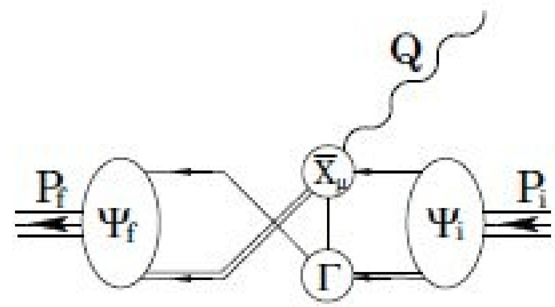
- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Takahashi identities





- DSE → BSE → Faddeev equation plus current → nucleon form factors



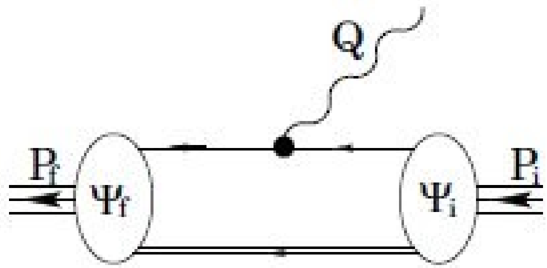


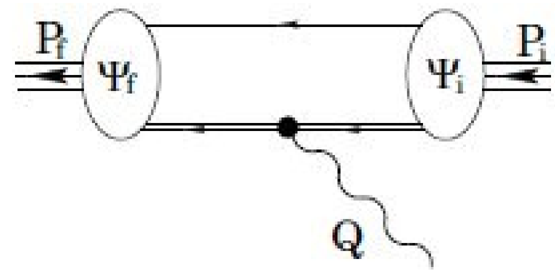
- In a realistic calculation, the last three diagrams represent 8-dimensional integrals, which can be evaluated using Monte-Carlo techniques

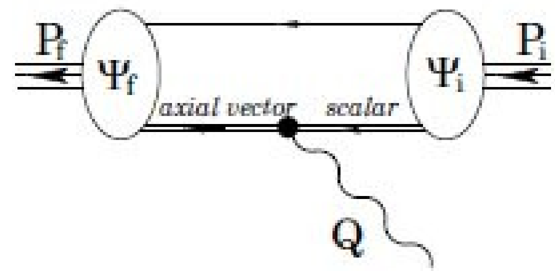
Oettel, Pichowsky, Smekal
Eur.Phys.J. A8 (2000) 251-281

Photon-nucleon current

- Owing to momentum-independence of the diquark Bethe-Salpeter and Faddeev amplitudes using the contact interaction in “static approximation”, the nucleon photon current simplifies
- Comparison between results from contact-interaction and realistic interaction can reveal a great deal







Just three terms survive

- **[arXiv:1112.2212 [nucl-th]](#)**, [Phys. Rev. C**85** (2012) 025205 [21 pages]](#) , *Nucleon and Roper electromagnetic elastic and transition form factors*, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts

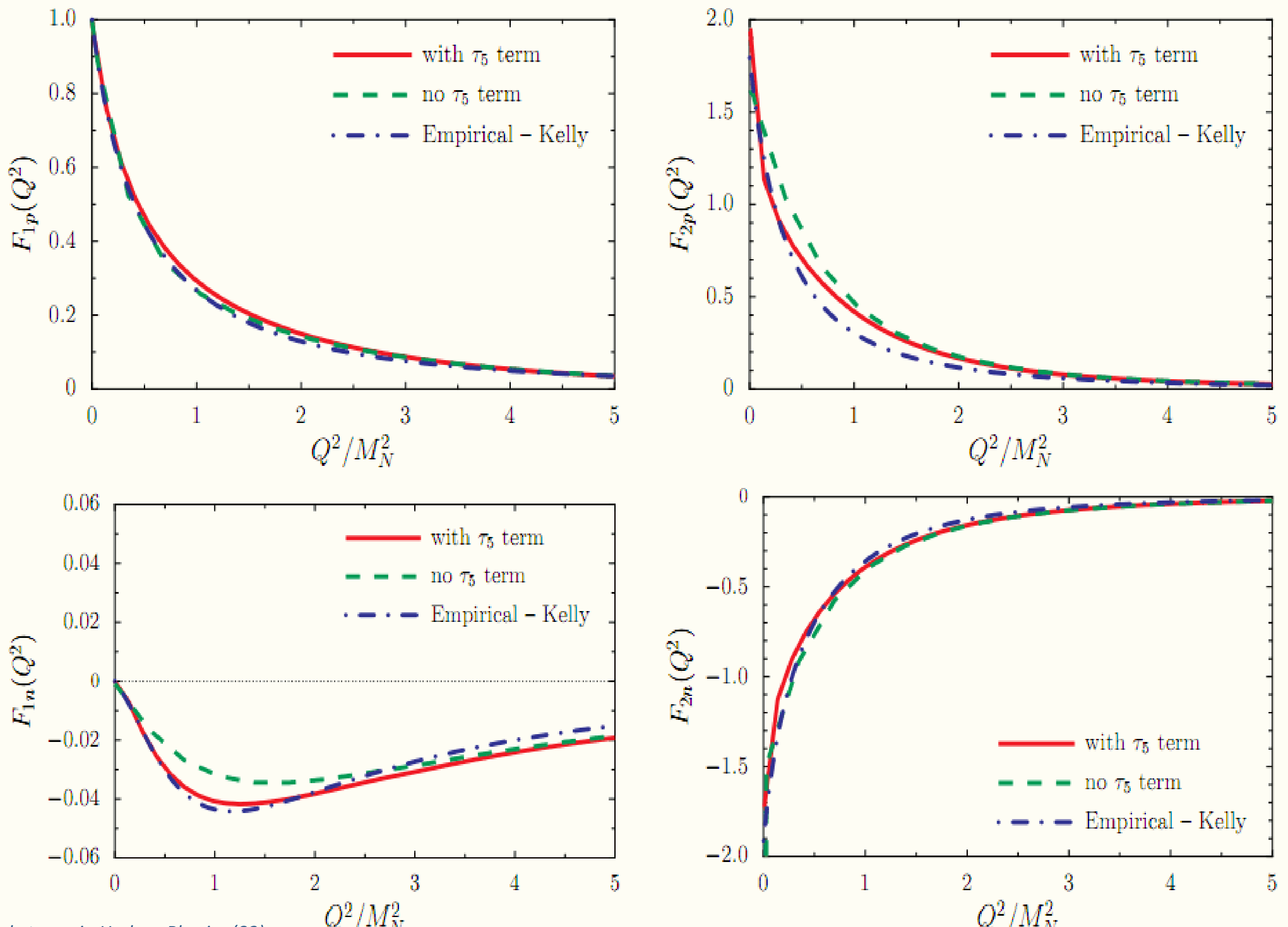
Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Nucleon Form Factors

Unification of meson and nucleon form factors.

Very good description.

Quark's momentum-dependent anomalous magnetic moment has observable impact & materially improves agreement in all cases.



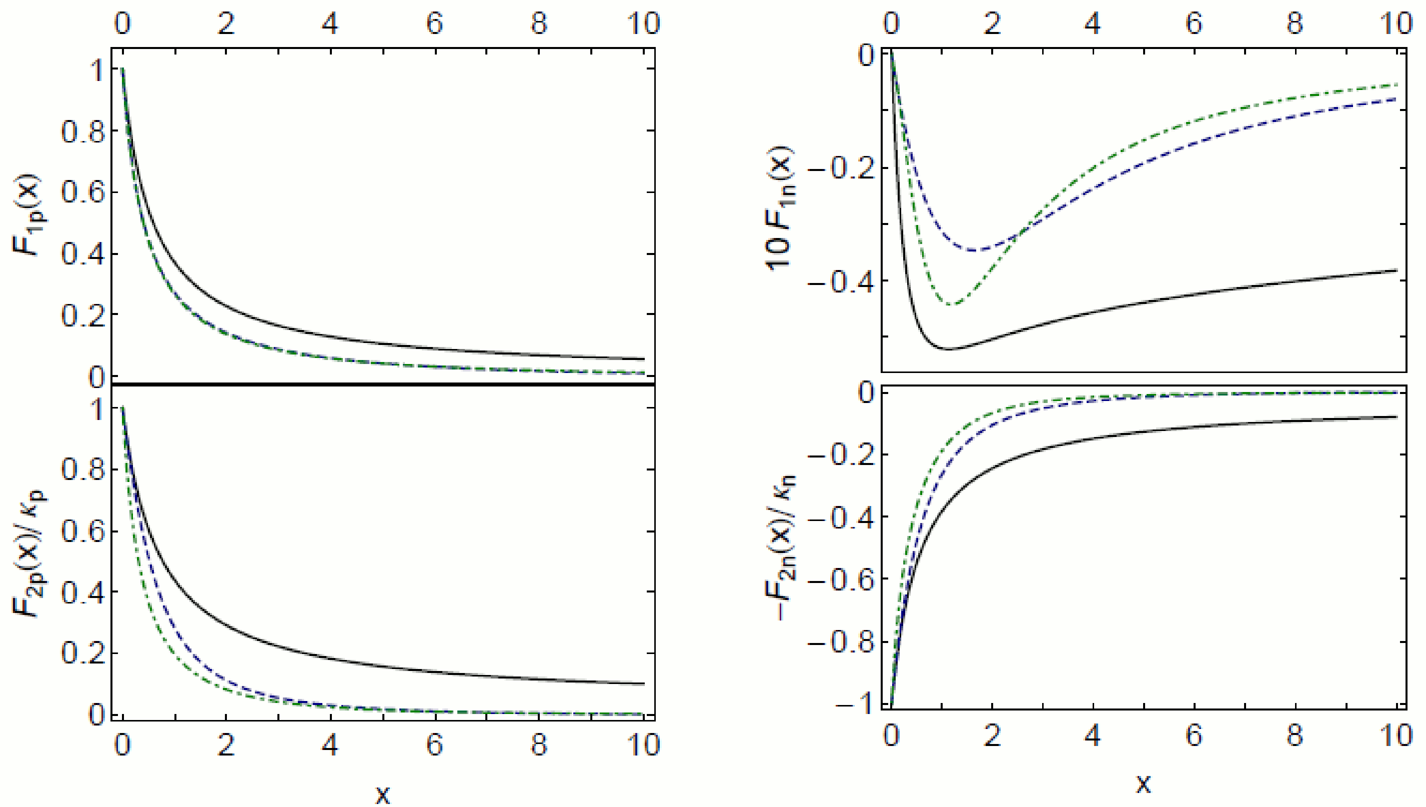
Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, [arXiv:1112.2212 [nucl-th]](), [Phys. Rev. C**85** (2012) 025205 [21 pages]]()

Nucleon Form Factors

Black solid curve = contact interaction

Blue dashed curve = momentum-dependent interaction

Green dot-dashed curve = parametrisation of experimental data



Momentum independent Faddeev amplitudes, paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Nucleon Form Factors

Black s
contact

Blue da
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interact

Green c
curve =
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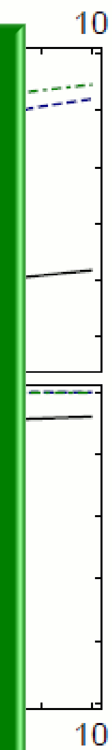
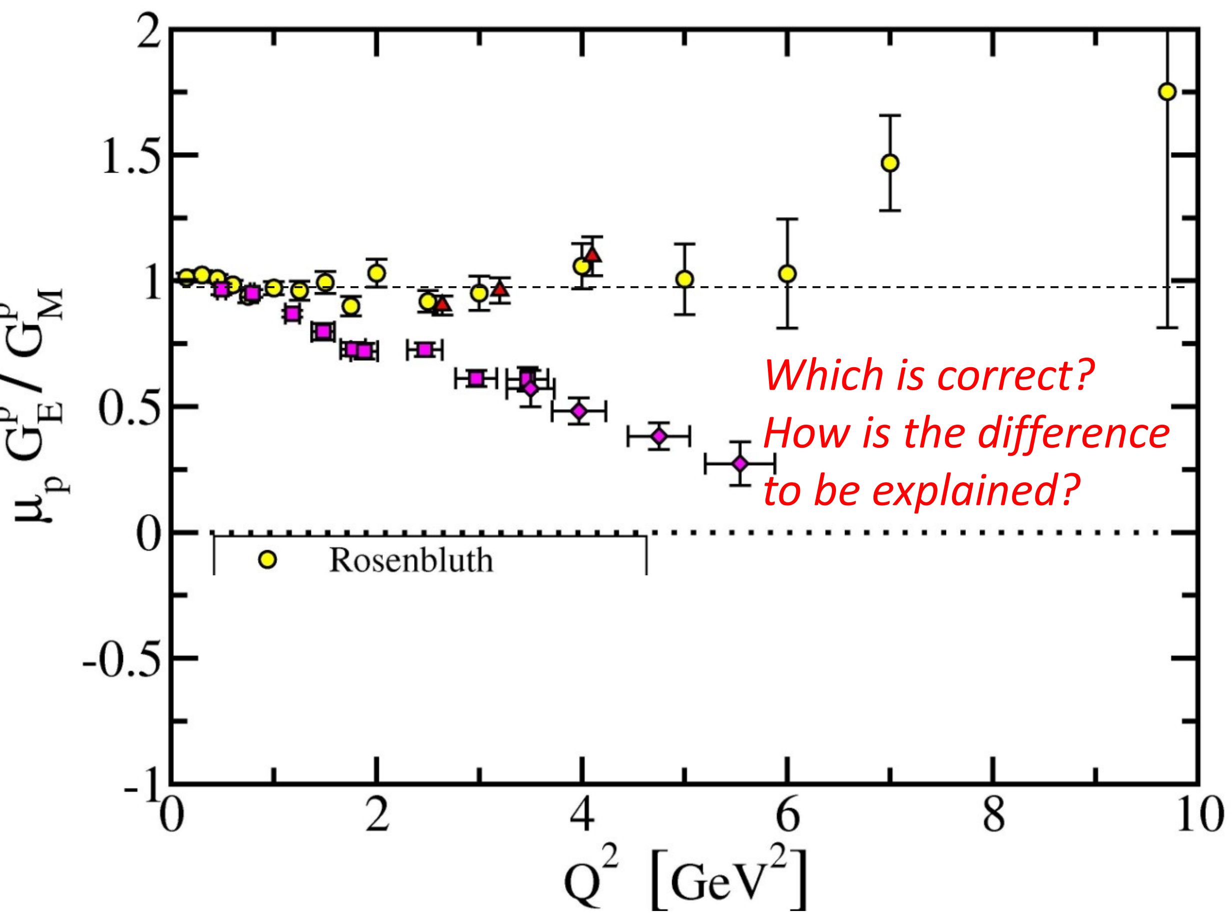


Completely unambigous!
Direct comparison between experiment and theory can distinguish between the momentum dependence of strong-interaction theory

