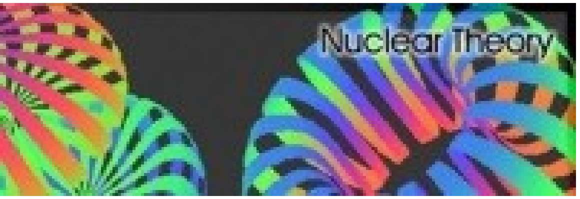
Emergence of DSEs in Real-World QCD

Craig Roberts



Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

National Science Foundation
WHERE DISCOVERIES BEGIN

U.S. DEPARTMENT OF ENERGY

Charting the interaction between light-quarks

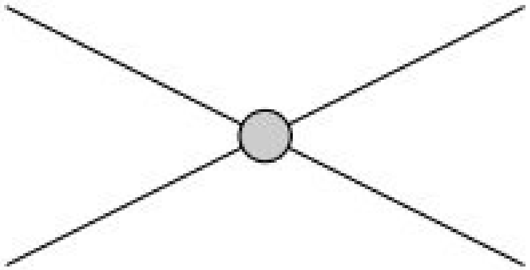
Let's see how that is done

- *Recall Lecture IB …*
- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines the pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors

  can be used to chart β-function's long-range behaviour.

H.L.L. Roberts, A. Bashir, L.X. Gutiérrez-Guerrero, C.D. Roberts and D.J. Wilson, arXiv:1102.4376 [nucl-th], Phys. Rev. C *in press*

Consider contact-interaction Kernel

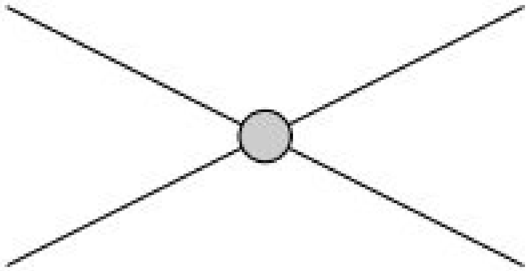


- Vector-vector contact interaction

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{1}{m_G^2}$$

m_G is a gluon mass-scale – dynamically generated in QCD

- Gap equation: $$M = m + \frac{M}{3\pi^2 m_G^2} \int_0^\infty ds\, s \frac{1}{s+M^2}$$
- DCSB: *M ≠ 0* is possible so long as $m_G < m_G^{critical}$ (*Lecture II*)
- *Studies of π & ρ static properties and π form factor establish that contact-interaction results are not realistically distinguishable from those of renormalisation-group-improved one-gluon exchange for processes with $Q^2<M^2$*

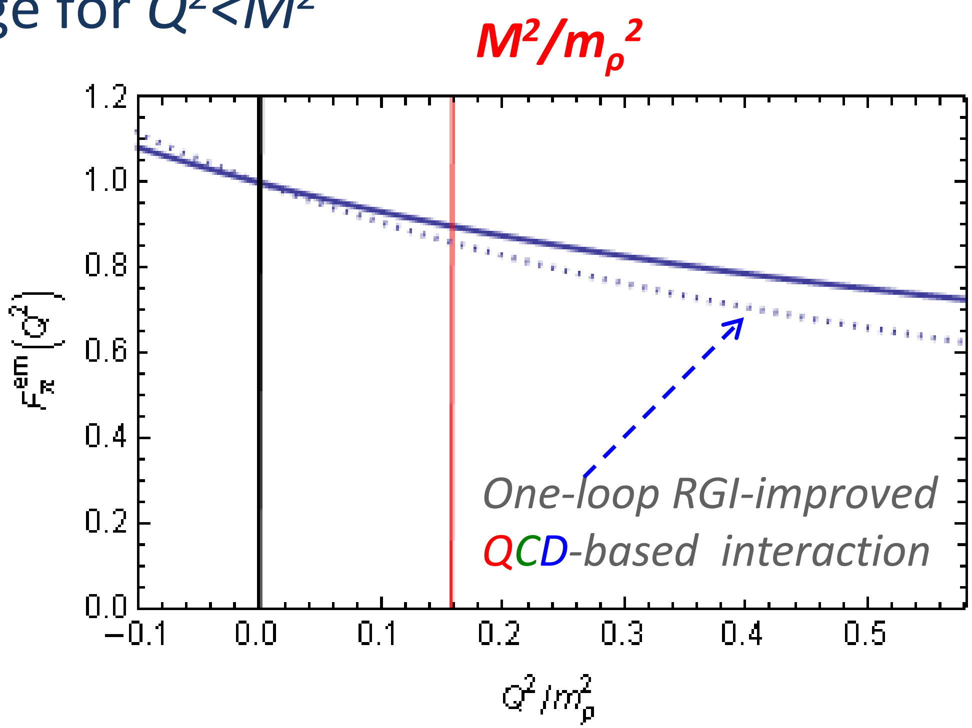


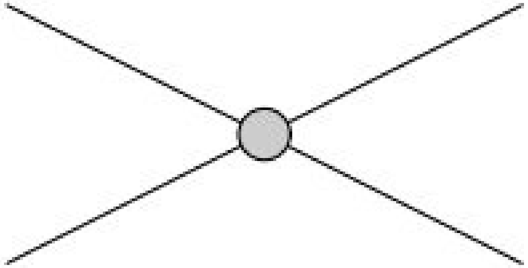
Interaction Kernel

- Studies of π & ρ static properties and π form factor establish that contact-interaction results are not realistically distinguishable from those of renormalisation-group-improved one-gluon exchange for $Q^2<M^2$

	contact interaction	QCD 1-loop RGI gluon
M	0.37	0.34
κ_π	0.24	0.24
m_π	0.14	0.14
m_ρ	0.93	0.74
f_π	0.10	0.093
f_ρ	0.13	0.15

cf. expt.
rms rel.err.=13%





Interaction Kernel - Regularisation Scheme

- Contact interaction is not renormalisable
- Must therefore introduce regularisation scheme
  - Use confining proper-time definition

D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B 388 (1996) 154.

No pole in propagator – DSE realisation of confinement

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, \mathrm{e}^{-\tau(s+M^2)} \to \int_{\tau_{uv}^2}^{\tau_{ir}^2} d\tau\, \mathrm{e}^{-\tau(s+M^2)} = \frac{\mathrm{e}^{-(s+M^2)\tau_{uv}^2} - e^{-(s+M^2)\tau_{ir}^2}}{s+M^2}$$

  - $\Lambda_{ir} = 0.24$GeV, $\tau_{ir} = 1/\Lambda_{ir} = 0.8$fm
    a confinement radius, which is not varied
  - Two parameters:
    $m_G=0.13$GeV, $\Lambda_{uv}=0.91$GeV
    fitted to produce tabulated results

	contact interaction
M	0.37
κ_π	0.24
m_π	0.14
m_ρ	0.93
f_π	0.10
f_ρ	0.13

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th], Phys.Rev. C **81** (2010) 065202

Pion's GT relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P)$$
$$+ \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)\Big]$$

(second line struck through)

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

$A(p^2) \to 1$, $B(p^2) \to M_Q$

- Bethe-Salpeter amplitude can't depend on relative momentum; & propagator can't be momentum-dependent
- Solved gap and Bethe-Salpeter equations

$P^2=0$: $M_Q=0.4$GeV, $E_\pi=0.098$, $F_\pi=0.5M_Q$

Nonzero and significant

Guttierez, Bashir, Cloët, Roberts
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Pion's GT relation Contact interaction

- Pion's Bethe-Salpeter amplitude

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- Dressed-quark propagator $S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

$A(p^2) \to 1$, $B(p^2) \to M_Q$

- Asymptotic form of $F_\pi^{em}(Q^2)$

$E_\pi^2(P) \Rightarrow F_\pi^{em}(Q^2) = M_Q^2/Q^2$

For 20+ years it was imagined that contact interaction produced a result that's indistinguishable from pQCD counting rule

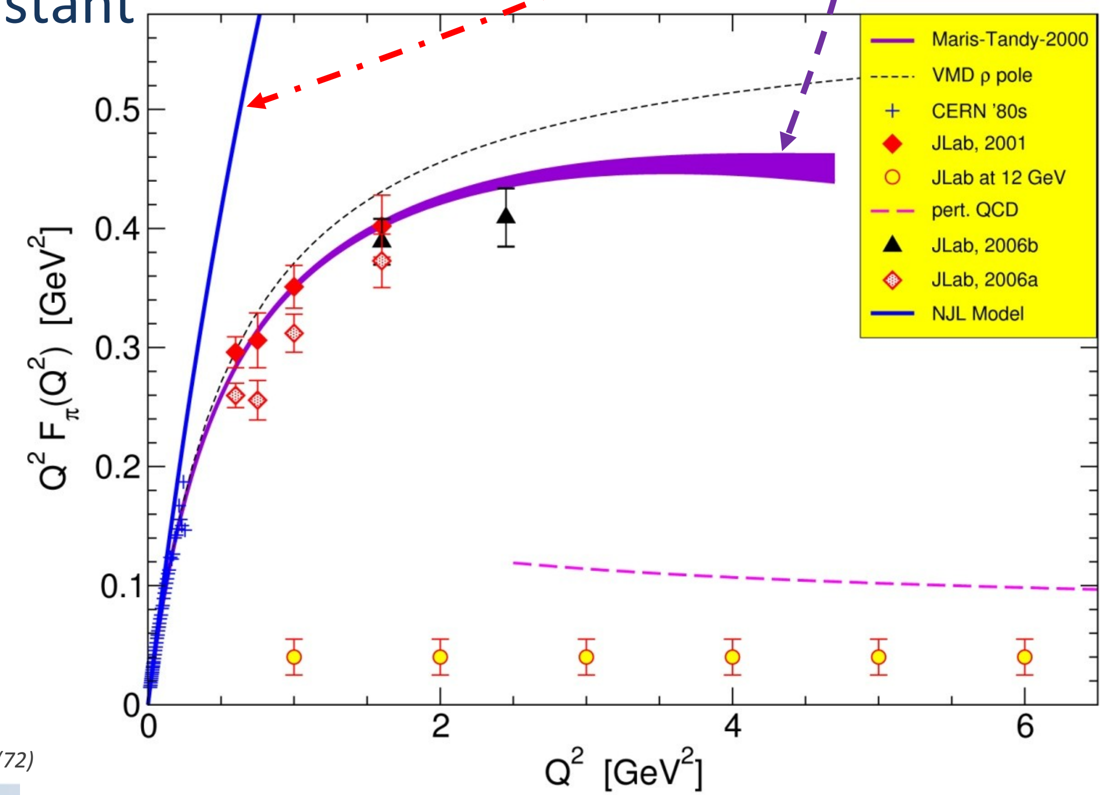
$E_\pi(P)\, F_\pi(P)$ – *cross-term*

$\rightarrow F_\pi^{em}(Q^2) = (Q^2/M_Q^2) * [E_\pi(P)/F_\pi(P)] * E_\pi^2(P)\text{-term} =$ *CONSTANT!*