M
dr
factors, which are readily distinguished from experiment

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

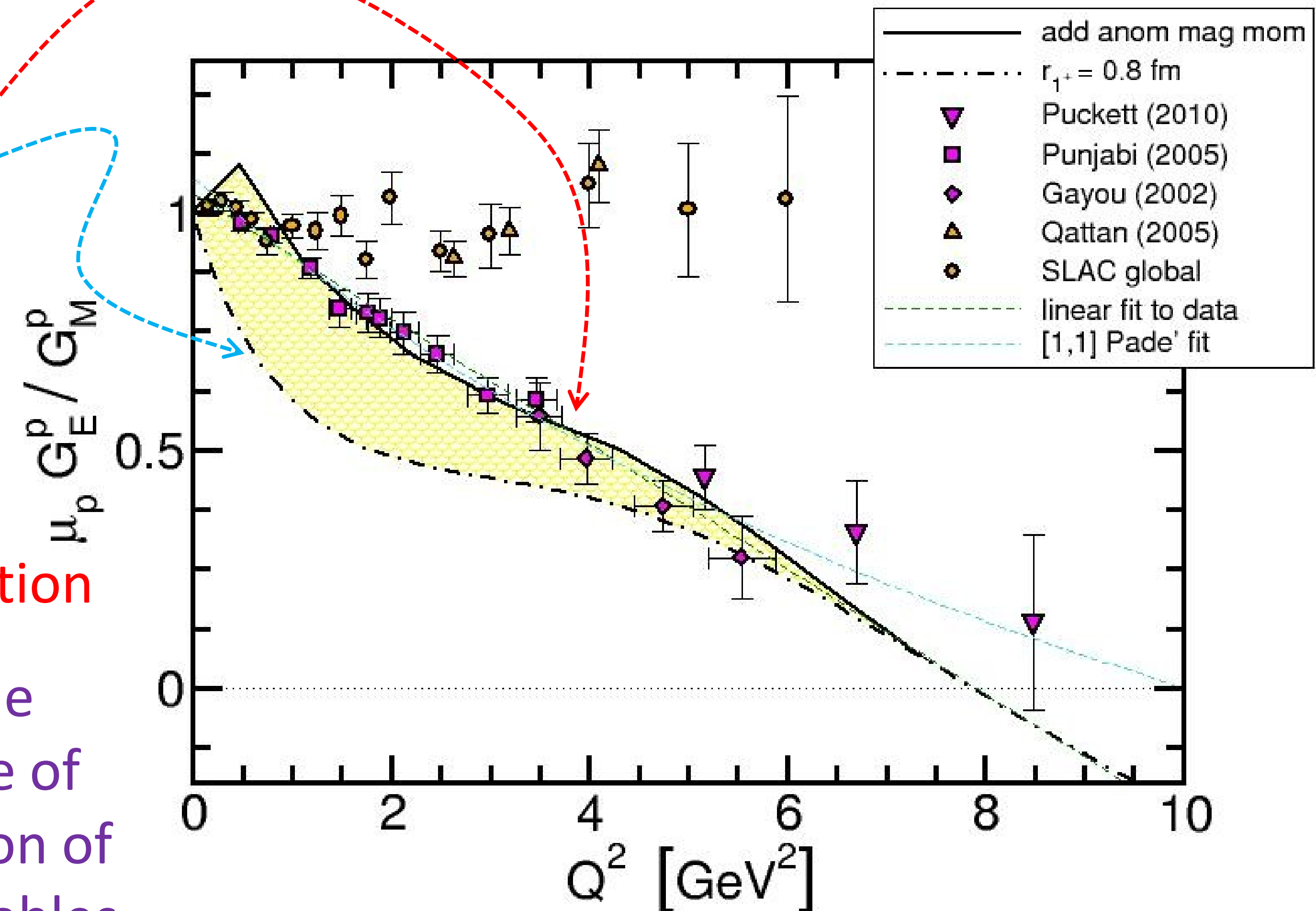
- Data before 1999
  - Looks like the structure of the proton is simple
- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE



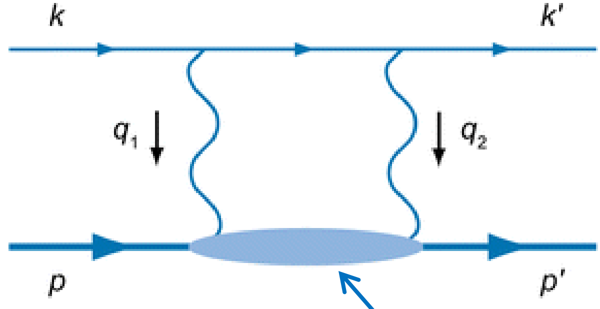
I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al*.
In progress

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

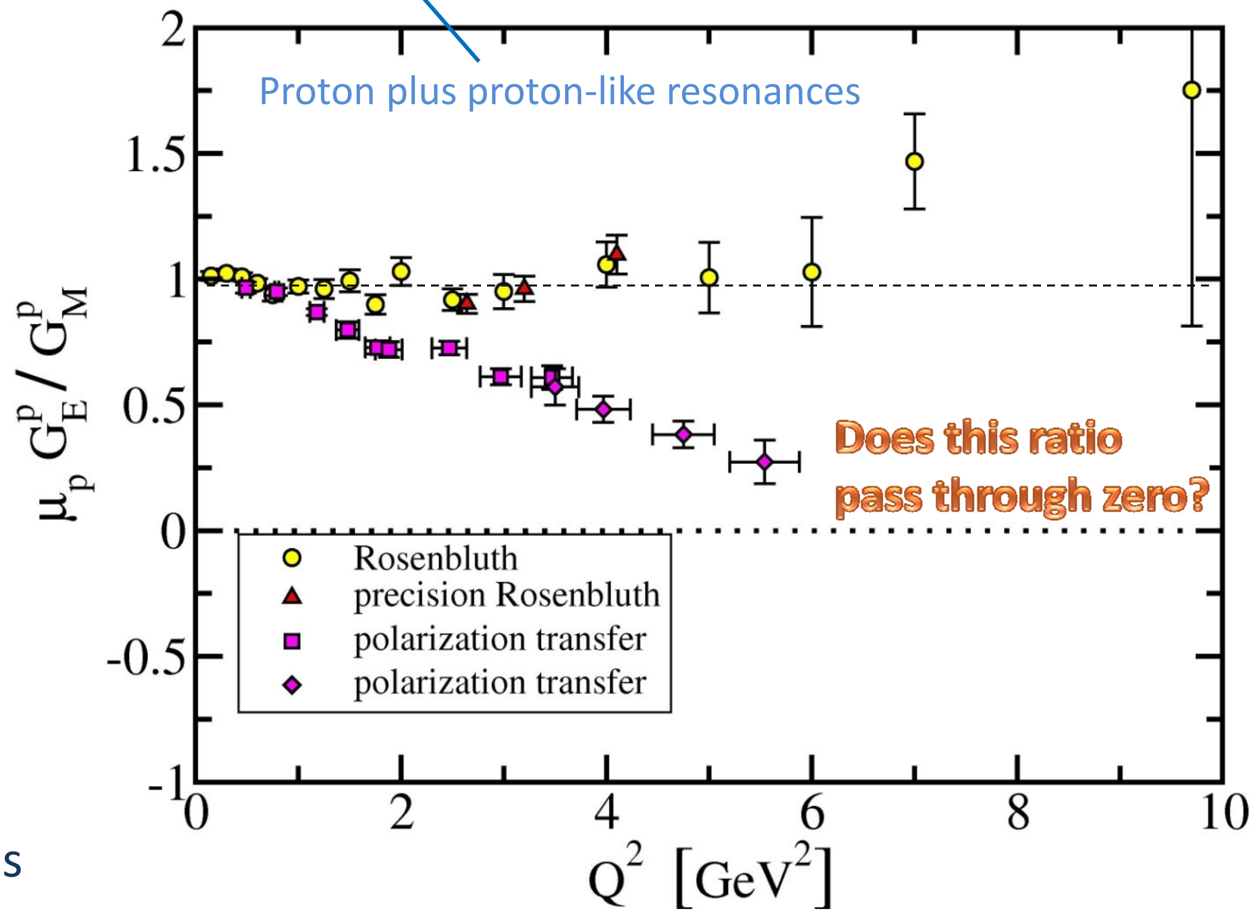
- DSE result Dec 08
- DSE result
  – including the anomalous magnetic moment distribution
- Highlights again the critical importance of DCSB in explanation of real-world observables.



$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$



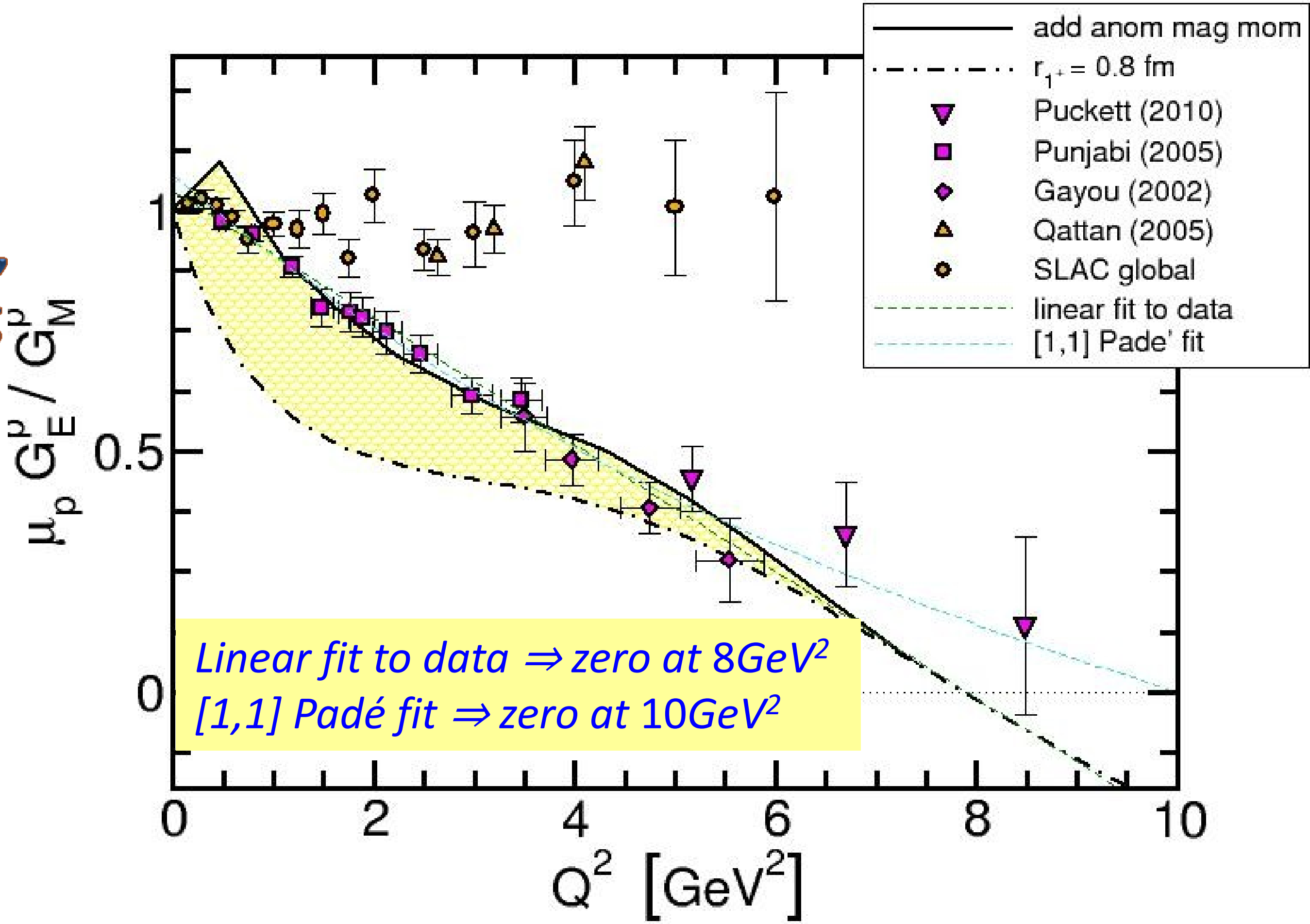
- DSE studies indicate that the *proton has a very rich internal structure*
- The JLab data, obtained using the polarisaton transfer method, are an accurate indication of the behaviour of this ratio
- The pre-1999 data (Rosenbluth) receive large corrections from so-called 2-photon exchange contributions



I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al*.
In progress

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

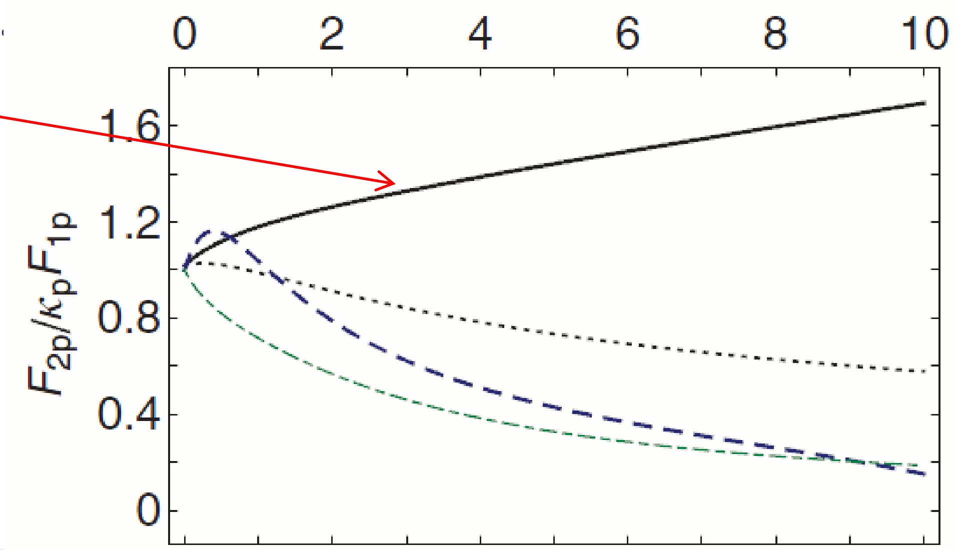
- **Does this ratio pass through zero?**
- **DSE studies say YES, with a zero crossing at *8*GeV^2, as a consequence of strong correlations within the nucleon**
- **Experiments at the upgraded JLab facility will provide the answer**

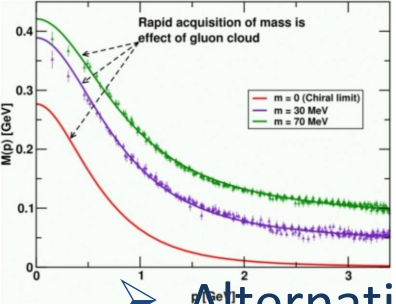


- **In the meantime, the DSE studies will be refined**

Origin of the zero & its location

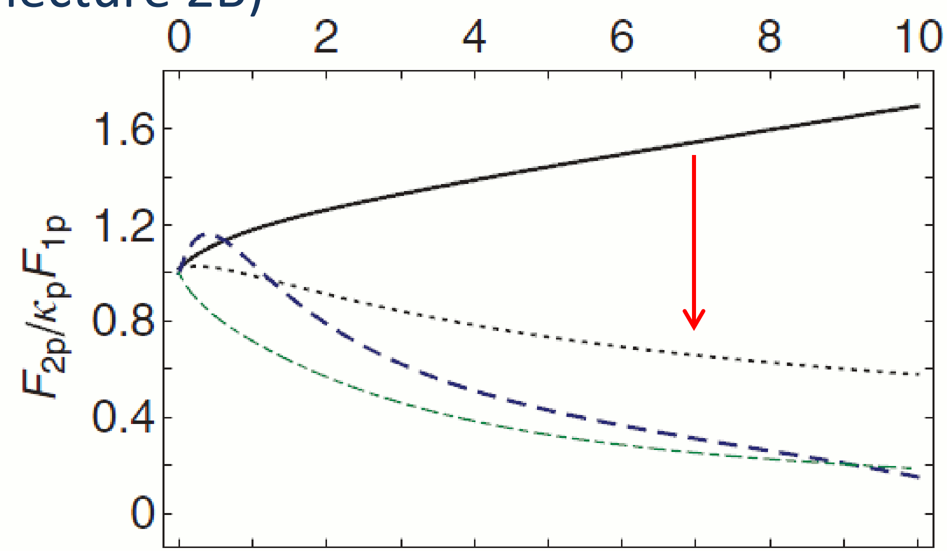
- The Pauli form factor is a gauge of the distribution of magnetization within the proton. Ultimately, this magnetisation is carried by the dressed quarks and influenced by correlations amongst them, which are expressed in the Faddeev wave function.
- If the dressed quarks are described by a momentum-independent mass function, *M*=constant, then they behave as Dirac particles with constant Dirac values for their magnetic moments and produce a hard Pauli form factor.





Origin of the zero & its location

- Alternatively, suppose that the dressed quarks possess a momentum-dependent mass function, $M=M(p^2)$, which is large at infrared momenta but vanishes as their momentum increases.
- At small momenta they will then behave as constituent-like particles with a large magnetic moment, but their mass and magnetic moment will drop toward zero as the probe momentum grows. (Remember: Massless fermions do not possess a measurable magnetic moment – lecture 2B)
- Such dressed quarks produce a proton Pauli form factor that is large for $Q^2 \sim 0$ but drops rapidly on the domain of transition between nonperturbative and perturbative QCD, to give a very small result at large Q^2.



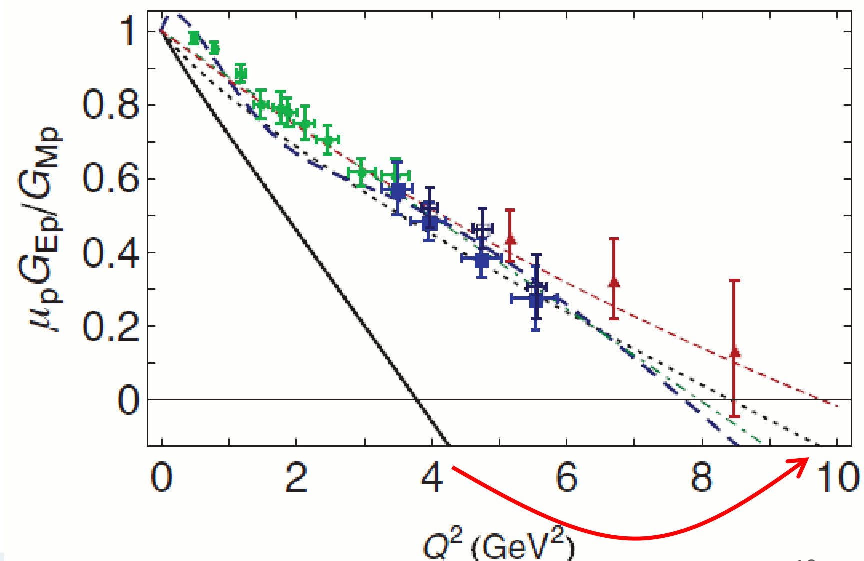
Origin of the zero & its location

- The precise form of the Q^2 dependence will depend on the evolving nature of the angular momentum correlations between the dressed quarks.
- From this perspective, existence, and location if so, of the zero in

  $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$

  are a fairly direct measure of the location and width of the transition region between the nonperturbative and perturbative domains of QCD as expressed in the momentum dependence of the dressed-quark mass function.
- Hard, M=constant

  → Soft, $M=M(p^2)$



Origin of the zero & its location

- One can anticipate that a mass function which rapidly becomes partonic—namely, is very soft—will not produce a zero
- We've seen that a constant mass function produces a zero at a small value of Q^2
- And also seen and know that a mass function which resembles that obtained in the best available DSE studies and via lattice-QCD simulations produces a zero at a location that is consistent with extant data.
- There is opportunity here for very constructive feedback between future experiments and theory.

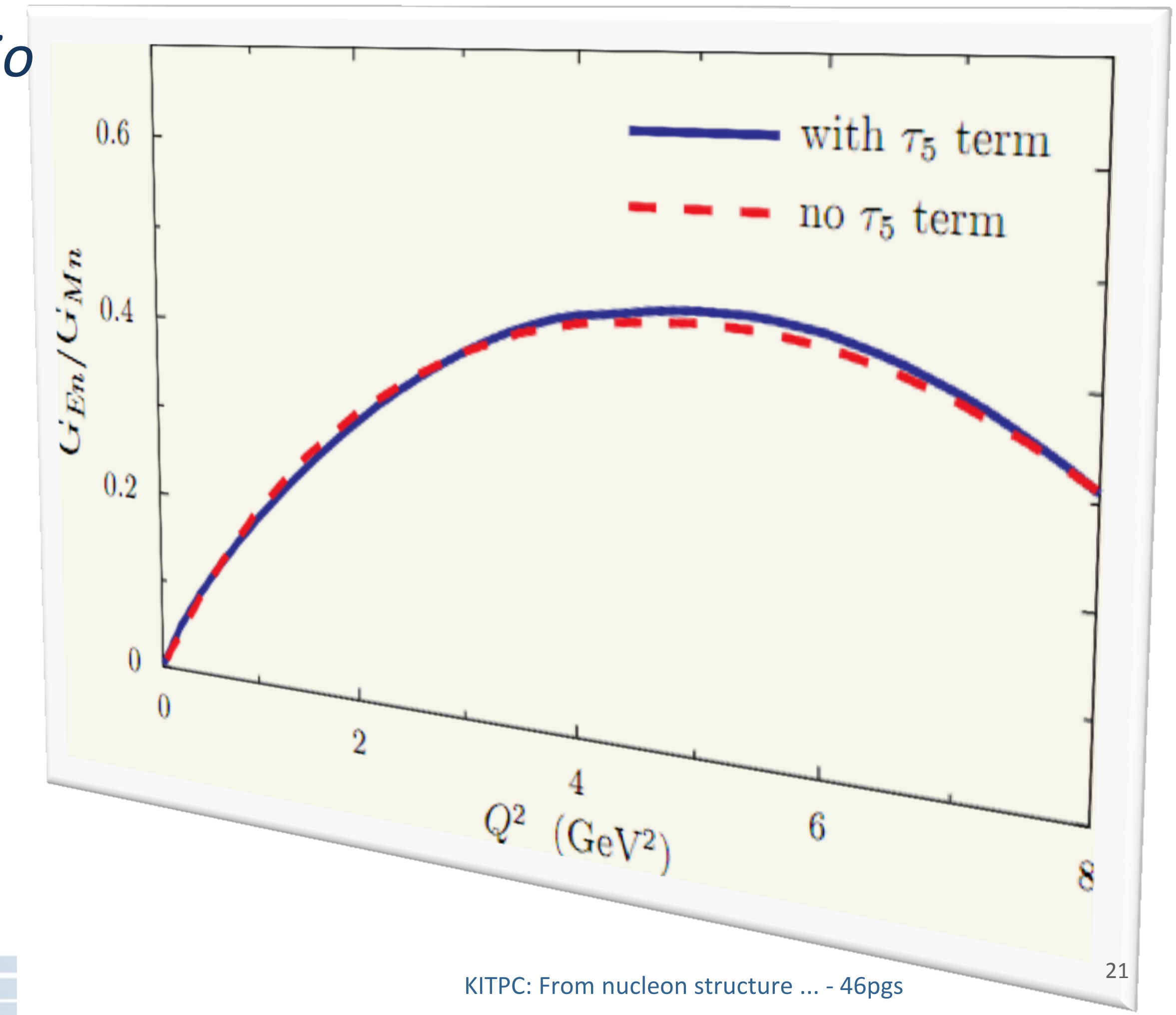
I.C. Cloët, C.D. Roberts, *et al.*
In progress

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

What about the same ratio for the neutron?

Quark anomalous magnetic moment has big impact on proton ratio

But little impact on the neutron ratio … because effect is focused near $Q^2=0$, at which G_{En} vanishes



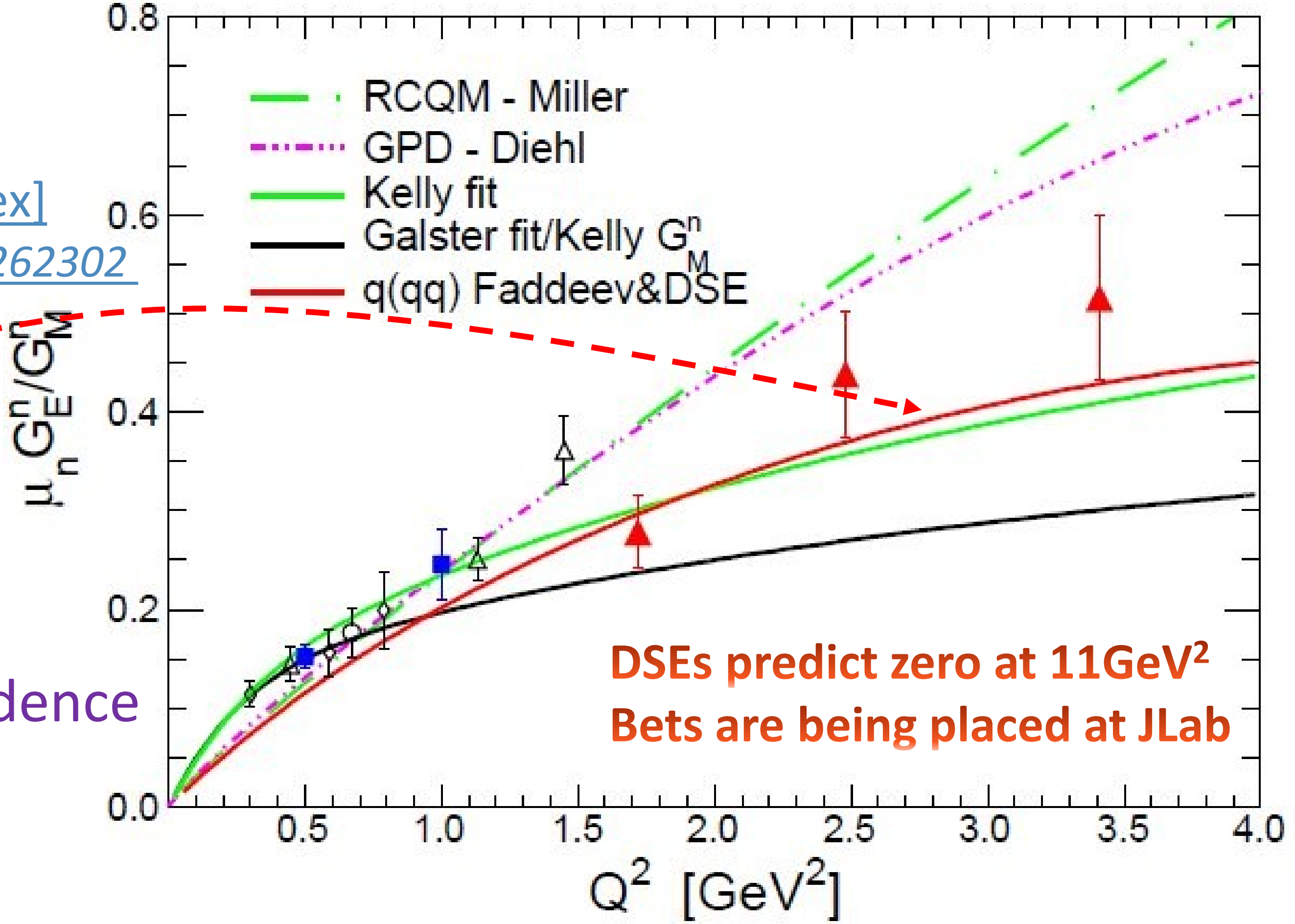
I.C. Cloët, C.D. Roberts, *et al*.
[arXiv:0812.0416 [nucl-th]]

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

- New JLab data: ▲
  S. Riordan *et al*.,
  arXiv:1008.1738 [nucl-ex]
  Phys.Rev.Lett. 105 (2010) 262302

- DSE-prediction

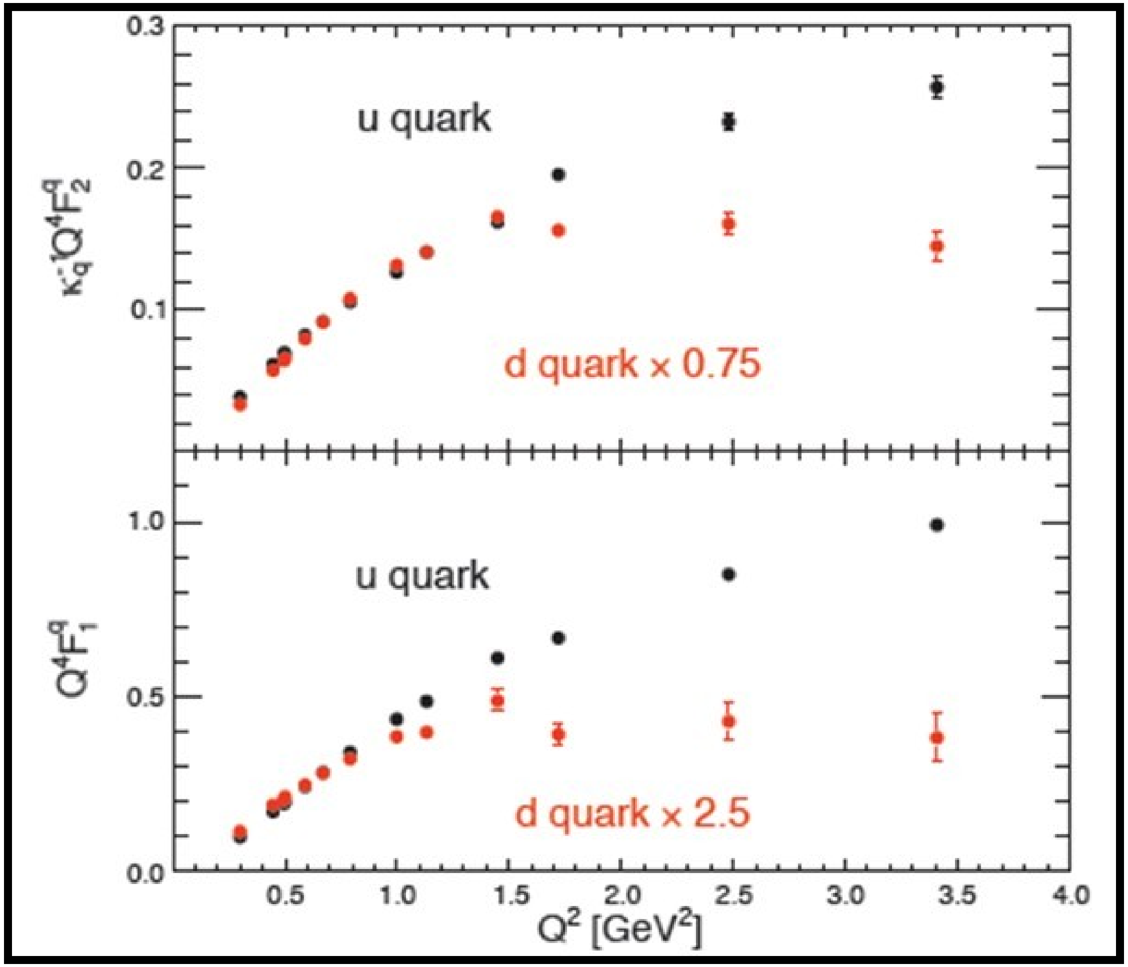
- This evolution is very sensitive to momentum-dependence of dressed-quark propagator



DSEs predict zero at 11GeV²
Bets are being placed at JLab

Flavor separation of proton form factors

$Q^4 F_2^q/\kappa$



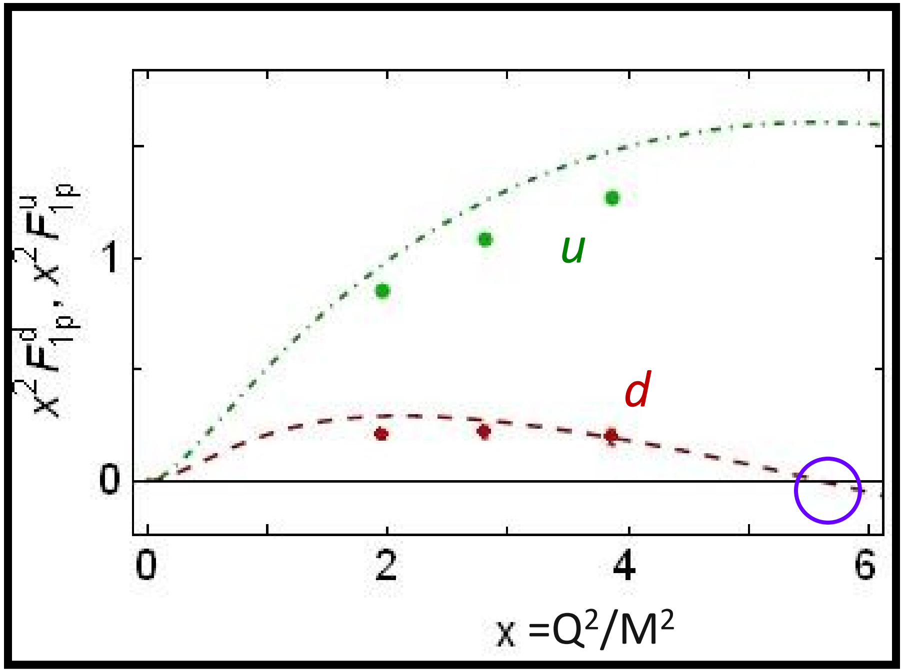
Cates, de Jager, Riordan, Wojtsekhowski, PRL 106 (2011) 252003

$Q^4 F_1^q$

- Very different behavior for *u* & *d* quarks
  Means apparent scaling in proton F2/F1 is *purely accidental*

Cloët, Eichmann, El-Bennich, Klähn, Roberts, Few Body Syst. 46 (2009) pp.1-36
Wilson, Cloët, Chang, Roberts, PRC 85 (2012) 045205

Diquark correlations!