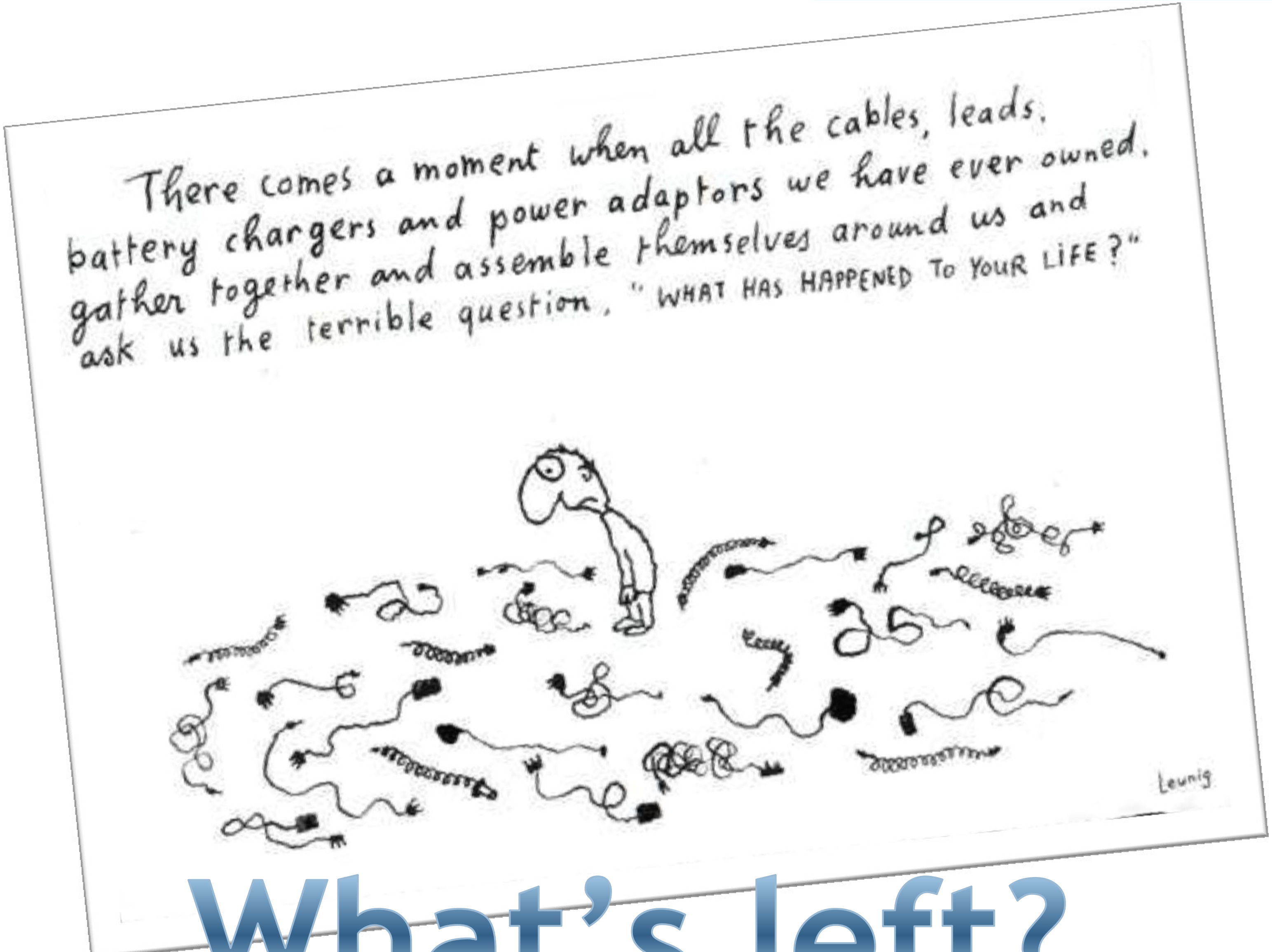
Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th], Phys.Rev. C **81** (2010) 065202

Pion's Electromagnetic Form Factor

- QCD-based DSE prediction: $D(x-y) = \frac{1}{(x-y)^2}$ produces $M(p^2)\sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta(x-y)$ produces $M(p^2)$=constant



- Single mass parameter in both studies
- Same predictions for $Q^2=0$ observables
- *Disagreement >20% for $Q^2>M_Q^2$*
- *Plainly, experiment IS, sensitive to the evolution of the interaction*


There comes a moment when all the cables, leads, battery chargers and power adaptors we have ever owned, gather together and assemble themselves around us and ask us the terrible question, "WHAT HAS HAPPENED TO YOUR LIFE?"
Leunig

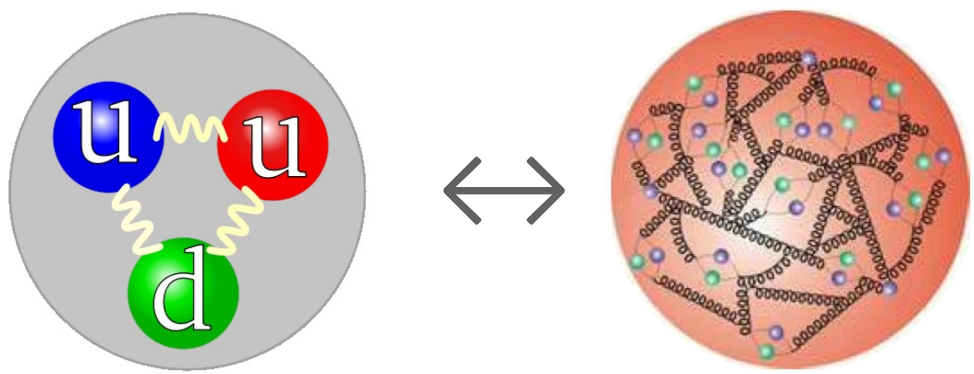
What's left?

New Challenges

- Computation of spectrum of hybrid and exotic mesons

  exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
  hybrid mesons: normal quantum numbers but non-quark-model decay pattern
  BOTH suspected of having "constituent gluon" content

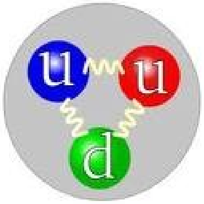
- Equally pressing, some might say more so, is the three-body problem; viz., baryons in QCD

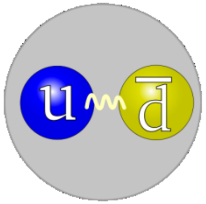


Grand Unification

Unification of Meson & Baryon Properties

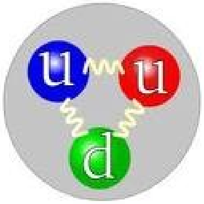
- Correlate the properties of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:
    Poincaré-covariant & satisfy relevant Ward-Takahashi identities
- Constituent-quark model has hitherto been the most widely applied spectroscopic tool; whilst its weaknesses are emphasized by critics and acknowledged by proponents, it is of continuing value because there is nothing better that is yet providing a bigger picture.
- Nevertheless,
  - no connection with quantum field theory & therefore not with QCD
  - not symmetry-preserving & therefore cannot veraciously connect meson and baryon properties

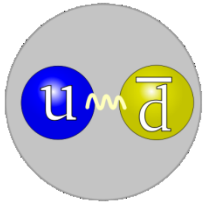




Unification of Meson & Baryon Spectra

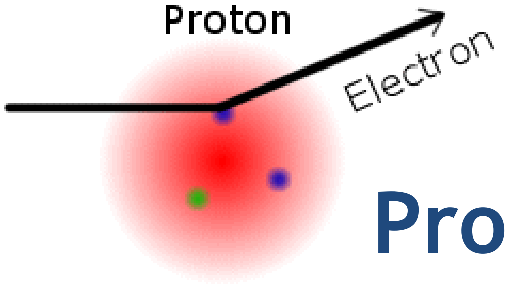
- Dyson-Schwinger Equations have been applied extensively to the spectrum and interactions of mesons with masses less than 1 GeV
- On this domain the *rainbow-ladder* approximation, which is the leading-order in a systematic & symmetry-preserving truncation scheme – nucl-th/9602012, is a well-understood tool that is accurate for pseudoscalar and vector mesons:
  e.g.,
  - Prediction of elastic pion and kaon form factors: nucl-th/0005015
  - Anomalous neutral pion processes – $\gamma\pi\gamma$ & BaBar anomaly: 1009.0067 [nucl-th]
  - Pion and kaon valence-quark distribution functions: 1102.2448 [nucl-th]
  - Unification of these and other observables – $\pi\pi$ scattering: hep-ph/0112015
- It can readily be extended to explain properties of the light neutral pseudoscalar mesons: 0708.1118 [nucl-th]





Unification of Meson & Baryon Spectra

- Some people have produced a spectrum of mesons with masses above 1GeV – but results have always been poor
  - For the bulk of such studies since 2004, this was a case of “*Doing what can be done, not what needs to be done*.”
- Now understood why rainbow-ladder is not good for states with material angular momentum
  - know which channels are affected – scalar and axial-vector;
  - and the changes to expect in these channels
- Task – Improve rainbow-ladder for mesons & build this knowledge into the study of baryon because, as we shall see, a formulation of the baryon bound-state problem rests upon knowledge of quark-antiquark scattering matrix



Nucleon Structure Probed in scattering experiments

- Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, F_1=1 & F_2=0, so that G_E=G_M and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$J_\mu(P',P) = ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,$$

$$= \quad ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)$$

F_1 = Dirac form factor

F_2 = Pauli form factor

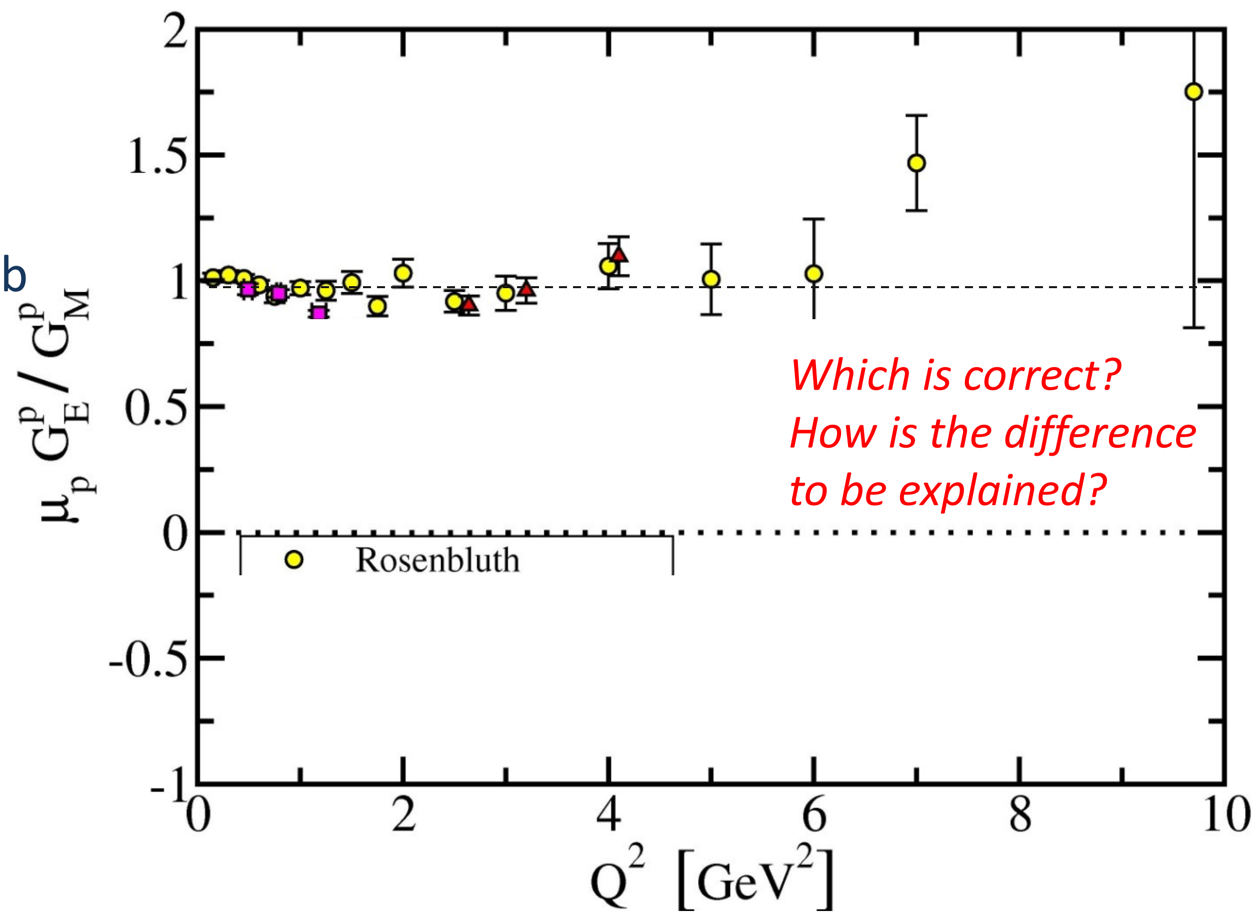
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magntic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

- Data before 1999
  - Looks like the structure of the proton is simple
- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE



Which is correct? How is the difference to be explained?

We were all agog

Fully-covariant computation of nucleon form factors

- First such calculations:
  - G. Hellstern *et al.*, Nucl.Phys. A627 (1997) 679-709 , restricted to Q^2<2GeV^2
  - J.C.R. Bloch *et al.*, Phys.Rev. C60 (1999) 062201(R) , restricted to Q^2<3GeV^2

  Exploratory:
  - Included some correlations within the nucleon, but far from the most generally allowed
  - Used very simple photon-nucleon interaction current
- Did not isolate and study $G_E^p(Q^2)/G_M^p(Q^2)$
- How does one study baryons in QCD?

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

DSEs & Baryons

- *Dynamical chiral symmetry breaking* (DCSB)
  – has enormous impact on meson properties.
    - *Must be included in description and prediction of baryon properties*.
- *DCSB* is essentially a quantum field theoretical effect.
  In quantum field theory
    - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
    - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is based on the observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

$SU_c(3)$: $3 \otimes \quad = \bar{3} \oplus$

R.T. Cahill *et al.*,
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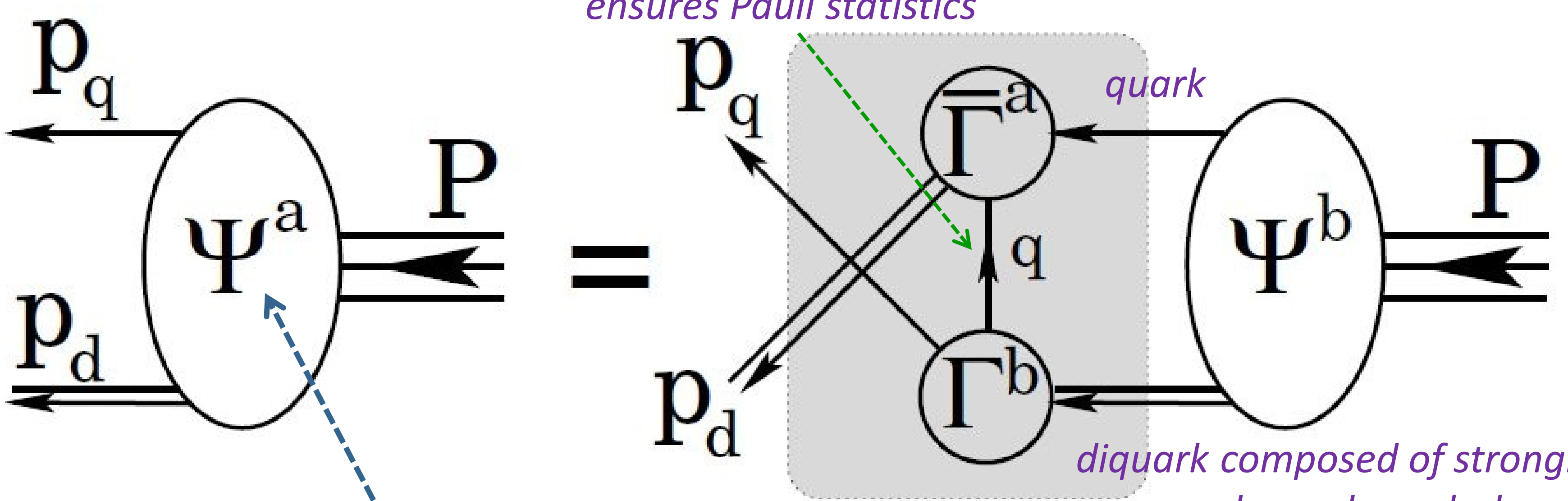
Faddeev Equation