- Poincaré covariant Faddeev equation
  - Predicts scalar and axial-vector diquarks
- Proton's singly-represented *d*-quark more likely to be struck in association with *1⁺* diquark than with *0⁺*
  - form factor contributions involving *1⁺* diquark are softer
- Doubly-represented *u*-quark is predominantly linked with harder *0⁺* diquark contributions
- Interference produces zero in Dirac form factor of *d*-quark in proton
  - Location of the zero depends on the relative probability of finding 1⁺ & 0⁺ diquarks in proton
  - Correlated, e.g., with valence *d/u* ratio at x=1

Nucleon Structure Functions

- Moments method will work here, too.
- Work about to begin: *Cloët, Roberts & Tandy*
  - Based on predictions for nucleon elastic form factors, which, e.g., predicted large-Q^2 behavior of $G_E^n(Q^2)$:

    Survey of nucleon electromagnetic form factors
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
    arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36
- Meantime, capitalise on connection between $x=1$ and $Q^2=0$ …

I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) 1-36
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Neutron structure function at high-x

- Valence-quark distributions at *x=1*
  - Fixed point under DGLAP evolution
  - Strong discriminator between models

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1}, \quad \text{where} \quad \frac{d_v(x)}{u_v(x)} = \frac{4\frac{F_2^n(x)}{F_2^p(x)} - 1}{4 - \frac{F_2^n(x)}{F_2^p(x)}}$$

- Algebraic formula

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1} = \frac{P_1^{p,d}}{P_1^{p,u}} = \frac{\frac{2}{3}P_1^{p,a} + \frac{1}{3}P_1^{p,m}}{P_1^{p,s} + \frac{1}{3}P_1^{p,a} + \frac{2}{3}P_1^{p,m}}$$

  - $P_1^{p,s}$ = contribution to the proton's charge arising from diagrams with a scalar diquark component in both the initial and final state
  - $P_1^{p,a}$ = kindred axial-vector diquark contribution
  - $P_1^{p,m}$ = contribution to the proton's charge arising from diagrams with a different diquark component in the initial and final state.

I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) 1-36
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Neutron structure function at high-x

- Algebraic formula

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1} = \frac{P_1^{p,d}}{P_1^{p,u}} = \frac{\frac{2}{3}P_1^{p,a} + \frac{1}{3}P_1^{p,m}}{P_1^{p,s} + \frac{1}{3}P_1^{p,a} + \frac{2}{3}P_1^{p,m}}$$

Contact interaction

	$P_1^{p,s}$	$P_1^{p,a}$	$P_1^{p,m}$	$\frac{d_v}{u_v}$	$\frac{F_2^n}{F_2^p}$
M = constant	0.78	0.22	0	0.18	0.41
$M(p^2)$	0.60	0.25	0.15	0.28	0.49

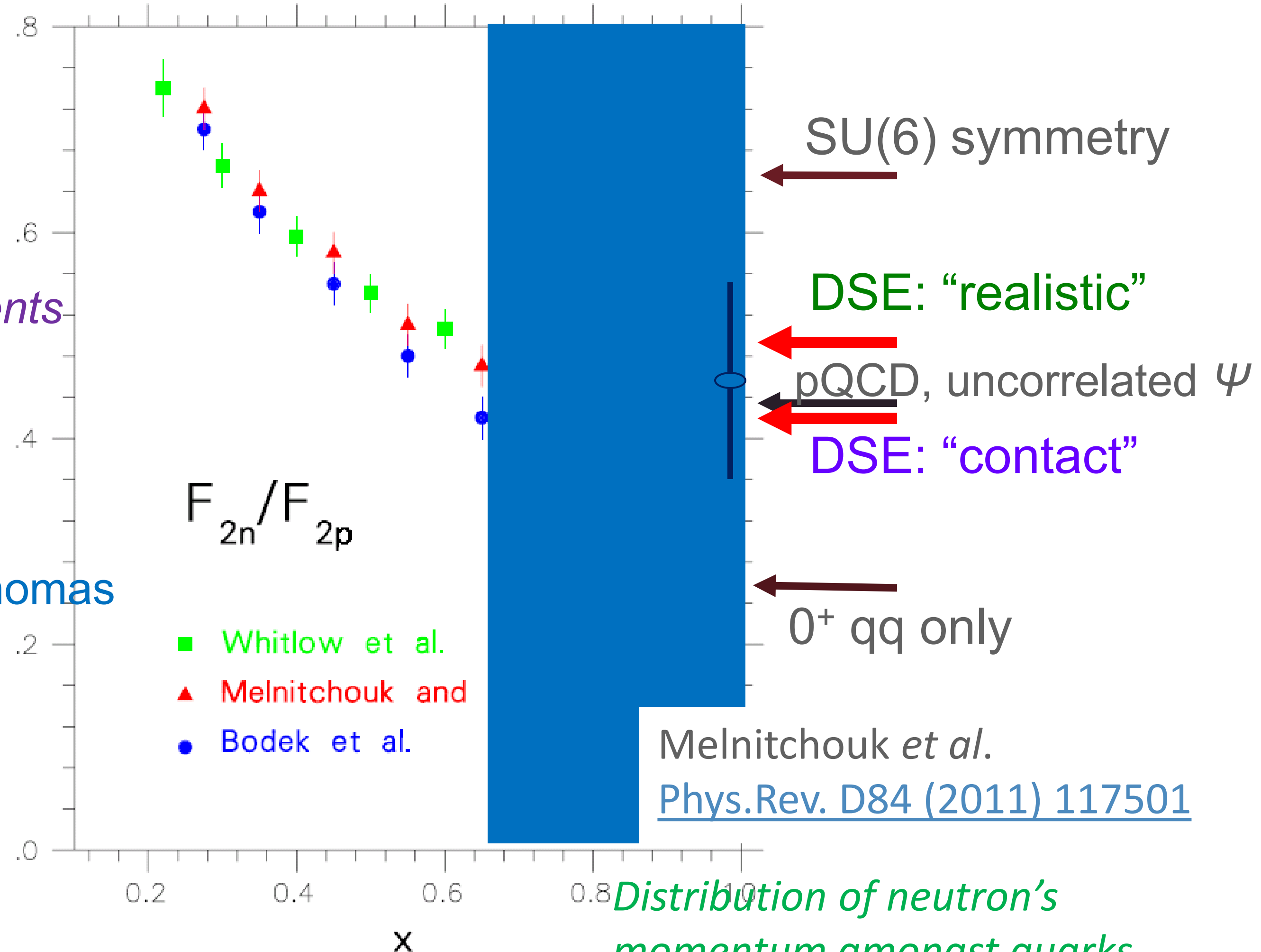
“Realistic” interaction

I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Neutron Structure Function at high x

Deep inelastic scattering – the Nobel-prize winning quark-discovery experiments

Reviews:
- S. Brodsky *et al.* NP B441 (1995)
- W. Melnitchouk & A.W.Thomas PL B377 (1996) 11
- N. Isgur, PRD 59 (1999)
- R.J. Holt & C.D. Roberts RMP (2010)



Melnitchouk *et al.*
Phys.Rev. D84 (2011) 117501

Distribution of neutron's momentum amongst quarks on the valence-quark domain

Nucleon to Excited-Nucleon Transition Form Factors

- Return to the Roper resonance.
  - High-quality data on nucleon to Roper transition form factor, from CLAS-Collaboration, is a challenge to theory
- As indicated in the latest Decadal Report and explicated via preceding transparencies, an understanding of the Roper is emerging through a constructive interplay between dynamical coupled-channels models and hadron structure calculations, particularly the symmetry preserving studies made using the tower of DSEs
- One must probe and further elucidate the possibility that πN final-state interactions play a critical role in understanding the Roper, through a simultaneous computation within the DSE framework of nucleon and Roper elastic form factors and the form factors describing the nucleon-to-Roper transition.

Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

Baryon Spectrum
diquark content

Fascinating suggestion:
nucleon's first excited state = almost entirely axial-vector diquark, despite the dominance of scalar diquark in ground state. In fact, because of it.

	m_N	m_{N^*}	$m_{N\frac{1}{2}^-}$	$m_{N^*\frac{1}{2}^-}$	m_Δ	m_{Δ^*}	$m_{\Delta\frac{3}{2}^-}$	$m_{\Delta^*\frac{3}{2}^-}$
PDG label	N	$N(1440)\,P_{11}$	$N(1535)\,S_{11}$	$N(1650)\,S_{11}$	$\Delta(1232)\,P_{33}$	$\Delta(1600)\,P_{33}$	$\Delta(1700)\,D_{33}$	$\Delta(1940)\,D_{33}$
This work	1.14	1.82±0.07	2.22	2.29 ± 0.02	1.39	1.85 ± 0.05	2.25	2.33 ± 0.02
EBAC		1.76	1.80	1.88	1.39	...	1.98	...
Jülich	1.24	none	2.05	1.92	1.46	...	2.25	...

Diquark content of bound state

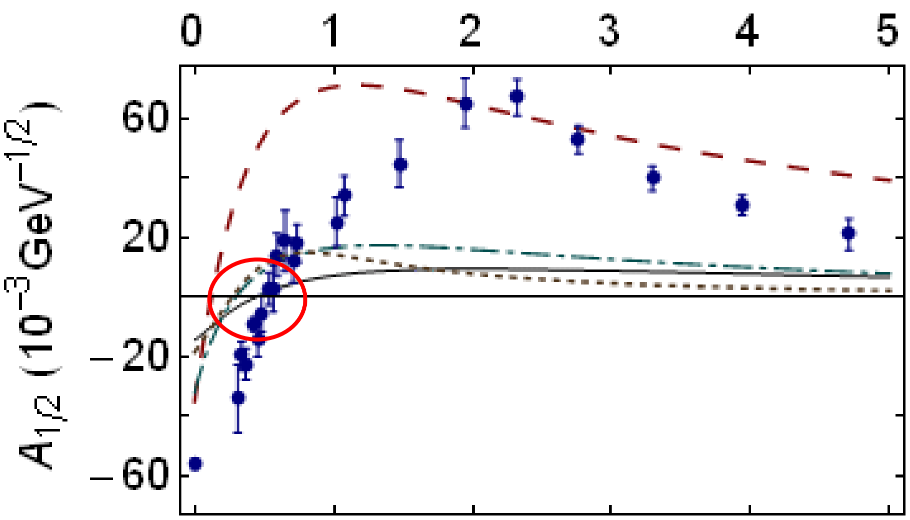
0^+	77%							
1^+	23%	100%			100%	100%		
0^-			51%	43%				
1^-			49%	57%			100%	100%

Owes fundamentally to close relationship between scalar diquark & pseudoscalar meson & this follows from dynamical chiral symmetry breaking

Nucleon to Roper Transition Form Factors

- Extensive CLAS @ JLab Programme has produced the first measurements of nucleon-to-resonance transition form factors

  I. Aznauryan et al.,
  Results of the N Program at JLab*
  [arXiv:1102.0597 [nucl-ex]](arXiv:1102.0597 [nucl-ex])



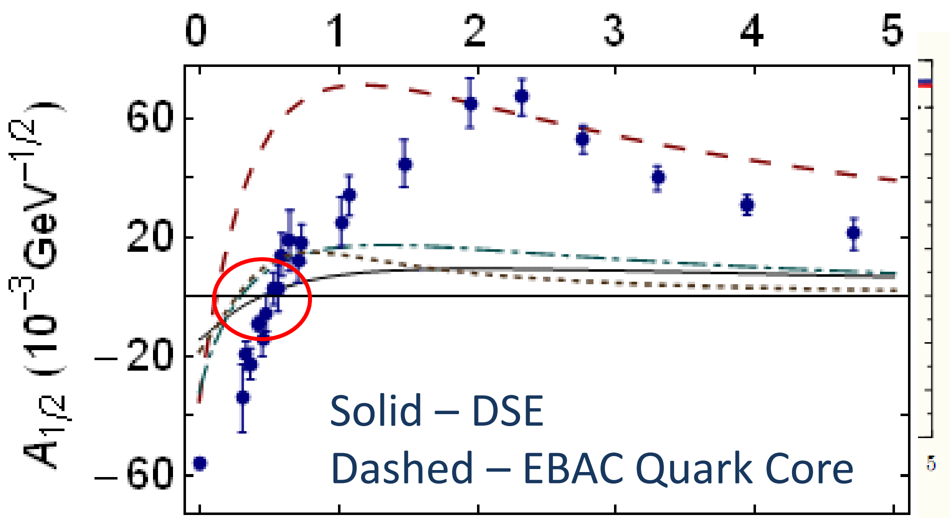
- Theory challenge is to explain the measurements
- Notable result is zero in $F_2^{p\to N*}$, explanation of which is a real challenge to theory.
- First observation of a zero in a form factor

Nucleon to Roper Transition Form Factors

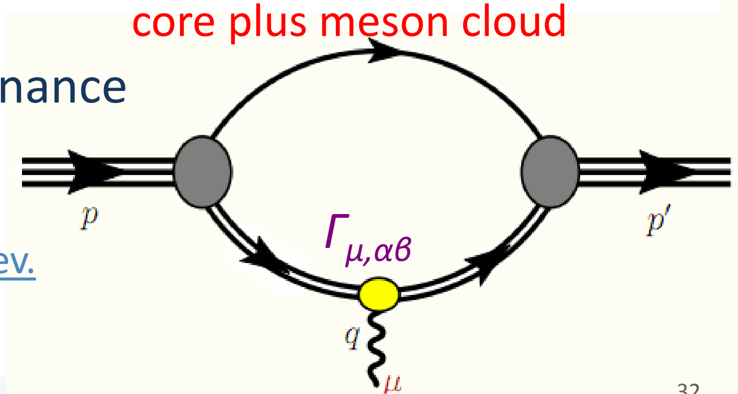
- Extensive CLAS @ JLab Programme has produced the first measurements of nucleon-to-resonance transition form factors

  I. Aznauryan et al.,
  Results of the N Program at JLab*
  [arXiv:1102.0597 [nucl-ex]]

- Theory challenge is to explain the measurements
- Notable result is zero in $F_2^{p\to N*}$, explanation of which is a real challenge to theory.
- DSE study connects appearance of zero in $F_2^{p\to N*}$ with
  - ✓ axial-vector-diquark dominance in Roper resonance
  - ✓ and structure of form factors of *J=1* state