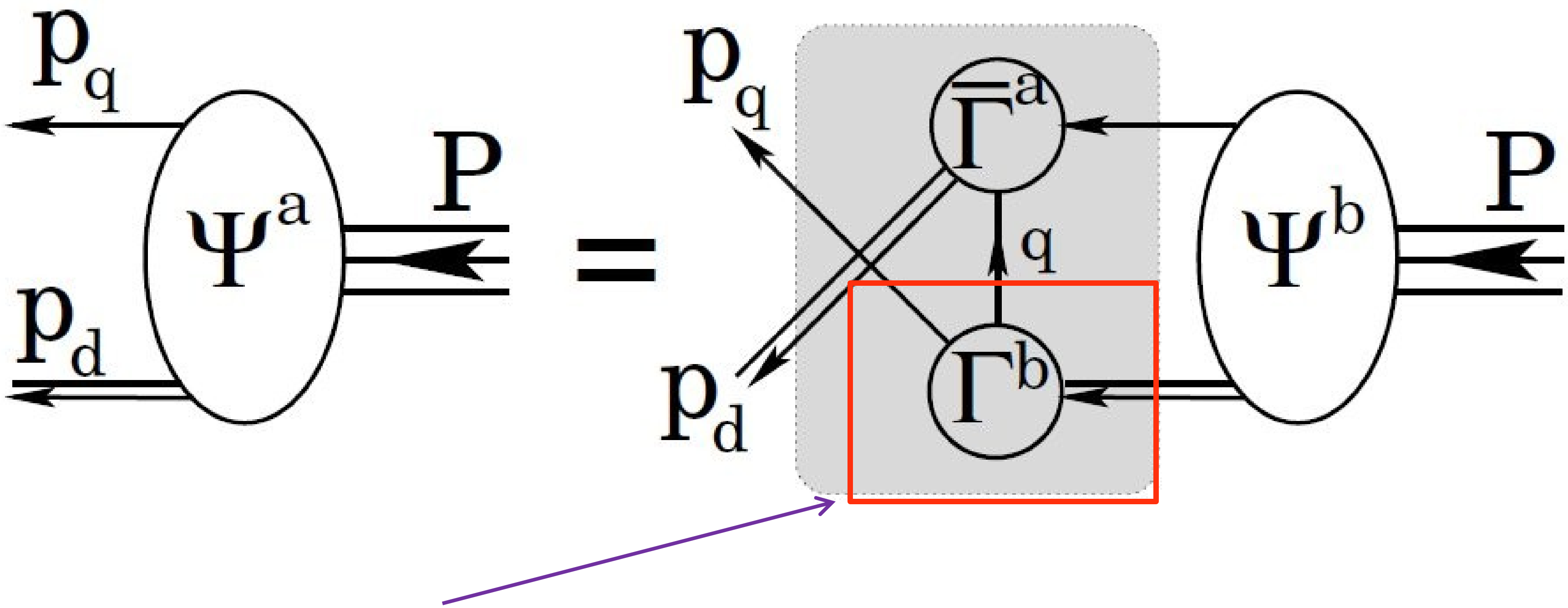


- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

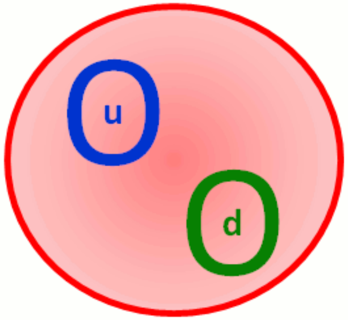
    s−, p− & d−wave correlations

Faddeev Equation

➤ Why should a pole approximation produce reliable results?



quark-quark scattering matrix
- a pole approximation is used to arrive at the Faddeev-equation



Calculation of diquark masses in QCD
R.T. Cahill, C.D. Roberts and J. Praschifka
Phys.Rev. D**36** (1987) 2804

Diquarks

Consider the rainbow-gap and ladder-Bethe-Salpeter equations

$$S(p)^{-1} = i\gamma\cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu(q,p),$$

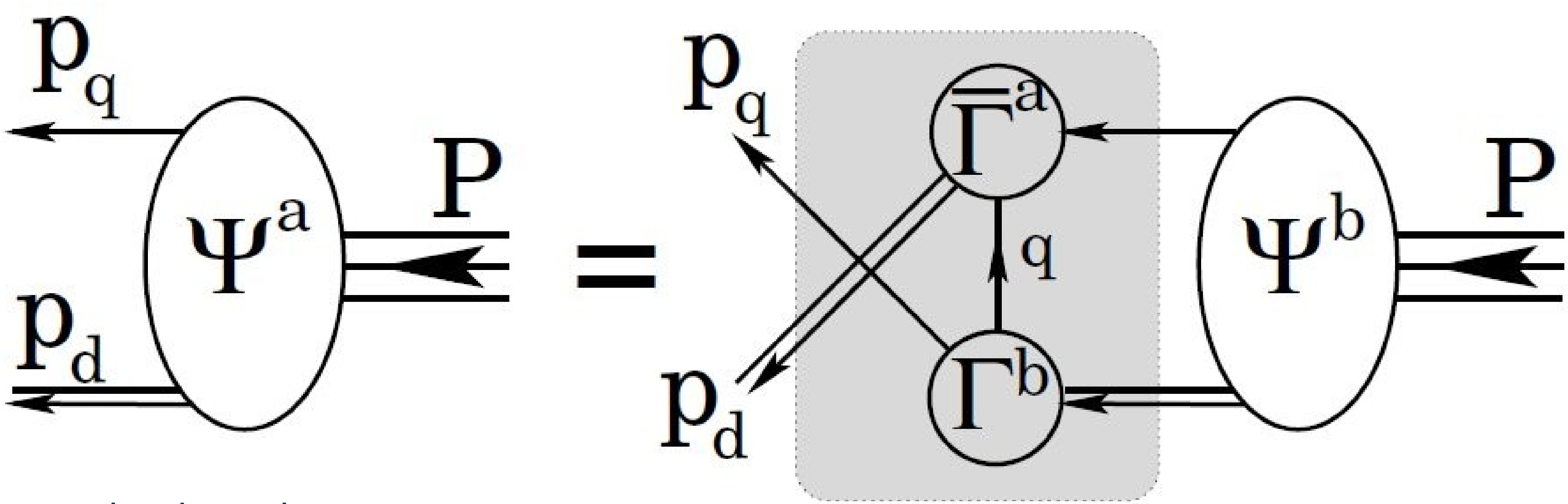
$$\Gamma(k;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma(q;P)S(q)\frac{\lambda^a}{2}\gamma_\nu\,.$$

- In this symmetry-preserving truncation, colour-antitriplet quark-quark correlations (diquarks) are described by a very similar homogeneous Bethe-Salpeter equation

$$\Gamma_{qq}(k;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\frac{\lambda^a}{2}\gamma_\nu$$

- Only difference is factor of ½
- Hence, an interaction that describes mesons also generates diquark correlations in the colour-antitriplet channel

Faddeev Equation



scalar diquark component

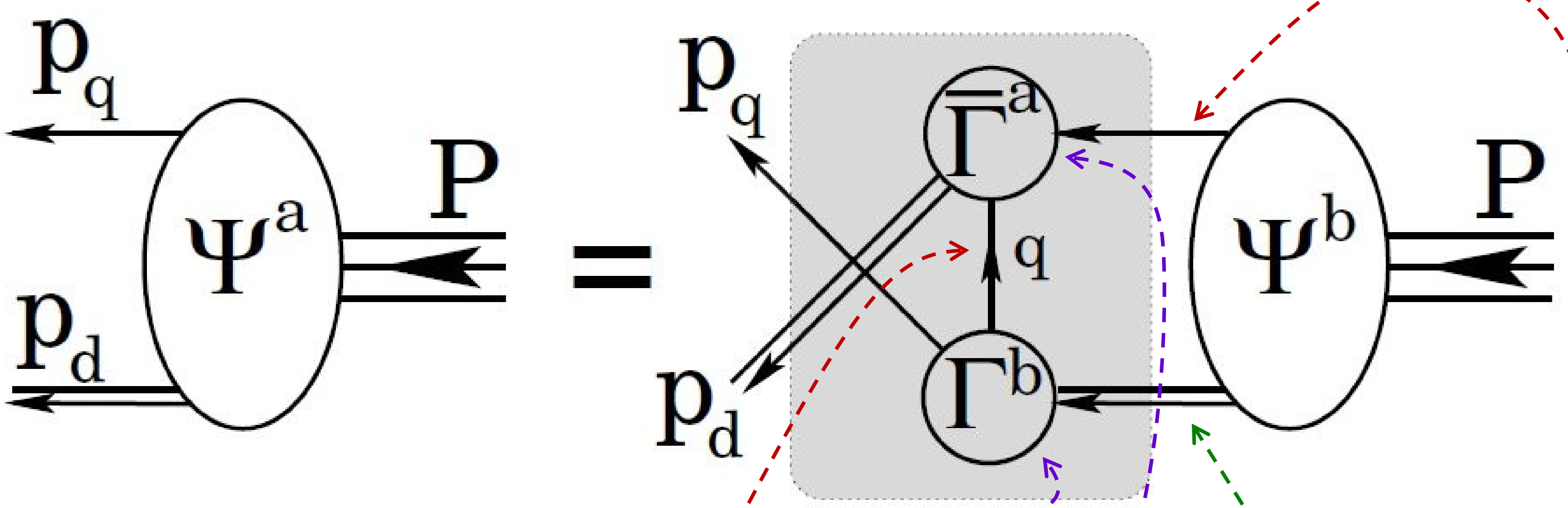
$$\begin{bmatrix} \mathcal{S}(k;P)\,u(P) \\ \mathcal{A}^i_\mu(k;P)\,u(P) \end{bmatrix} = -4 \int \frac{d^4\ell}{(2\pi)^4}\, \mathcal{M}(k,\ell;P) \begin{bmatrix} \mathcal{S}(\ell;P)\,u(P) \\ \mathcal{A}^j_\nu(\ell;P)\,u(P) \end{bmatrix}$$

axial-vector diquark component

$$\mathcal{M}(k,\ell;P) = \begin{bmatrix} \mathcal{M}_{00} & (\mathcal{M}_{01})^j_\nu \\ (\mathcal{M}_{10})^i_\mu & (\mathcal{M}_{11})^{ij}_{\mu\nu} \end{bmatrix}$$

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Faddeev Equation



$$\mathcal{M}_{00} = \Gamma^{0^+}(k_q - \ell_{qq}/2; \ell_{qq})\, S^{\mathrm{T}}(\ell_{qq} - k_q)$$
$$\times\, \bar{\Gamma}^{0^+}(\ell_q - k_{qq}/2; -k_{qq})\, S(\ell_q)\, \Delta^{0^+}(\ell_{qq})$$

$$\ell_q = \ell + P/3,\ k_q = k + P/3,$$
$$\ell_{qq} = -\ell + 2P/3,\ k_{qq} = -k + 2P/3$$

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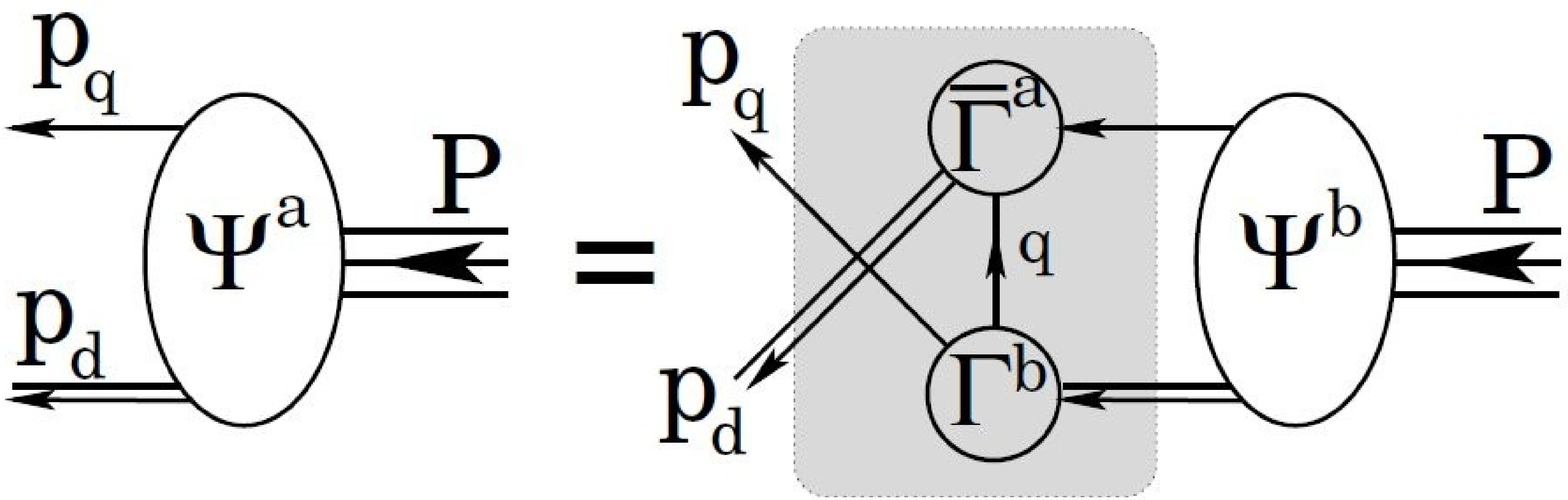
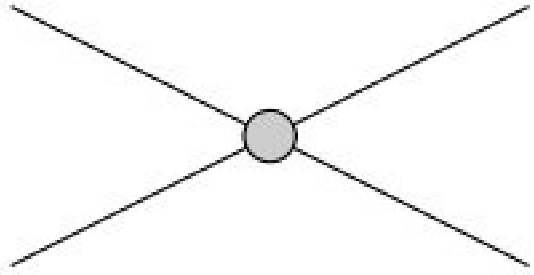
Faddeev Equation



- Every one of these entries has a simple matrix structure
- Similar form for the kernel entries that involve axial-vector diquark correlations
- Combining everything, one arrives at a linear homogeneous matrix equation for the amplitudes

  $S(k;P)u(P)$, $A(k;P)u(P)$

Voyage of Discovery



Contact-Interaction Kernel

- Vector-vector contact interaction

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{IR}}{m_G^2}$$

m_G = 800MeV is a gluon mass-scale

– dynamically generated in QCD

- Gap equation: $M_f = m_f + M_f \frac{4\alpha_{IR}}{3\pi m_G^2} \int_0^\infty ds\, s\, \frac{1}{s+M_f^2}$
- DCSB: *$M \neq 0$* is possible so long as $\alpha_{IR} > \alpha_{IR}^{critical} = 0.4\pi$
- Observables require $\alpha_{IR} = 0.93\pi$

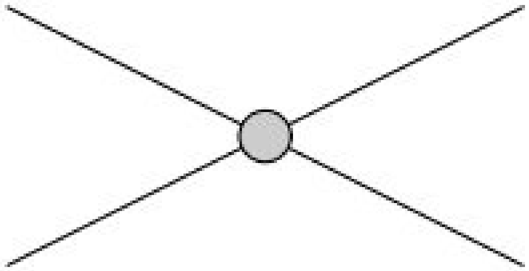
Symmetry-preserving treatment of vector-vector contact-interaction: series of papers establishes strengths & limitations.

Contact Interaction

- **arXiv:1204.2553 [nucl-th]**, Few Body Syst. (2012) DOI: 10.1007/s00601-012-0466-3
  Spectrum of Hadrons with Strangeness,
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson
- **arXiv:1112.2212 [nucl-th]**, Phys. Rev. C**85** (2012) 025205 [21 pages]
  Nucleon and Roper electromagnetic elastic and transition form factors,
  D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
- **arXiv:1102.4376 [nucl-th]**, Phys. Rev. C **83**, 065206 (2011) [12 pages] ,
  π- and ρ-mesons, and their diquark partners, from a contact interaction,
  H.L.L. Roberts, A. Bashir, L.X. Gutiérrez-Guerrero, C.D. Roberts and David J. Wilson
- **arXiv:1101.4244 [nucl-th]**, Few Body Syst. **51** (2011) pp. 1-25
  Masses of ground and excited-state hadrons
  H.L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts
- **arXiv:1009.0067 [nucl-th]**, Phys. Rev. C**82** (2010) 065202 [10 pages]
  Abelian anomaly and neutral pion production
  Hannes L.L. Roberts, C.D. Roberts, A. Bashir, L. X. Gutiérrez-Guerrero & P. C. Tandy
- **arXiv:1002.1968 [nucl-th]**, Phys. Rev. C **81** (2010) 065202 (5 pages)
  Pion form factor from a contact interaction
  L. Xiomara Gutiérrez-Guerrero, Adnan Bashir, Ian C. Cloët and C. D. Roberts

Contact Interaction

- Symmetry-preserving treatment of vector×vector contact interaction is useful tool for the study of phenomena characterised by probe momenta less-than the dressed-quark mass.
- Whilst this interaction produces form factors which are too hard, interpreted carefully, even they can be used to draw valuable insights; e.g., concerning relationships between different hadrons.
- Studies employing a symmetry-preserving regularisation of the contact interaction serve as a useful surrogate, exploring domains which analyses using interactions that more closely resemble those of QCD are as yet unable to enter.
- They're critical at present in attempts to use data as tool for charting nature of the quark-quark interaction at long-range; i.e., identifying signals of the running of couplings and masses in QCD.



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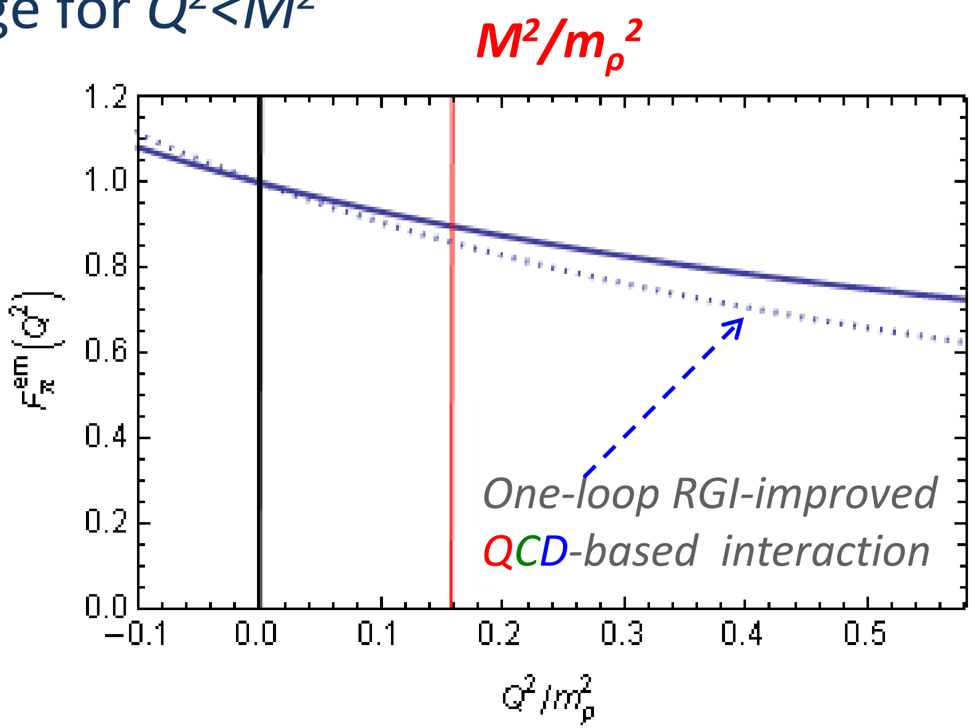