Near match supports picture of Roper as quark core plus meson cloud

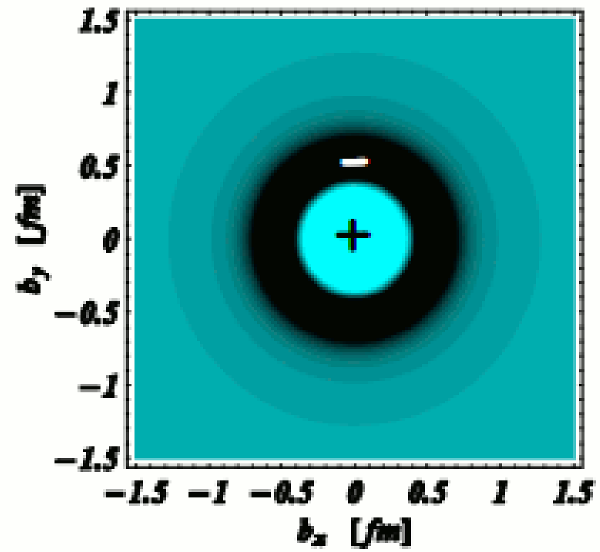


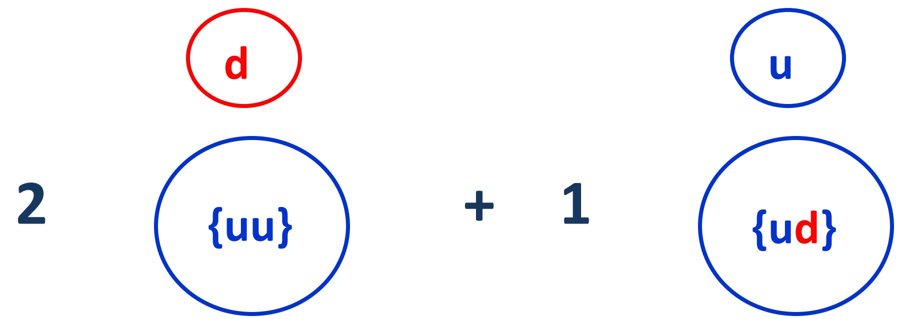
Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, [Phys. Rev. C85 (2012) 025205 [21 pages]]

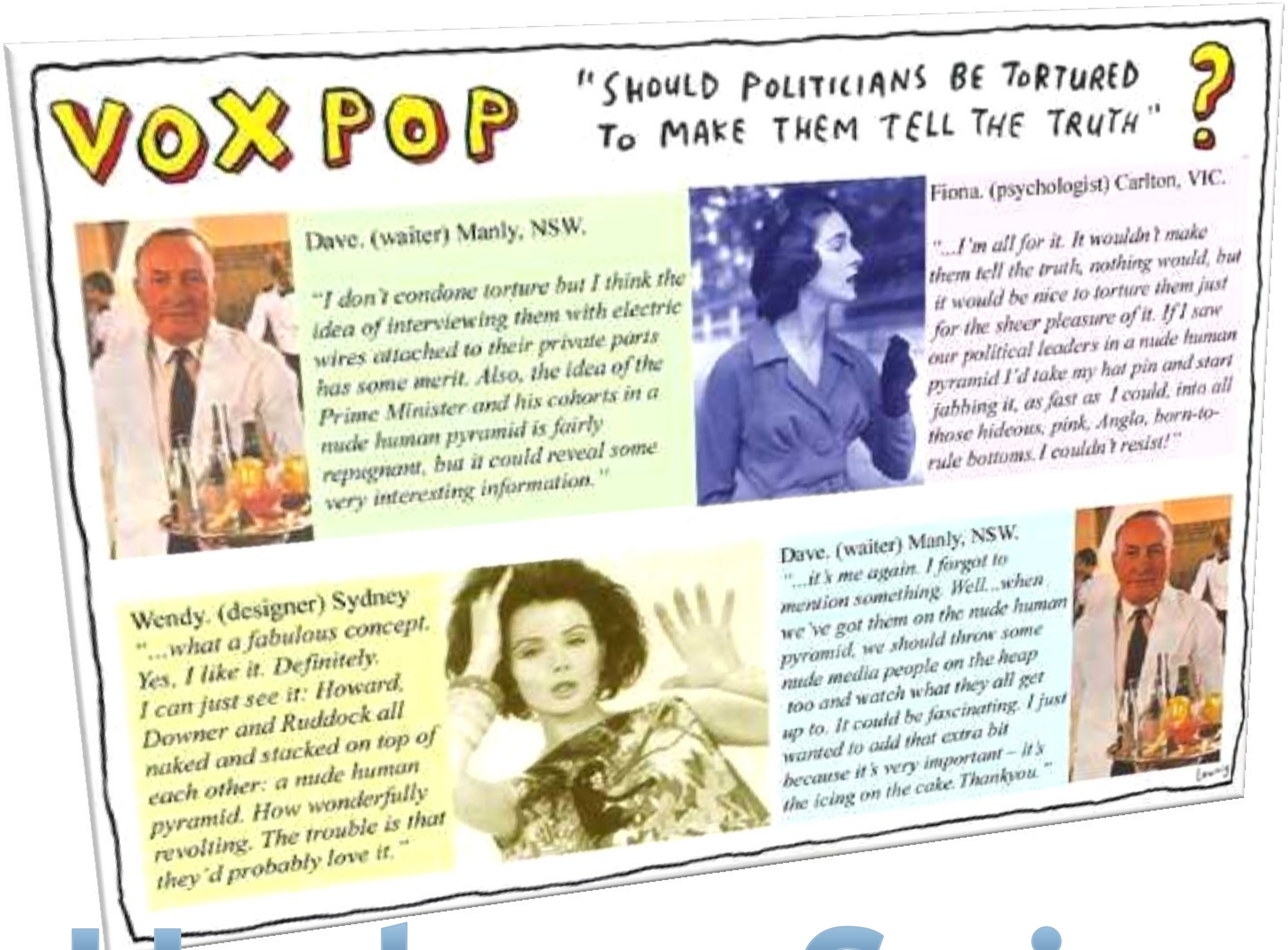
Nucleon to Roper Transition Form Factors

- Tiator and Vanderhaeghen – *in progress*
  - Empirically-inferred *light-front-transverse* charge density
  - Positive core *plus* negative annulus
- *Readily explained by dominance of axial-vector diquark configuration in Roper*
  - Considering isospin and charge
    Negative *d*-quark twice as likely to be delocalised from the always-positive core than the positive *u*-quark

p -> N* (1440)






VOX POP
"SHOULD POLITICIANS BE TORTURED TO MAKE THEM TELL THE TRUTH"?
Dave. (waiter) Manly, NSW.
"I don't condone torture but I think the idea of interviewing them with electric wires attached to their private parts has some merit. Also, the idea of the Prime Minister and his cohorts in a nude human pyramid is fairly repugnant, but it could reveal some very interesting information."
Fiona. (psychologist) Carlton, VIC.
"...I'm all for it. It wouldn't make them tell the truth, nothing would, but it would be nice to torture them just for the sheer pleasure of it. If I saw our political leaders in a nude human pyramid I'd take my hat pin and start jabbing it, as fast as I could, into all those hideous, pink, Anglo, born-to-rule bottoms. I couldn't resist!"
Wendy. (designer) Sydney
"...what a fabulous concept. Yes, I like it. Definitely. I can just see it: Howard, Downer and Ruddock all naked and stacked on top of each other: a nude human pyramid. How wonderfully revolting. The trouble is that they'd probably love it."
Dave. (waiter) Manly, NSW.
"...it's me again. I forgot to mention something. Well...when we've got them on the nude human pyramid, we should throw some nude media people on the heap too and watch what they all get up to. It could be fascinating. I just wanted to add that extra bit because it's very important – it's the icing on the cake. Thankyou."

Hadron Spin

Hadron Spin

- Relativistic quantum mechanics
  10 generators of the Poincaré algebra
  rotations = 3, boosts = 3, translations = 4
  but only two Casimir operators constructed therefrom:
  - P^2 & W^2

  where W is the Pauli-Lubanski operator

  $$W_\mu = \varepsilon_{\mu\alpha\beta\lambda} J^{\alpha\beta} P^\lambda \,, \quad W^2|M,J\rangle = M^2 J(J+1)$$

  with $J^{\alpha\beta}$ the Noether charge for rotations and boosts

  and P^λ is the four-momentum operator

Itzykson and Zuber, Secs. 1-2-2 & 2-1-3

- Hence, Poincaré invariance entails that an isolated hadron is characterised by two properties; namely, eigenvalues –
  - Eigenvalue of P^2 = Mass-squared, M^2
  - Eigenvalue of W^2 = Total angular momentum, J^2

  Truly Poincaré invariants

  States are represented by ket-vectors: |M,J>
- Pion = |m_π, 0>, Nucleon = |m_N,½>

Hadron Spin

- Questions:
  - How is the total *J* distributed over the hadron's constituents and in what form?
- In answering these questions, one must specify a reference frame because $J=L+S$, and whilst J^2 is Poincaré invariant, that is not true of L^2 and S^2 separately.
  - Compare this with nonrelativistic quantum mechanics, where states of the hydrogen atom are labelled by angular momentum, *L*, and its projection on the z-axis, *m*.
  - NB. Spin is truly a relativistic concept and understanding hyperfine splitting requires quantum electrodynamics
- Consider the pion: $|m_\pi, 0>$

  Here $J=0$, so what is the question?

Pion spin

- To study *J*, one must consider the pion's Bethe-Salpeter wave function; i.e., the quantity that can be connected with the Schroedinger wave function when a nonrelativistic limit is possible

$$\begin{aligned}\chi_\pi(k;P) &= S(k_+)\Gamma_\pi(k;P)S(k_-)\\ &= \gamma_5\left[i\mathcal{E}_\pi(k;P) + \gamma\cdot P\,\mathcal{F}(k;P) + \gamma\cdot k\,k\cdot P\,\mathcal{G}_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\,H_\pi(k;P)\right]\end{aligned}$$

- Return to the Pauli-Lubanski operator

$$W_\mu = \varepsilon_{\mu\alpha\beta\lambda}\, J^{\alpha\beta}\, P^\lambda$$

- X_π is an eigenstate of W^2 with eigenvalue $J=0$.
  This is not news.
- However, the pion is a composite, with constituents that possess spin. Wave function might be nontrivial.

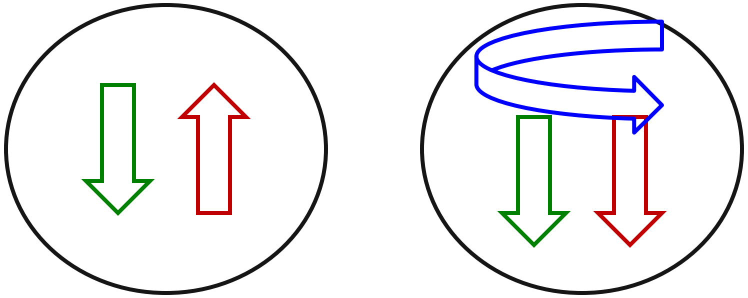
Pion spin

- Consider the rest frame, wherein the Pauli-Lubanski operator for an isolated hadron simplifies:

  $W^0 = 0$, $\boldsymbol{W} = \tfrac{1}{2} M \gamma^5 \gamma^0 \boldsymbol{\gamma} = \tfrac{1}{2} M \boldsymbol{\Sigma}$,

  $$\Sigma = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

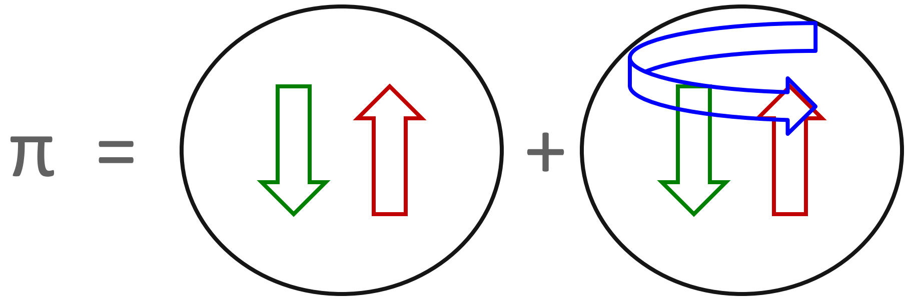
  viz., this is the usual spin operator
- In the rest frame of a composite, particle+antiparticle system, *W* separates into a sum of two terms, *W=L+S*, and in this frame one can show
  - E_π, F_π correspond to *L=0, S=0, L+S=0* …
  - G_π, H_π correspond to *L=1, S=1, L+S=0* …



- In vector exchange theories with interactions that possess nontrivial momentum dependence, G_π, H_π are necessarily nonzero.

Pion spin

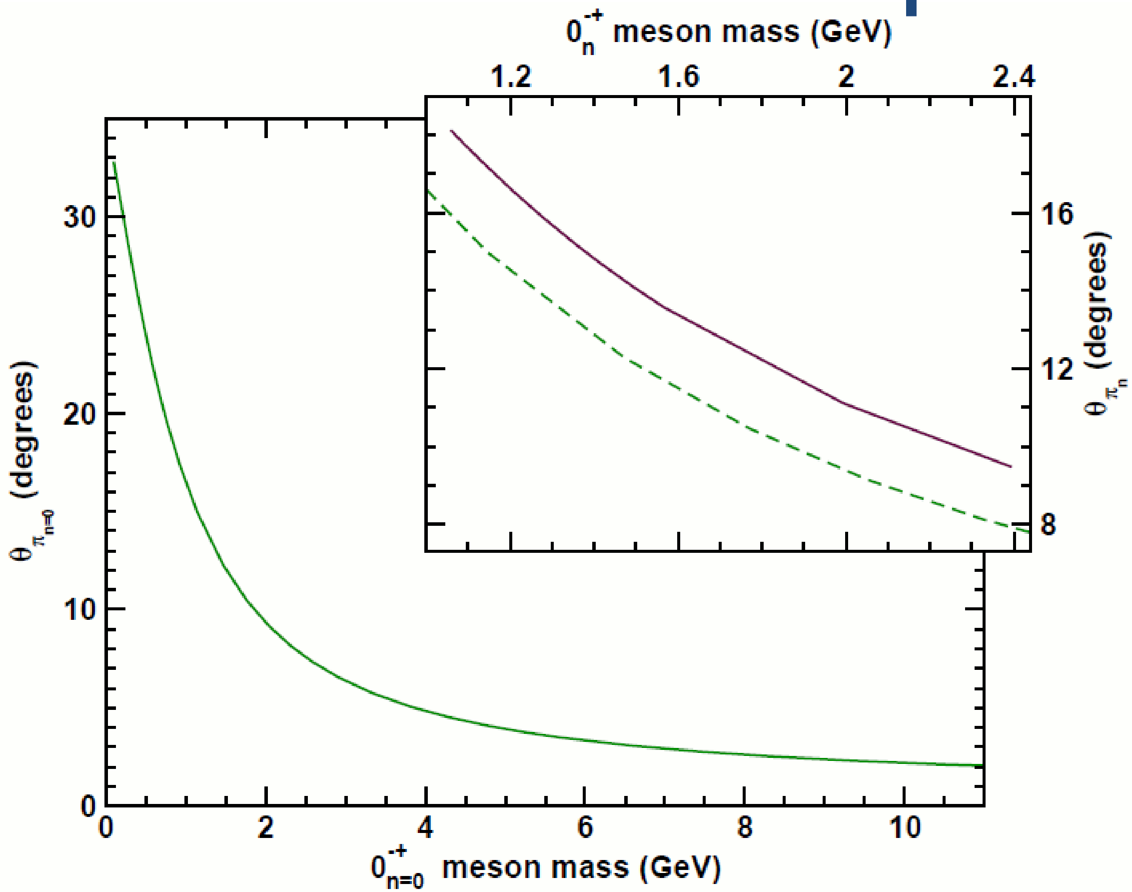
- This means that pseudoscalar mesons necessarily contain P-wave (*L=1*) components in their rest frame wave function and hence in (almost) every frame.
- Indeed, it is *impossible* for the pion's spin simply to result from the sum of the spins of its constituents
  - That would violate Poincaré covariance



Pion spin

Mind the gap - Bhagwat, Krassnigg, Maris, Roberts, nucl-th/0612027, Eur.Phys. J. A**31** (2007) 630-637

- Explicit calculation
  - A Bethe-Salpeter amplitude is canonically normalised.
  - There are sixteen distinguishable terms in the associated sum; viz., an $E_\pi E_\pi$ contribution plus an $E_\pi F_\pi$ contribution, etc.
  - In the sum of the squares of these terms we associate sinΘ with the nondiagonal contributions, in which case sinΘ gauges the role played by *L = 1*



By this measure, the pion is roughly 25% *L=1*

Angular momentum component decreases with increasing current-quark mass – for obvious reasons; viz., the quarks are getting heavier and they can't orbit as easily within the confines of the hadron!