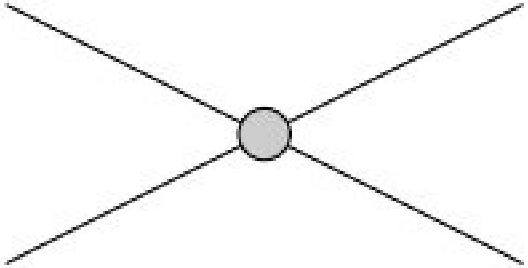

Interaction Kernel - Regularisation Scheme

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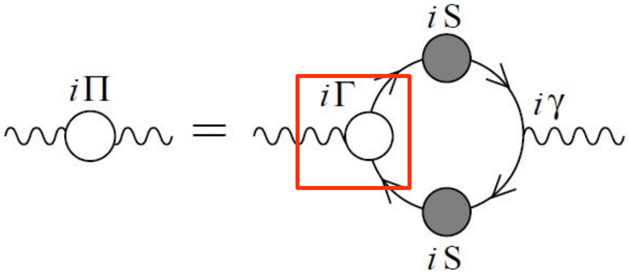
- Must therefore introduce regularisation scheme
  - Use confining proper-time definition

D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B 388 (1996) 154.
No pole in propagator
– DSE realisation of confinement

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, \mathrm{e}^{-\tau(s+M^2)} \to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, \mathrm{e}^{-\tau(s+M^2)} = \frac{\mathrm{e}^{-(s+M^2)\tau_{\rm uv}^2} - e^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}$$

- $\Lambda_{ir} = 0.24$GeV, $\tau_{ir} = 1/\Lambda_{ir} = 0.8$fm
  a confinement radius, which is not varied
- Two parameters:
  $m_G=0.13$GeV, $\Lambda_{uv}=0.91$GeV
  fitted to produce tabulated results

	contact interaction
M	0.37
κ_π	0.24
m_π	0.14
m_ρ	0.93
f_π	0.10
f_ρ	0.13



Regularisation & Symmetries

- In studies of the hadron spectrum it's critical that an approach satisfy the vector and axial-vector Ward-Takahashi identities.
  - Without this it is impossible to preserve the pattern of chiral symmetry breaking in QCD & hence a veracious understanding of hadron mass splittings is not achievable.
- Contact interaction should & can be regularised appropriately
- Example: dressed-quark-photon vertex
  - Contact interaction plus rainbow-ladder entails general form

$$\Gamma_\mu(k;Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\mu^L P_L(Q^2)$$

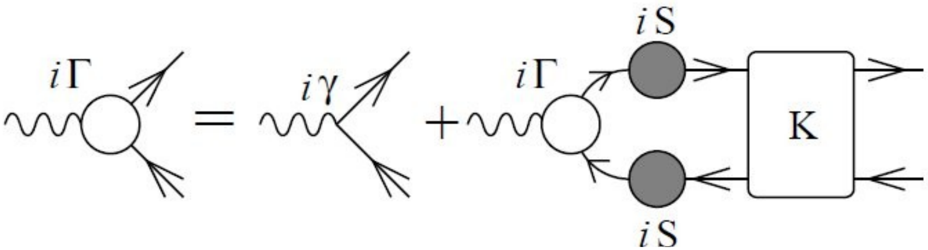
  - Vector Ward-Takahashi identity

$$Q_\mu i\Gamma_\mu(k;Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$$

  - With symmetry-preserving regularisation of contact interaction, Ward Takahashi identity requires

$P_L(Q^2)=1$ & $P_T(Q^2=0)=1$

Interactions cannot generate an on-shell mass for the photon.



BSE - inhomogeneous vector vertex

$$\Gamma_\mu(Q) = \gamma_\mu - \frac{4}{3}\frac{1}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\,\gamma_\alpha \chi_\mu(q_+,q)\gamma_\alpha$$

$$\Gamma_\mu(k;Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\mu^L P_L(Q^2)$$

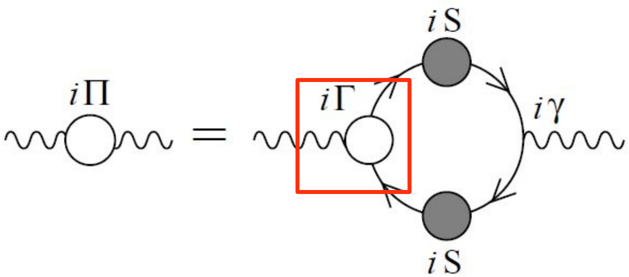
- Solution: $P_L(Q^2) = 1$
  Readily established using vector Ward-Takahashi identity

$$P_T(Q^2) = \frac{1}{1+K_\gamma(Q^2)}$$

$$K_\gamma(Q^2) = \frac{1}{3\pi^2 m_G^2} \times \int_0^1 d\alpha\, \alpha(1-\alpha) Q^2\, \overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,Q^2))$$

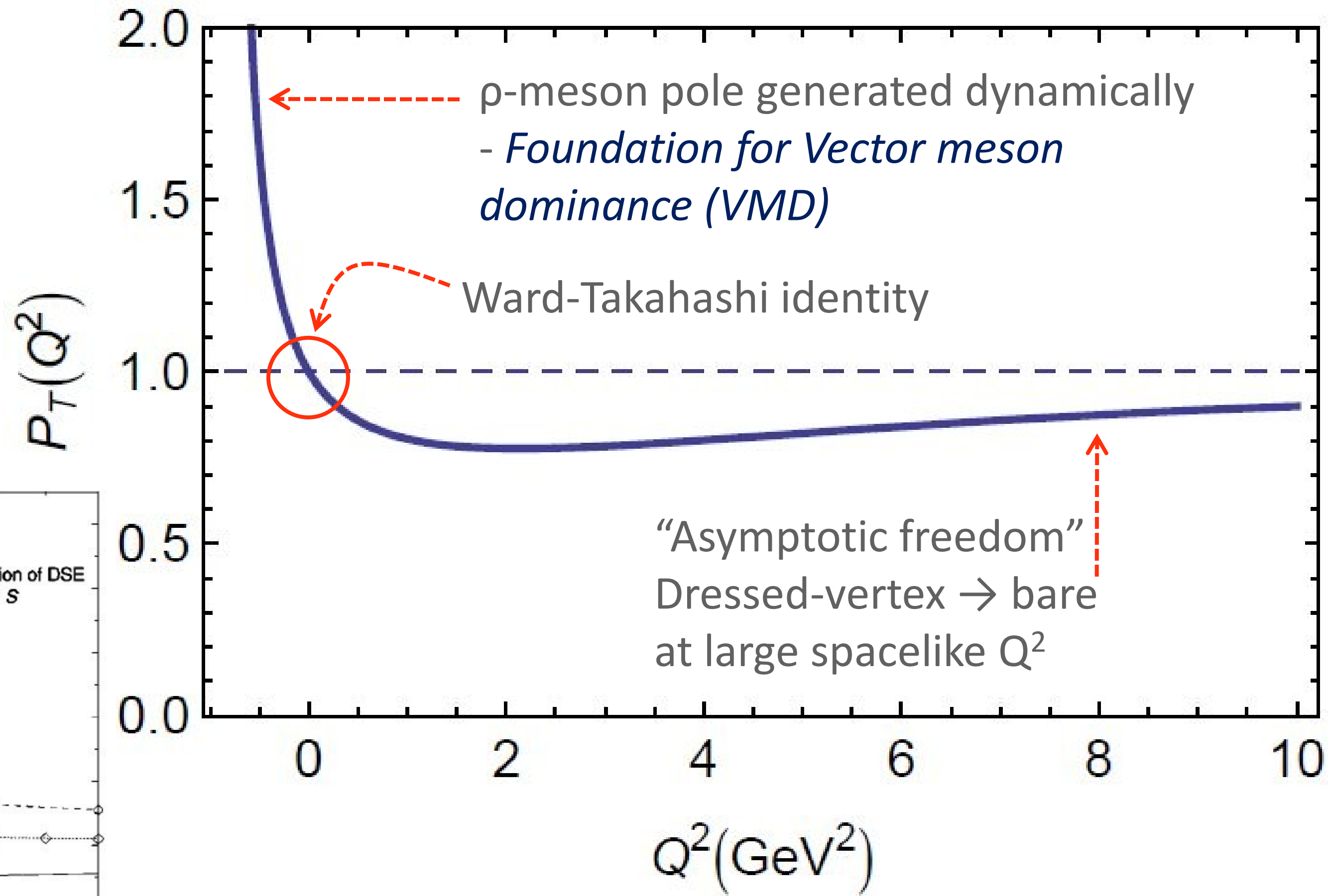
$\omega(x,\alpha,z) = x + \alpha(1-\alpha)z$

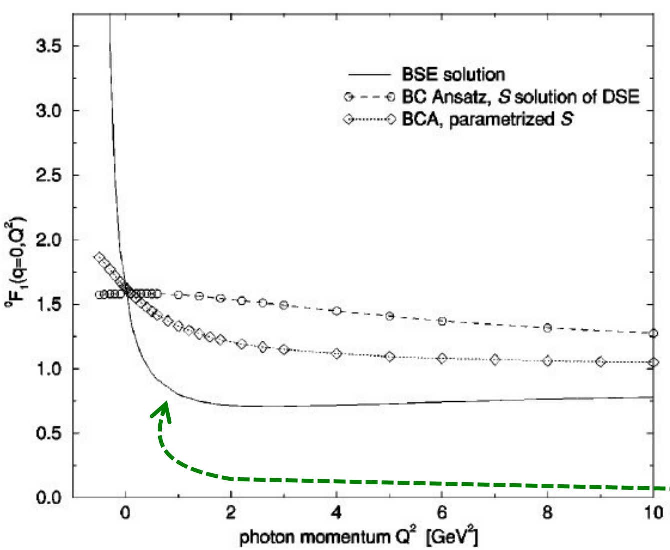
- Plainly: $P_T(Q^2=0) = 1$ because $K_\gamma(Q^2=0)=1$
  Again, power of vector Ward-Takahashi identity revealed