Physics Letters B

Volume 206, Issue 2, 19 May 1988, Pages 364–370

ELSEVIER

A measurement of the spin asymmetry and determination of the structure function g_1 in deep inelastic muon-proton scattering

European Muon Collaboration

Proton Spin Crisis

Proton Spin Crisis

- The preceding observations, *so straightforward*, make extremely puzzling the furore that a erupted 25 years ago …

  "The European Muon Collaboration made a startling discovery: only a portion of a proton's spin comes from the quarks that make up the proton.

  The revelation was a bit of a shock for physicists who had believed that the spin of a proton could be calculated simply by adding the spin states of the three constituent quarks.

  This is often described as the "proton spin crisis."
- Perhaps the high-energy physicists should have asked a nuclear physicist? Correlations have long been known in nuclei – but that would have meant a particle physicist thinking about something other than perturbation theory …

Proton Spin

- Faddeev wave function for the *J=½* nucleon is expressed through eight scalar functions
  - no more are needed, no number fewer is complete.
- Two are associated with the 0^+ diquark correlation: $S_{1,2}$, and six with the 1^+ correlation: $A_{1,\ldots,6}$.
- In the rest frame, one can derive the following "good" angular momentum and spin assignments, which add vectorially to give *J=½*

S-wave: quark carries all nucleon spin

Two types of P-wave: with L either aligned or opposed to nucleon spin

D-wave: all quark spins are antiparallel to that of nucleon

$L=0\,,S=\frac{1}{2}$	$L=1\,,S=\frac{1}{2}$	$L=1\,,S=\frac{3}{2}$	$L=2\,,S=\frac{3}{2}$
$\mathcal{S}_1\,,\mathcal{A}_2\,,\mathcal{B}_1$	$\mathcal{S}_2\,,\mathcal{A}_1\,,\mathcal{B}_2\,,$	$\mathcal{C}_2$	$\mathcal{C}_1$

$$\begin{aligned} B_1 &= \tfrac{1}{3}A_3+\tfrac{2}{3}A_5\,, & B_2 &= \tfrac{1}{3}A_4+\tfrac{2}{3}A_6\,,\\ C_1 &= A_3-A_5\,, & C_2 &= A_4-A_6\,. \end{aligned}$$

Craig Roberts: The Issue of Substance in Hadron Physics (83)

Dynamics, symmetries and hadron properties.
I.C. Cloet, A. Krassnigg, C.D. Roberts
arXiv:0710.5746 [nucl-th], MENU-2007 eConf C070910

Proton Spin

- To exhibit the importance of the various *L-S* correlations within the nucleon's Faddeev wave-function, one may report the breakdown of contributions to the nucleon's canonical normalization, which is equivalent to it's Dirac form factor at $Q^2=0$:

	$\mathcal{S}_1$	$\mathcal{A}_2$	$\mathcal{B}_1$	$\mathcal{S}_2$	$\mathcal{A}_1$	$\mathcal{B}_2$	$\mathcal{C}_2$	$\mathcal{C}_1$
$\mathcal{S}_1$	0.62	−0.01	0.07	0.25				−0.02
$\mathcal{A}_2$	−0.01		−0.06		0.05	0.04	0.02	−0.16
$\mathcal{B}_1$	0.07	−0.06	−0.01		0.01	0.13	−0.01	
$\mathcal{S}_2$	0.25			0.06				
$\mathcal{A}_1$		0.05	0.01			−0.07	−0.07	0.02
$\mathcal{B}_2$		0.04	0.13		−0.07	−0.10	−0.02	0.13
$\mathcal{C}_2$		0.02	−0.01		−0.07	−0.02	−0.11	0.37
$\mathcal{C}_1$	−0.02	−0.16			0.02	0.13	0.37	−0.15

Dynamics, symmetries and hadron properties.
I.C. Cloet, A. Krassnigg, C.D. Roberts
arXiv:0710.5746 [nucl-th], MENU-2007 eConf C070910

Proton Spin

- Largest single entry is associated with $S_1 \otimes S_1$, which represents the quark outside the scalar diquark correlation carrying all the nucleon's spin. That is the u-quark in the proton.
- However, it is noteworthy that a contribution of similar magnitude is associated with the axial-vector diquark correlations, expressing mixing between P- and D-waves; viz., $C_1 \otimes C_2 + C_2 \otimes C_1$.
  - With C_2 all quark spins are aligned with that of the nucleon and the unit of angular momentum is opposed, whilst with C_1 all quark spins are opposed and the two units of angular momentum are aligned.
  - This contribution is more important than those associated with S_2; namely, scalar diquark terms with the bystander quark's spin antiparallel.
- *Finally, one single number is perhaps most telling: the contribution to the normalization from $(L = 0) \otimes (L = 0)$ terms is only 37% of the total.*

Dynamics, symmetries and hadron properties.
I.C. Cloet, A. Krassnigg, C.D. Roberts
arXiv:0710.5746 [nucl-th], MENU-2007 eConf C070910

Proton Spin

- The first EMC result was that the proton's spin received zero contribution from the quark spins
  - *That was a bit extreme*
- Modern experiments indicate that, in the light-front frame, quarks contribute approximately 33% of the nucleon's spin
- The rest-frame result described previously as 37% provided by the dressed-quarks, with the remainder expressed in correlations within the nucleon's Faddeev wave function
  - More to be done in order to establish quantitative connection with the light-front result
- *Notwithstanding this, qualitatively and semiquantitatively there is not and never was a proton spin crisis*

Weak interactions of strong-interaction bound-states

Weak interactions of strong-interaction bound-states: nucleon's axial charge

- The prototypical weak interaction is nuclear β^- decay, which explains the instability of neutron-rich nuclei and proceeds via the transition

  $n \to p + e^- +$ anti-ν_e

- Fermi made first attempt at its explanation, using a contact current-current interaction, modulated by a constant

  $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$.

- Electroweak gauge theory replaces the contact interaction by exchange of a heavy gauge boson and expresses

  $G_F \sim g^2 / M_W^2$,

  where $M_W \sim 80$ GeV is the gauge-boson's mass and g is a universal dimensionless coupling; namely, it is the same for all interactions between gauge-bosons, leptons and current-quarks.

Nucleon's axial charge

- Neutron β^- decay and kindred processes play a crucial role in many domains, e.g.:
  - Big-Bang nucleosynthesis, constraining the abundance of deuterium;
  - supernovae explosions, producing a vast amount of energy through neutrino production;
  - testing the Standard Model, placing constraints on extensions via low-energy experiments;
  - and in many real-world applications, such as ^{14}C-dating and positron emission tomography.
- Notwithstanding this widespread importance, a connection between the coupling, *g*, that describes weak processes involving current-quarks and that between weak bosons and the dressed-quarks produced by nonperturbative interactions in QCD has not been elucidated.

Nucleon's axial charge

- Neutron β⁻ decay may be studied via the quark-based axial-vector matrix element:

$$\Lambda_{5\mu}^{pn}(p_f,p_i) = \langle p(p_f,\lambda_f)|\,\bar{u}\gamma_5\gamma_\mu d\,|n(p_i,\lambda_i)\rangle$$

  where $p_{i,f}$, $\lambda_{i,f}$ are, respectively, initial/final momenta and helicities associated with the initial-state neutron and final-state proton.

- If one assumes isospin symmetry, then Poincaré covariance entails this matrix element is completely described by two form factors:

$$\Lambda_{5\mu}^{pn}(p_f,p_i) = \bar{u}_p(p_f,\lambda_f)\left[\gamma_5\gamma_\mu\, G_A(q^2) + i\gamma_5 \frac{1}{2M_N}\, q_\mu\, G_P(q^2)\right] u_n(p_i,\lambda_i)$$

  where $q=p_f-p_i$; $G_A(q^2)$ is nucleon's axial-vector form factor, $G_P(q^2)$ is its induced pseudoscalar form factor; M_N is average nucleon mass.

- Nucleon's nonsinglet axial charge

  $g_A := G_A(q^2=0)$

Axial charge & spin physics

- Given assumption of isospin symmetry, then

$$\langle p(p_f,\lambda_f)|\,\bar{u}\gamma_5\gamma_\mu d\,|n(p_i,\lambda_i)\rangle = 2\,\langle p(p_f,\lambda_f)|\,\bar{u}\gamma_5\gamma_\mu u - \bar{d}\gamma_5\gamma_\mu d\,|p(p_i,\lambda_i)\rangle$$

- In the forward scattering limit; i.e., $p_f = p_i = p$, with $\lambda_f = \lambda_i = \lambda$ and $\lambda \bullet p = ½$, then in the infinite-momentum (or light-front) frame

$$2M_N\,\lambda_\mu\,\langle q_\uparrow\rangle = \langle p(p,\lambda)|\,\bar{q}\gamma_5\gamma_\mu q\,|p(p,\lambda)\rangle$$

where

$$\langle q_\uparrow\rangle = \int_0^1 dx\,[\Delta q(x) + \Delta\bar{q}(x)]$$

$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$

is the light-front helicity distribution for a quark q carrying a fraction x of the proton's light-front momentum.

- $\Delta q(x)$ measures difference between the light-front number-density of quarks with helicity parallel to proton and the density of quarks with helicity antiparallel.

Axial charge & spin physics

- Connection between axial matrix element & helicity won't be surprising, given relationship that may be drawn between matrix structure $\gamma_5\gamma_\mu$ & Pauli-Lubanski four-vector $\boldsymbol{W} = ½\, M\, \gamma^5\, \gamma^0\, \boldsymbol{\gamma} = ½\, M\, \boldsymbol{\Sigma}$
- Follows that

$$g_A = \int_0^1 dx\, \left[\Delta u(x) + \Delta\bar{u}(x) - \Delta d(x) - \Delta\bar{d}(x)\right]$$

- Namely, nonsinglet axial charge measures difference, in light-front frame, between *u*- and *d*-quark contributions to proton's helicity

Goldberger Treiman Relation

- The induced pseudoscalar form factor, $G_P(q^2)$, holds its own fascinations, owing particularly to its connection with pion-nucleon interactions.
- Fundamental to the character and strength of such interactions is DCSB, the phenomenon responsible for both
  - 98% of the visible mass in the universe
  - and masslessness of the chiral-limit pion

Recurrent themes throughout

- The existence of such a pion entails

Pion pole saturates form factor

$$\frac{q^2}{2M_N} G_P(q^2) \overset{q^2 \sim 0}{=} 2 f_\pi^0 g_{\pi NN}^0$$

where f_π^0 is the pion's leptonic decay constant and $g^0_{\pi NN}$ is the pion-nucleon coupling constant. (The superscript ``0'' indicates a quantity evaluated in the chiral limit.)

Goldberger Treiman Relation

- Using the equations of motion for on-shell nucleons – Gordon Identities , then chiral-limit axial-vector current conservation at the nucleon-level; viz.,

  $$q_\mu \Lambda_{5\mu}^{pn}(p_f,p_i) \;= 0$$

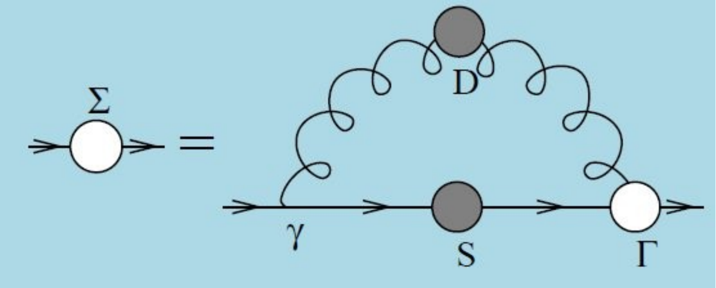
  delivers the Goldberger-Treiman relation:

  $$\boxed{M_N^0\, g_A^0 = f_\pi^0\, g^0_{\pi NN}}$$

- This identity has some curious implications
  - In the absence of DCSB, $f_\pi^0 = 0$ and hence no pseudoscalar meson couples to the weak interaction.
  - Follows then from GT relation that if a nucleon exists with a nonzero mass in a universe without DCSB, then $g_A^0 = 0$ for that nucleon; i.e., such nucleons, too, decouple from the weak interaction.

Lecture 2A

- In these circumstances then g_A^0 appears to serve as an order parameter for DCSB and a nonzero value of g_A signals the presence of in-hadron quark condensates



g_A and dressed quarks

- What is the connection between g_A and the strong physics of dressed-quarks, which are described in QCD by the gap equation?
- Axial-vector Ward-Takahashi identity and its consequences are obviously relevant

$$P_\mu \boxed{\Gamma^l_{5\mu}(k;P)} = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

Axial-vector vertex describes how dressed-quark couples to axial-vector current

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

- In particular, these three quark-level Goldberger-Treiman relations

$$\begin{aligned} F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Lecture 2A

Axial charge of a dressed quark

- These three key identities involve elements from the dressed-quark – axial-vector vertex, which has the general form

$$\Gamma_{5\mu}(k;P) = \gamma_5\left[\gamma_\mu F_R(k;P) + k_\mu \gamma\cdot k G_R(k;P) - \sigma_{\mu\nu}k_\nu H_R(k;P)\right] + \tilde{\Gamma}_{5\mu}(k;P) + \frac{P_\mu}{P^2+m_\pi^2}\, 2\, f_\pi\, \Gamma_\pi(k;P)\,,$$

  with the possibility of a pion pole made explicit.
- In this vertex, only *$F_R(k;P)$*, the function associated with the Dirac structure $\gamma_5\,\gamma_\mu$, possesses an ultraviolet divergence in QCD perturbation theory; and in Landau gauge the renormalised amplitude *$F_R = 1$*, up to next-to-leading-order perturbative corrections
  - One-loop corrections vanish.
  - All other functions in the axial-vector vertex are k^2 power-law suppressed in the ultraviolet.
- In perturbation theory, therefore, the distribution of a current-quark's axial-charge is expressed by $\boxed{g_A^q(k^2) := F_R(k;P=0)}$
  It remains close to unity: $g_A^q(k^2) \approx 1$

Axial charge of a dressed quark

- Nonperturbatively, however, the situation is very different!
- Consider the contact interaction.

With $\alpha_{IR}/\pi \approx 1$, DCSB does not occur, so $f_\pi^0 = 0$ and hence, from the quark-level Goldberger-Treiman relation,

$g^q_{ACN} = 1$

On the other hand, with $\alpha_{IR}/\pi \approx 1$; one obtains
$A^0(k^2)=1$, $M^0(k^2) = M^0 = 0.358$GeV, $f_\pi^0 = 0.1$GeV, $F_\pi^0(k;0) = 0.46$,
and hence,

$$g^q_{A_{\rm CN}} = F_R^0(k;0) = 1 - 2\frac{f_\pi^0}{M^0} F_\pi^0(k;0) = 0.74$$

Thus the quantity associated with the current-quark's axial-charge is markedly suppressed in the infrared owing to the nonperturbative phenomenon of DCSB.

Axial charge of a dressed quark

- Is this result model specific? NO
- Use best rainbow-ladder interaction available, then

  $g^q{}_{ARL} = 0.81$

Qin et al., Phys. Rev. C 84 042202(Rapid Comm.) (2011) Rainbow-ladder truncation

- Use best kernel available, with many features of DCSB built into the Bethe-Salpeter equation, like dressed-quark anomalous magnetic moment,

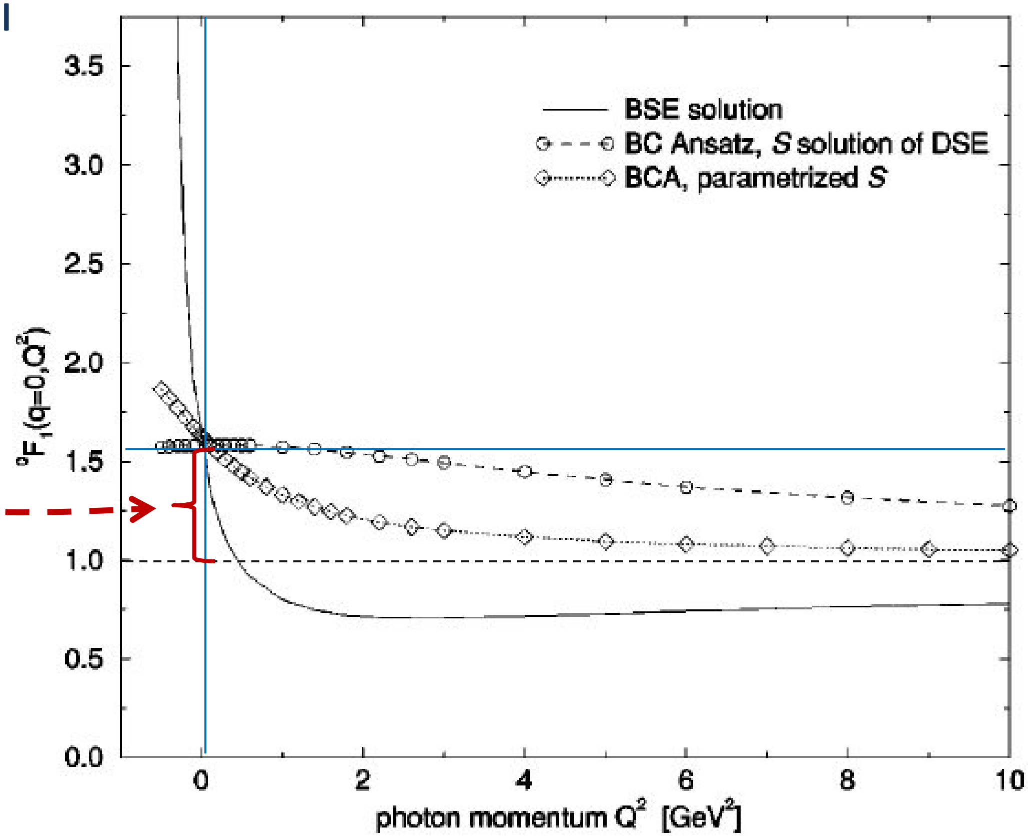
  $g^q{}_{ADB} = 0.87 = 1.06\, g^q{}_{ARL}$

Lei Chang & C.D. Roberts, arXiv:1104.4821 [nucl-th], Phys. Rev. C 85, 052201(R) (2012) [5 pages], Tracing massess of ground-state light-quark mesons

- The infrared suppression is thus seen to be a generic feature of the axial-vector vertex.

Vector charge of a dressed quark

- NB. This contrasts markedly with the effect of dressing on the vector vertex, which is bounded below by unity at *(k=0;P=0)* owing to the vector Ward-Takahashi identity.
- Indeed, with a momentum-dependent interaction the vector vertex is always enhanced



Elucidating the Connection with DCSB

- NB. Poincaré covariance demands that the general form for a pseudoscalar meson Bethe-Salpeter amplitude possess four components.

  F_π is going to be nonzero in (almost) every frame!

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Inspection of the Bethe-Salpeter equation for pseudoscalar mesons shows that a nonzero value for E_π is the force behind $F_\pi \neq 0$, with the coupling fixed by the DCSB mass-scale
  - To be clear, $M \neq 0$ in the chiral limit entails $E_\pi \neq 0$, and together these results require $F_\pi \neq 0$.
- Contact interaction, $m=0=P^2$

$$\begin{bmatrix} E_\pi(P) \\ F_\pi(P) \end{bmatrix} = \frac{1}{3\pi^2 m_G^2}\begin{bmatrix} \mathcal{K}_{EE} & \mathcal{K}_{EF} \\ \boxed{\mathcal{K}_{FE}} & \mathcal{K}_{FF} \end{bmatrix}\begin{bmatrix} \boxed{E_\pi(P)} \\ F_\pi(P) \end{bmatrix}$$

$$\mathcal{K}_{EE} = \mathcal{C}(M^2;\tau_{\rm ir}^2,\tau_{\rm uv}^2)\,, \quad \mathcal{K}_{EF} = 0\,,$$
$$2\mathcal{K}_{FE} = \mathcal{C}_1(M^2;\tau_{\rm ir}^2,\tau_{\rm uv}^2)\,, \quad \mathcal{K}_{FF} = -2\mathcal{K}_{FE}$$

$C_1 = - M^2\, C'(M^2) > 0$

Elucidating the Connection with DCSB

- General result.
  - Independent of the function chosen to represent the dressed-gluon and the *Ansatz* for the dressed-quark-gluon vertex in the gap equation
- Explains why appearance of $F_\pi \neq 0$ is a necessary consequence of DCSB.
  - Given an interaction with nontrivial momentum dependence, then G_π and H_π are also necessarily nonzero for the same reason.
- Demonstrates that a complete expression of DCSB is not achieved merely by producing nonzero values for the in-pion condensate and pion leptonic decay constant.

  The full structure of the Goldstone mode must also be described.

$$F_R(k;0) + 2\, f_\pi F_\pi(k;0) \;=\; A(k^2)$$

Elucidating the Connection with DCSB

- Finally, positivity of f_π^0 guarantees that of $F_\pi(k;0)$, and hence the second term on the right-hand-side of Goldberger-Treiman relation is positive.
- This means that
  - $F_\pi(k^2;P=0)$ is bounded above by $A^0(k^2)$
  - $F_\pi(k^2;P=0)$ approaches $A^0(k^2)$ from below as $k^2 \to$ infinity.
- However, does not, however guarantee $F_R(0;0)<1$.
  - That is a consequence of the dynamics which produces the Goldstone pion and sets the mass-scale for DCSB.

$$F_R(k;0) = A(k^2)$$

Elucidating the Connection with DCSB

- Now *a propos* to reconsider the role of g_A in connection with DCSB.
- In chiral-limit DSE studies, chiral symmetry restoration and deconfinement are coincident, no matter which control parameter is varied: temperature, density, number quark flavours, etc.
- This supports a view that DCSB and confinement are intimately related
  - In the presence of some agent which undermines the interaction strength required for DCSB, confinement is also lost.
- In these circumstances none of the interactions in the Standard Model is strong – one is within an essentially perturbative domain of the theory – and $f_\pi^0=0$.
- Consequently, $F_R(k^2;0) = A^0(k^2)$, and both functions are unity up to perturbative corrections.

$M_N^0\, g_A^0 = f_\pi^0\, g^0_{\pi NN}$

g_A and DCSB - an order parameter?

- Chiral symmetry restored, so $f_\pi^0 = 0$ and hence

  $\boxed{M_N^0\, g_A^0 = 0}$
- But we've just seen that a connection between the restoration of chiral symmetry and g_A vanishing is not driven by changes at the level of the axial-vector dressed-quark vertex because this vertex dressing is perturbative as chiral symmetry is restored:
  - $F_R(k^2;0) = A^0(k^2)$, and both functions are unity up to perturbative corrections.
- In the chiral limit, the relationship above must therefore be connected with dissolution of the nucleon bound-state at a point of coincident chiral symmetry restoration and deconfinement:
  - No nucleon survives this limit

Axial charge and spin physics

$$g_A = \int_0^1 dx \left[\Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x)\right]$$

- A vanishing of g_A entails that the right-hand-side of this expression is zero.
- This expression is normally described as expressing the difference in the light-front frame between the *u*- and *d*-quark contributions to the proton's helicity.
- *How can that vanish?*

Axial charge and spin physics

$$g_A = \int_0^1 dx \, \left[\Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x)\right]$$

- One is here considering the chiral limit.
  - Absent a DCSB mechanism, a chiral limit theory with massless quarks separates into two distinct, non-communicating theories: one for positive helicity states and another for negative helicity.
  - Each sub-theory has identical interactions and hence each will produce the same quark number distributions, labelled, however, by opposite helicities.
  - Since there is no mechanism in the total theory that can flip helicity, the number of positive helicity states will always match the number with negative helicity.
- *Hence the result g_A=0 is achieved because each of the four terms in the above expression vanishes individually, irrespective of whether or not they are associated with a bound-state.*

$$g^q_{A_{\rm CN}} = F^0_R(k;0) = 1 - 2\frac{f^0_\pi}{M^0} F^0_\pi(k;0) = 0.74$$

$g^q{}_{ARL} = 0.81$

$g^q{}_{ADB} = 0.87 = 1.06\, g^q{}_{ARL}$

Quark models and g_A

- Related to constituent-quark model phenomenology, these results are curious.
- Textbook knowledge that constituent-quark models with spin-flavour wave-functions based on *SU(6)* symmetry produce

$$g_A = \frac{5}{3} g_A^Q \int d^3x \left[u^2(x) - \frac{1}{3} v^2(x) \right]$$
$$= \frac{5}{3} g_A^Q \left[1 - \frac{4}{3} \int d^3x\, v^2(x) \right],$$

where $g_A{}^Q$ is the axial-charge of a constituent-quark, and *u(x), v(x)* are, respectively, the upper and lower components of the nucleon's constituent-quark wave-function.

$$g^q_{A_{\rm CN}} = F^0_R(k;0) = 1 - 2\frac{f^0_\pi}{M^0}F^0_\pi(k;0) = 0.74$$

$g^q_{ARL} = 0.81$

$g^q_{ADB} = 0.87 = 1.06\, g^q_{ARL}$

Quark models and g_A

$$g_A = \frac{5}{3}\, g_A^Q \int d^3x \left[u^2(x) - \frac{1}{3}v^2(x)\right]$$
$$= \frac{5}{3}\, g_A^Q \left[1 - \frac{4}{3}\int d^3x\, v^2(x)\right] ,$$

- In a nonrelativistic model, $v(x)=0$ so that

  $g_A = (5/3)\, g_A^Q$
- Hence, reproducing the empirical value of

  $g_A=1.27$

  requires

  $g_A{}^Q{}_{NR} = 0.76$
- *Of course, the origin of the empirical value of g_A is more complicated but nonperturbative dressing of $g_A{}^q$ plays a part.*

$$g^q_{A_{\rm CN}} = F^0_R(k;0) = 1 - 2\frac{f^0_\pi}{M^0} F^0_\pi(k;0) = 0.74$$

$g^q{}_{ARL} = 0.81$

$g^q{}_{ADB} = 0.87 = 1.06\, g^q{}_{ARL}$

Quark models and g_A

$$g_A = \frac{5}{3} g_A^Q \int d^3x \left[u^2(x) - \frac{1}{3}v^2(x)\right]$$
$$= \frac{5}{3} g_A^Q \left[1 - \frac{4}{3}\int d^3x\, v^2(x)\right] ,$$

- A full explanation is suggested by elucidating the nature of the terms in this expression, which has two key features.
- As described above, the first is dressing of the axial-vector vertex, an effect that modifies the strength with which a dressed-quark couples to the *W*-boson.
- The other is indicated by the second term within the parentheses:

  $\int d^3x \;\; v^2(x)$

  This represents the appearance of P-wave quark orbital angular momentum in a relativistic constituent-quark model; *namely, the presence of correlations in the nucleon's wave function.*

Correlations and g_A

- As we've seen, in QCD the nucleon is properly described by a Poincaré covariant Faddeev equation. (Lecture 3A)
- In this context, *v(x)* may be interpreted as signifying the impact of correlations within the nucleon's Faddeev wave-function, which possesses *S*-, *P*- and *D*-wave dressed-quark orbital angular momentum components in the nucleon's rest frame.
- In the presence of DCSB, such correlations are strong; e.g., the *S*-wave-only contribution to the nucleon's normalisation is just 37%
- *Thus empirical value of g_A embodies the outcome of interference between dressing the quark–W-boson vertex and angular momentum correlations within the nucleon's Faddeev amplitude.*
- Not too surprising given the connection between g_A and the *u*- and *d*-quark helicity distributions
- *NB. The magnitude of the suppression of g_A^q and the strength of orbital angular momentum correlations in bound-state wave functions are both driven by DCSB.*

Correlations and g_A

- The strength of orbital angular momentum correlations in bound-state wave functions is driven by DCSB.
- In the absence of DCSB,
  - the amplitudes F_π, G_π, H_π in the pion's Bethe-Salpeter amplitude vanish identically in the chiral limit,
  - as does $M(p^2)$ in the dressed-quark propagator
- It follows that any correlation which survives is described by a Bethe-Salpeter wave-function:

$$X_\pi(k;P) = S(p)\,\Gamma_\pi(k;P)\,S(p) = \gamma_5 * \text{scalar-function}$$

This is a purely *S*-wave structure in rest-frame kinematics.

$$f_\pi E_\pi(k;P=0) = B(p^2)$$

$$\boxed{\begin{aligned} F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}}$$

Faddeev equation and g_A

- It is known that Faddeev equation models can be constructed to reproduce the empirical value of g_A, unifying it in the process with other nucleon observables. *Aspects of hadron physics*, C.D. Roberts, M.S. Bhagwat, A. Höll, S.V. Wright, arXiv:0802.0217 [nucl-th], Eur. Phys. J. ST 140 (2007) 53
- However, such studies employed axial-vector vertices that do not respect the last three quark-level Goldberger-Treiman relations
- This is mended in Eichmann & Fischer, arXiv:1111.2614 [hep-ph], which solves all elements of the problem:
  - gap, Bethe-Salpeter and Faddeev equations
  - rainbow-ladder truncation.
- That study, however, produces $g_{A\,RL} = 0.99(2)$, underestimating the empirical value by 22%.
  - Naturally, with $M_N^{\,0}$ and $f_\pi^{\,0}$ near to their experimental values, the Goldberger-Treiman relation entails that this study underestimates $g_{\pi NN}$ by a similar amount.

$$M_N^{\,0}\, g_A^{\,0} = f_\pi^{\,0}\, g^0_{\pi NN}$$

Faddeev equation and g_A

- The magnitude of the error is typical of rainbow-ladder truncation in those channels for which it is known and understood *a priori* to be adequate.
  - In the sector of light-quark vector and flavour nonsinglet pseudoscalar mesons, over an illustrative basket of thirty-one calculated quantities, the truncation delivers a standard-deviation of 15% in the relative error between experiment and theory
- Part of the remedy to this quantitative error lies in going beyond the leading-order truncation when solving the gap and Bethe-Salpeter equations.
- This is now possible in a symmetry-preserving manner, as described in Lecture 2B.
  - Illustrate that now …

$$\Gamma_{5\mu}(k;P) = \gamma_5\,[\gamma_\mu F_R(k;P) + k_\mu \gamma\cdot k G_R(k;P) - \sigma_{\mu\nu} k_\nu H_R(k;P)] + \tilde{\Gamma}_{5\mu}(k;P) + \frac{P_\mu}{P^2+m_\pi^2}\, 2\, f_\pi\, \Gamma_\pi(k;P)\,,$$