Regularisation & Symmetries

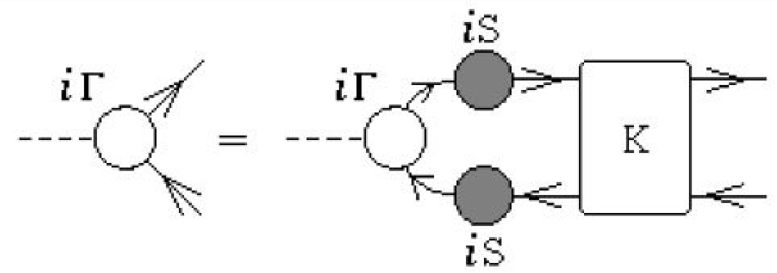
- Solved Bethe-Salpeter equation for dressed-quark photon vertex, regularised using symmetry-preserving scheme





Renormalisation-group-improved one-gluon exchange
Maris & Tandy prediction of $F_\pi(Q^2)$

Craig Roberts: Emergence of DSEs in Real-World QCD 3A (72)



Bethe-Salpeter Equations

- Ladder BSE for *ρ*-meson

$$1 + K^\rho(-m_{1-}^2) = 0\,,\; K^\rho(P^2) = \frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\alpha(1-\alpha)P^2\,\overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,P^2))$$

$$\overline{C}_1^{iu}(\omega \;=\;),M^2 r_{uv}^2) - \;),M^2 r_{ir}^2),C_1^{iu}(\omega \;=\; \omega C_1^{iu}(\omega$$

ω(M²,α,P²)= M² + α(1- α)P²

  - *Contact interaction, properly regularised, provides a practical simplicity & physical transparency*

- Ladder BSE for *a₁*-meson

$$1 + K^{a_1}(-m_{1+}^2) = 0\,,\; K^{a_1}(P^2) = -\frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\mathcal{C}_1^{\rm iu}(\omega(M^2,\alpha,P^2))$$

- All BSEs are one- or –two dimensional eigenvalue problems, eigenvalue is P^2= - (mass-bound-state)²

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Meson Spectrum -Ground-states

- Ground-state masses
  - Computed very often, always with same result

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

But, we know how to fix that (Lecture IV)
viz., DCSB – beyond rainbow ladder
- increases scalar and axial-vector masses
- leaves π & ρ unchanged

- Namely, with rainbow-ladder truncation

$m_{a1} - m_\rho = 0.15\ \text{GeV} \approx \frac{1}{3} \times 0.45_{\text{experiment}}$

	Experiment	Rainbow-ladder	One-loop corrected	Full vertex
a1	1230	759	885	1230
ρ	770	644	764	745
Mass splitting	455	115	121	485

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts, arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Meson Spectrum -Ground-states

- Ground-state masses
  - Correct for omission of DCSB-induced spin-orbit repulsion

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

$m_\sigma^{qq} \approx 1.2$ GeV is location of quark core of σ-resonance:

- Pelaez & Rios (2006)
- Ruiz de Elvira, Pelaez, Pennington & Wilson (2010)

First novel post-diction

- Leave π- & ρ-meson BSEs unchanged but introduce repulsion parameter in scalar and axial-vector channels; viz.,

$$1 + K^{a1}(-m_{1+}^2) = 0\,,\; K^{a1}(P^2) = -\frac{g_{SO}^2}{3\pi^2 m_G^2}\int_0^1 d\alpha\, \mathcal{C}_1^{\rm iu}(\omega(M^2,\alpha,P^2))$$

- $g_{SO}=0.24$ fitted to produce $m_{a1} - m_\rho = 0.45_{\rm experiment}$

Pseudoscalar meson radial excitations
A. Höll, A. Krassnigg & C.D. Roberts,
Phys.Rev. C**70** (2004) 042203(R), nucl-th/0406030

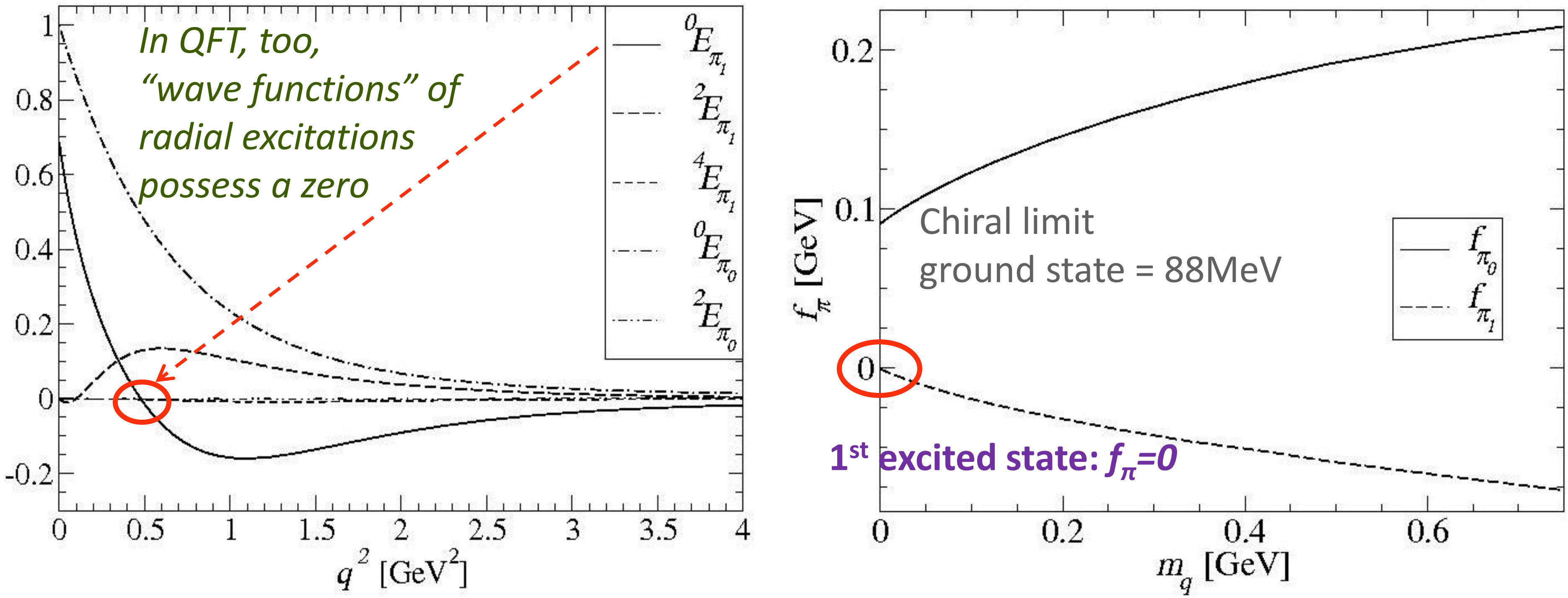
Radial Excitations

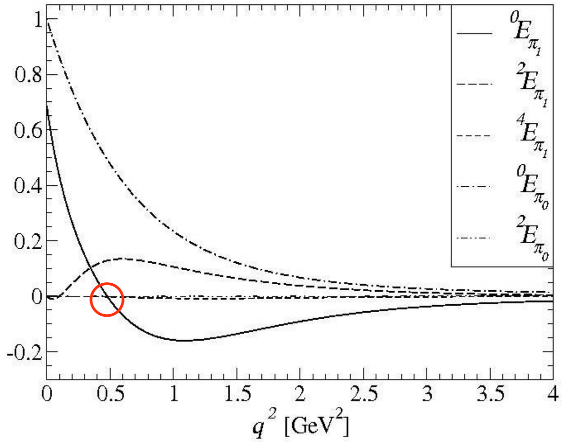
- ***Lecture III***: Goldstone modes are the only pseudoscalar mesons to possess a nonzero leptonic decay constant, f_π , in the chiral limit when chiral symmetry is dynamically broken.
- The decay constants of their radial excitations vanish.
  - In quantum mechanics, decay constants suppressed by factor of roughly ⅓
  - But only a symmetry can ensure that something vanishes completely
- Goldstone's Theorem for non-ground-state pseudoscalars
- These features and aspects of their impact on the meson spectrum were illustrated using a manifestly covariant and symmetry-preserving model of the kernels in the gap and Bethe-Salpeter equations.

see also
A Chiral Lagrangian for excited pions
M.K. Volkov & C. Weiss
Phys. Rev. D**56** (1997) 221, hep-ph/9608347

Radial Excitations

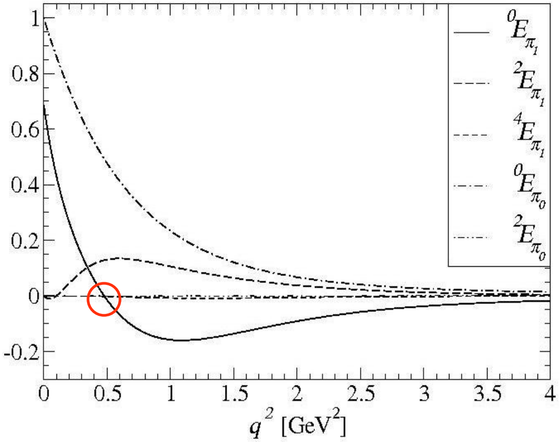
- These features and aspects of their impact on the meson spectrum were illustrated using a manifestly covariant and symmetry-preserving model of the kernels in the gap and Bethe-Salpeter equations.





Meson Spectrum - Radial Excitations

- *Key*: in proceeding with study of baryons – 1st radial excitation of a bound state possess a single zero in the relative-momentum dependence of the leading Tchebychev-moment in their bound-state amplitude
- The existence of radial excitations is therefore obvious evidence against the possibility that the interaction between quarks is momentum-independent:
  - *A bound-state amplitude that is independent of the relative momentum cannot exhibit a single zero*
- One may express this differently; viz.,
  - If the location of the zero is at k_0^2, then a momentum-independent interaction can only produce reliable results for phenomena that probe momentum scales $k^2 < k_0^2$.
  - In QCD, $k_0^2 \approx 2M^2$.