Transverse part of axial-vector vertex

- Only part that contributes to directly to $g_A{}^q$

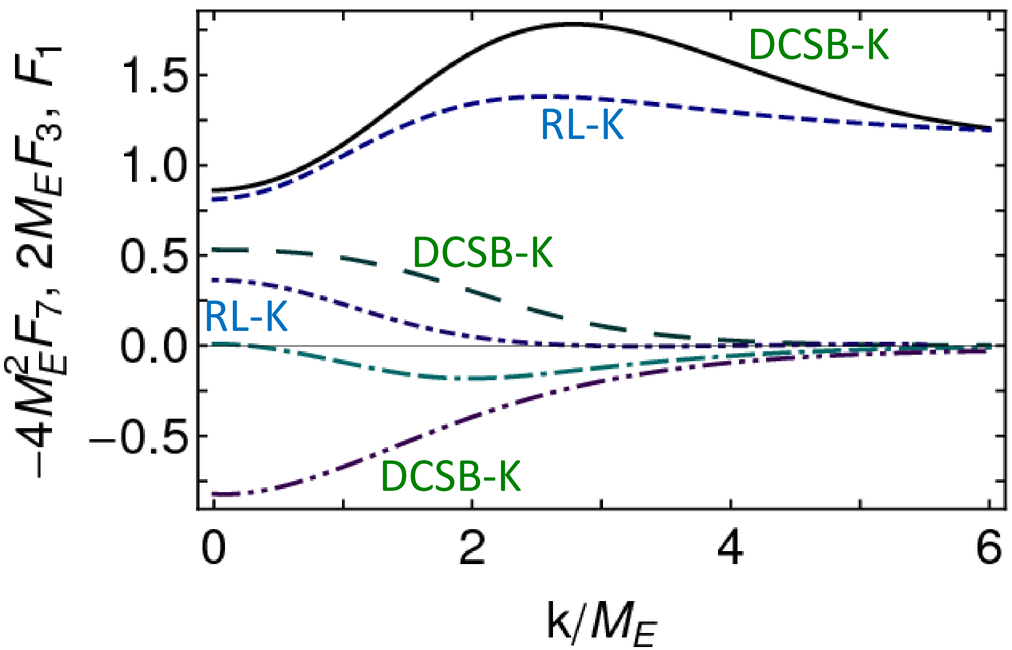
$$\Gamma^{\perp}_{5\mu}(k;P) = \gamma_5 [\gamma^{\perp}_{\mu} F_1 + i\gamma^{\perp}_{\mu}\gamma\cdot\hat{P} k\cdot\hat{P} F_2 + T_{\mu\nu}\sigma_{\nu\rho}k_\rho F_3 + [k^{\perp}_{\mu}\gamma\cdot\hat{P} + i\gamma^{\perp}_{\mu}\sigma_{\nu\rho}k_\nu\hat{P}_\rho] F_4 - ik^{\perp}_{\mu} k\cdot\hat{P} F_5 + k^{\perp}_{\mu}\gamma\cdot\hat{P} k\cdot\hat{P} F_6 + k^{\perp}_{\mu}\gamma\cdot k F_7 + k^{\perp}_{\mu}\sigma_{\nu\rho}k_\nu\hat{P}_\rho k\cdot\hat{P} F_8]$$

  – $\{F_i \mid i=1,\ldots,8\}$ are scalar functions of $(k^2, k.P, P^2)$ that are even under $k.P \to (-\,k.P)$; P^2=1; $T_{\mu\nu} = [\delta_{\mu\nu} - P_\mu P_\nu/P^2]$, $T_{\mu\nu} + L_{\mu\nu} = \delta_{\mu\nu}$; and $a_\mu^{\dagger} = T_{\mu\nu}\, a_\nu$.

- Since the transverse vertex may simply be obtained from this general form through contraction with $T_{\mu\nu}$, one has the following correspondences:
  – $F_1 \leftrightarrow F_R$

Transverse part of axial-vector vertex

- Given that a dressed-quark anomalous chromomagnetic moment produces a large dressed-quark anomalous magnetic moment (Lecture 2B), one should at least expect that F_3, with its similar tensor structure, is significantly altered when proceeding beyond rainbow-ladder truncation.
- In fact, all the scalar functions are materially modified on a domain $0 < |k|/M_E < 5$, where the Euclidean constituent quark mass is $M_E = \{\surd s \mid s = M^2(s), s > 0\}$:
  - $F_{1,2,3,5,6,8}$ magnitudes are enhanced, with F_5 also changing sign;
  - $F_{4,7}$ magnitudes are suppressed.



Transverse part of axial-vector vertex

- With at least eight quantities reacting markedly to improvements in the DSE kernels, it is natural to seek a single measure that can illustrate the plausible consequences for g_A
- Consider $\bar{u}(p_f)\Gamma^{\perp}_{5\mu}(k;P)u(p_i)$

where, at each value of $p^2 > 0$, the Euclidean spinors satisfy $\gamma\cdot p\, u(p) = \varsigma_p\, u(p)$, $\bar{u}(p)\gamma\cdot p = \bar{u}(p)\,\varsigma_p$, $\varsigma_p = M(p^2)$

- Focusing on the case *(k.P=0, P^2=0)* and using the appropriate Euclidean-Gordon identities, one obtains an axial-charge distribution $g_A^{E_q}(k^2) = F_1(\varsigma_k^2;0) + i\varsigma_k\, F_3(\varsigma_k^2;0)$

  With this kinematic arrangement, no other functions from the transverse vertex contribute.

Transverse part of axial-vector vertex

- This single measure confirms the result described previously; viz. DB/RL = 1.06
- Suggests DCSB-kernel will improve Faddeev equation result
  g_A = 1.05(2)
- *Modest size of the improvement is good because the utility of rainbow-ladder truncation would have been much reduced if the magnification were too large.*

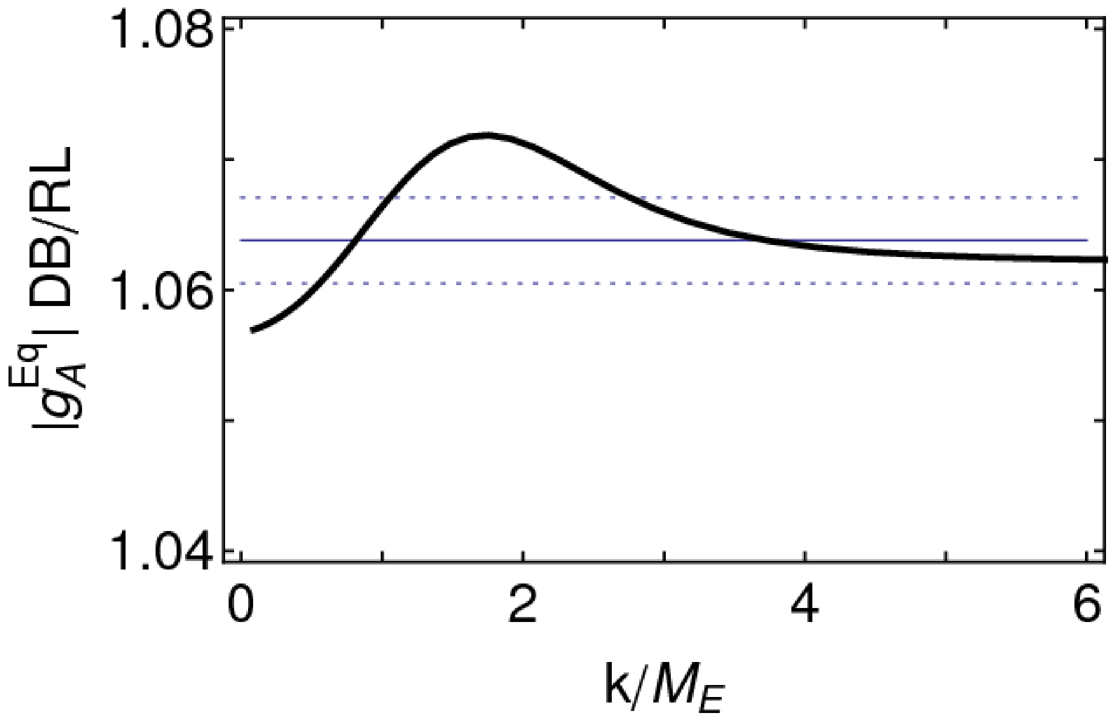
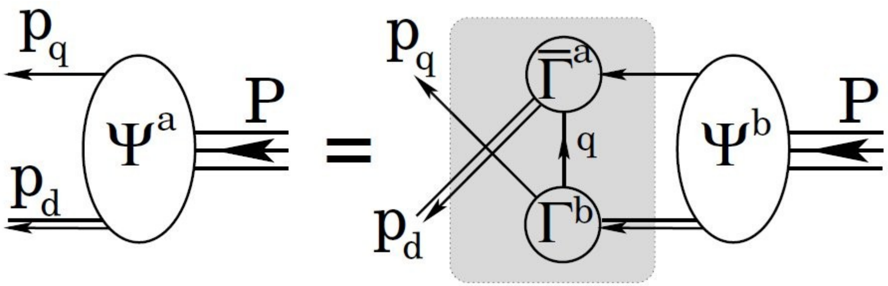
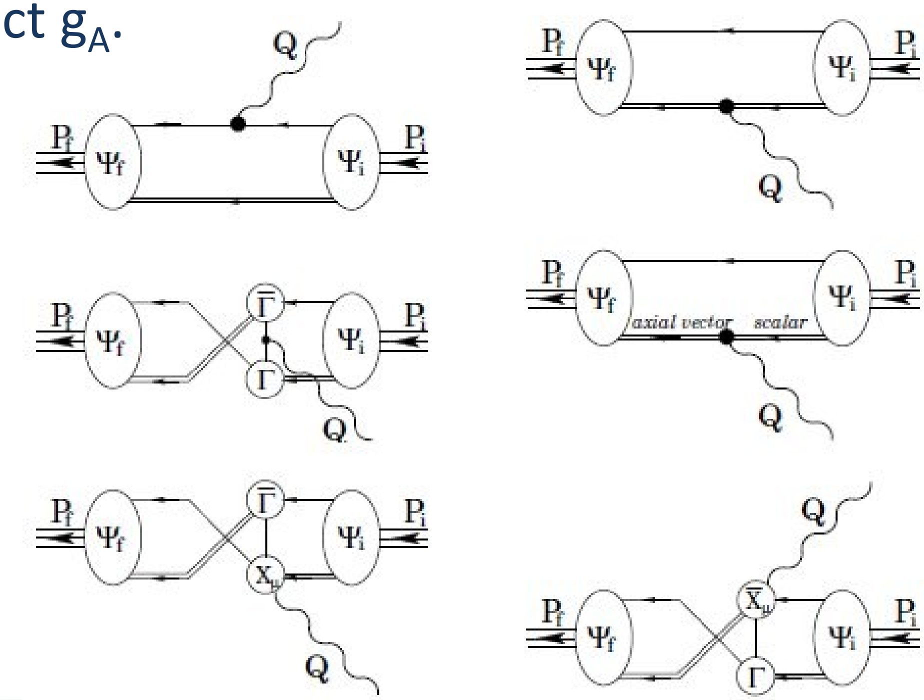


FIG. 2. With the elements defined in association with Eq. (33), the ratio $|g_A^{qDB}(k^2)|/|g_A^{qRL}(k^2)|$ (*solid curve*). The straight line with dotted outliers represents the band 1.064 ± 0.003.

Faddeev equation and g_A

- One should bear in mind, however, that correcting the gap and Bethe-Salpeter equation kernels is not the complete picture.
- The Faddeev equation kernel and associated interaction current should also be modified.
- Such modifications must also affect g_A.



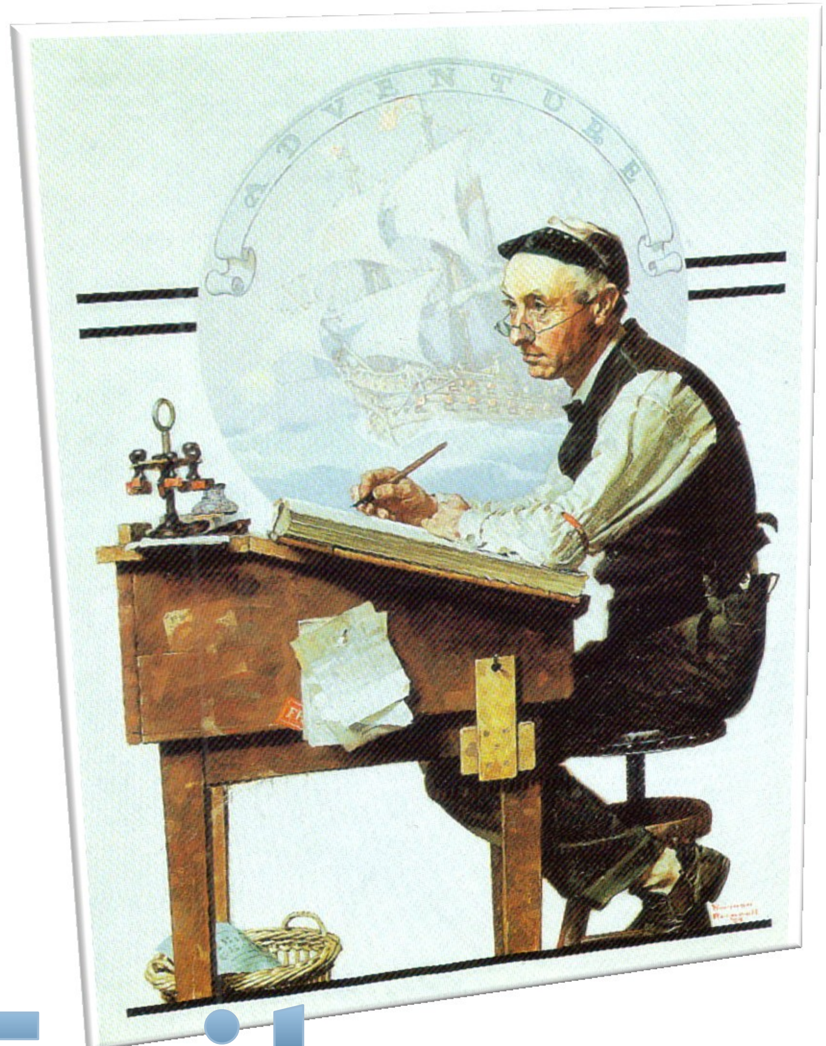


These RL expressions must both be modified, too.

Faddeev equation and r_A

- Improvement of the gap and Bethe-Salpeter kernels should also affect the result for the nucleon's axial radius, r_A:
  - quite simply because the rainbow-ladder truncation is unable to explain the location of the a_1-meson pole in the axial-vector vertex, whereas the DCSB-corrected kernels resolve this longstanding problem (Lecture 2B)
- In underestimating the mass of the a_1 meson, the rainbow-ladder truncation overestimates the contribution to r_A from the associated pole.
  - Likely, therefore, to overestimate this radius or, equally, understate the mass-scale, m_A, that characterises evolution of the nucleon's axial form factor in the neighbourhood of $P^2=0$.
- Curious, however, because RL produces $m_A^{RL}=1.28(6)$, which is already at the upper limit of values inferred from experiment. (*This corresponds to a value of r_A at the lower limit of experiment.*)
- NB. Corrections to the Faddeev kernel and associated interaction current can plausibly magnify correlations within the nucleon and their impact on interactions.
  - We know they do for quark-antiquark systems.

Such effects would serve to increase r_A.



Epilogue

Epilogue

DSEs: A practical, predictive, unifying tool for fundamental physics

- Exact results proved in QCD, amongst them:
  - ✓ Quarks are not Dirac particles and gluons are nonperturbatively massive
  - ✓ Dynamical chiral symmetry breaking is a fact.
    It's responsible for 98% of the mass of visible matter in the Universe
  - ✓ Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel
  - ✓ Confinement is a dynamical phenomenon
    It cannot in principle be expressed via a potential
  - ✓ The list goes on …

McLerran & Pisarski
arXiv:0706.2191 [hep-ph]

- DSEs are a single framework, with IR model-input turned to advantage, *"almost unique in providing an unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."*

It’s been a journey

Craig Rober

This is not the end