Meson Spectrum - Radial Excitations

- Nevertheless, there exists an established expedient ; viz.,
  - Insert a zero by hand into the Bethe-Salpeter kernels

$(1-k^2/2M^2)$

$$1 + K^{\rho}(-m_{1-}^2) = 0,\ K^{\rho}(P^2) = \frac{1}{3\pi^2 m_G^2} \int_0^1 d\alpha\, \alpha(1-\alpha) P^2\, \overline{\mathcal{C}}_1^{iu}(\omega(M^2, \alpha, P^2))$$

A Chiral Lagrangian for excited pions
M.K. Volkov & C. Weiss
Phys. Rev. D**56** (1997) 221, hep-ph/9608347

- Plainly, the presence of this zero has the effect of reducing the coupling in the BSE & hence it increases the bound-state's mass.
- Although this may not be as transparent with a more sophisticated interaction, a qualitatively equivalent mechanism is always responsible for the elevated values of the masses of radial excitations.
- Location of zero fixed at "natural" location: $k^2=2M^2$ – not a parameter

plus predicted diquark spectrum

Meson Spectrum Ground- & Excited-States

- Complete the table …

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
RL $* g_{SO}^2$	0.14	0.93	1.29	1.38
experiment	0.14	0.78	0.4 – 1.2	1.24

- Error estimate for radial excitations:

  Shift location of zero by ±20%

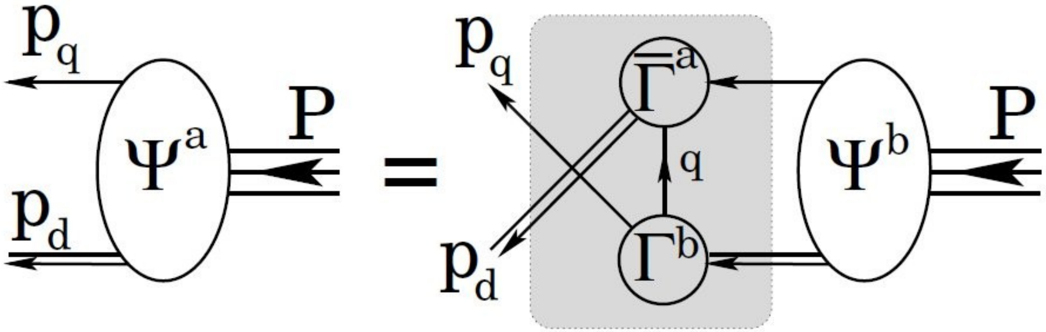
- *rms-relative-error/degree-of-freedom = 13%*

FIRST results for other *qq* quantum numbers

They're critical for excited states of N and Δ

	$m_{qq_{0+}}$	$m_{qq_{1+}}$	$m_{qq_{0-}}$	$m_{qq_{1-}}$	$m_{qq^*_{0+}}$	$m_{qq^*_{1+}}$	$m_{qq^*_{0-}}$	$m_{qq^*_{1-}}$
RL	0.78	1.06	0.93	1.16	1.39 ± 0.06	1.32 ± 0.05	1.42 ± 0.05	1.33 ± 0.05
RL $* g_{SO}^2$	0.78	1.06	1.37	1.45	1.39 ± 0.06	1.32 ± 0.05	1.50 ± 0.03	1.52 ± 0.02

- *No parameters*
- *Realistic* DSE estimates: $m_{0+}=0.7\text{-}0.8$, $m_{1+}=0.9\text{-}1.0$
- Lattice-QCD estimate: $m_{0+}=0.78 \pm 0.15$, $m_{1+}-m_{0+}=0.14$



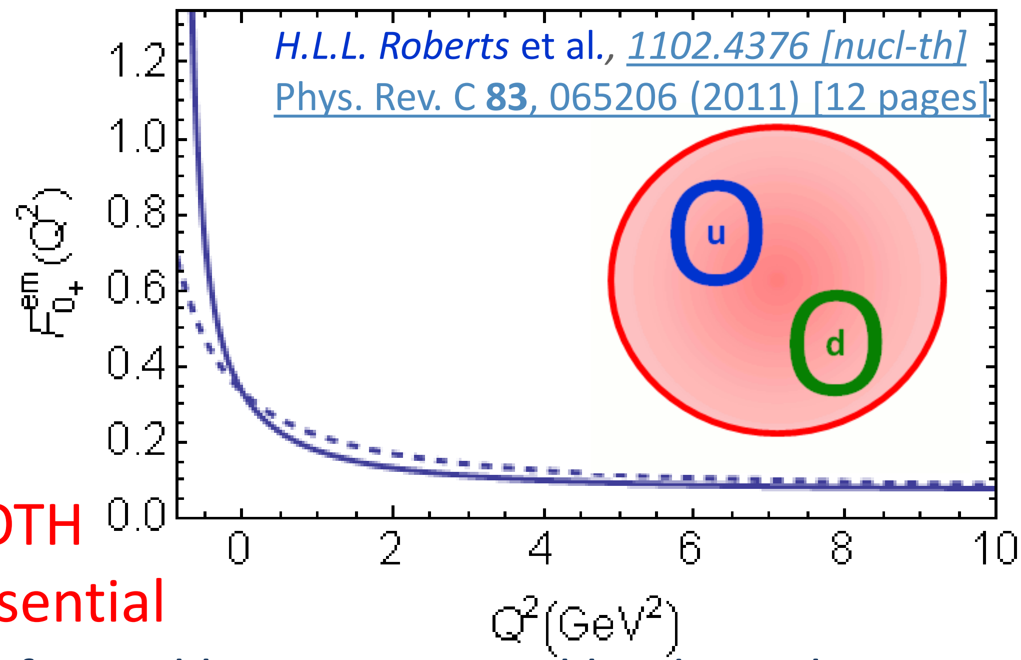
Diquarks in QCD

Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

	$m_{qq_{0+}}$	$m_{qq_{1+}}$	$m_{qq_{0-}}$	$m_{qq_{1-}}$	$m_{qq^*_{0+}}$	$m_{qq^*_{1+}}$	$m_{qq^*_{0-}}$	$m_{qq^*_{1-}}$
RL	0.78	1.06	0.93	1.16	1.39 ± 0.06	1.32 ± 0.05	1.42 ± 0.05	1.33 ± 0.05
RL $* g^2_{SO}$	0.78	1.06	1.37	1.45	1.39 ± 0.06	1.32 ± 0.05	1.50 ± 0.03	1.52 ± 0.02

- "Spectrum" of *nonpointlike* quark-quark correlations
- Observed in
  - DSE studies in QCD
  - 0^+ & 1^+ in Lattice-QCD
- Scalar diquark form factor
  - $r_{0+} \approx r_\pi$
- Axial-vector diquarks
  - $r_{1+} \approx r_\rho$

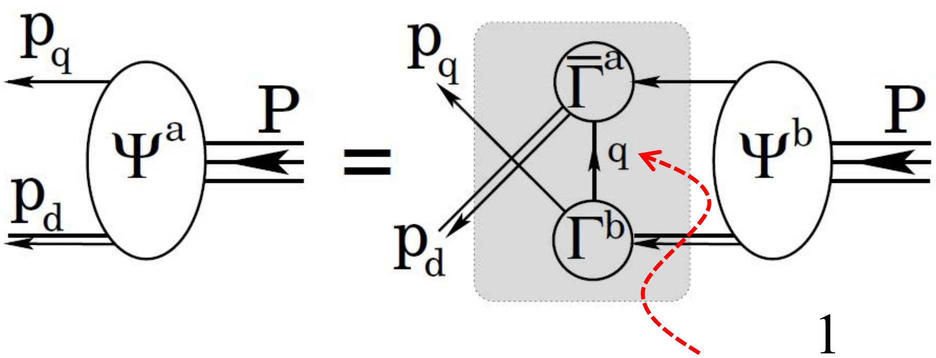
BOTH essential



- *Zero* relation with old notion of pointlike constituent-like diquarks

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Spectrum of Baryons



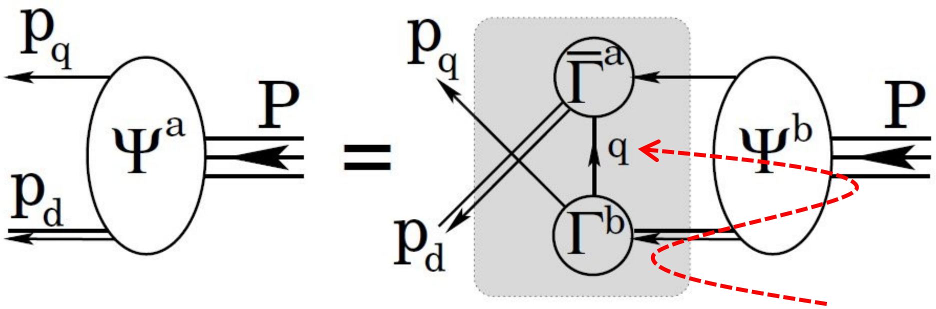
$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

Variant of:
A. Buck, R. Alkofer & H. Reinhardt, Phys. Lett. ***B286*** *(1992) 29.*

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- In combination with contact-interaction diquark-correlations, generates Faddeev equation kernels which themselves are momentum-independent
- The merit of this truncation is the *dramatic simplifications* which it produces
- Used widely in hadron physics phenomenology; e.g., Bentz, Cloët, Thomas *et al.*

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Spectrum of Baryons

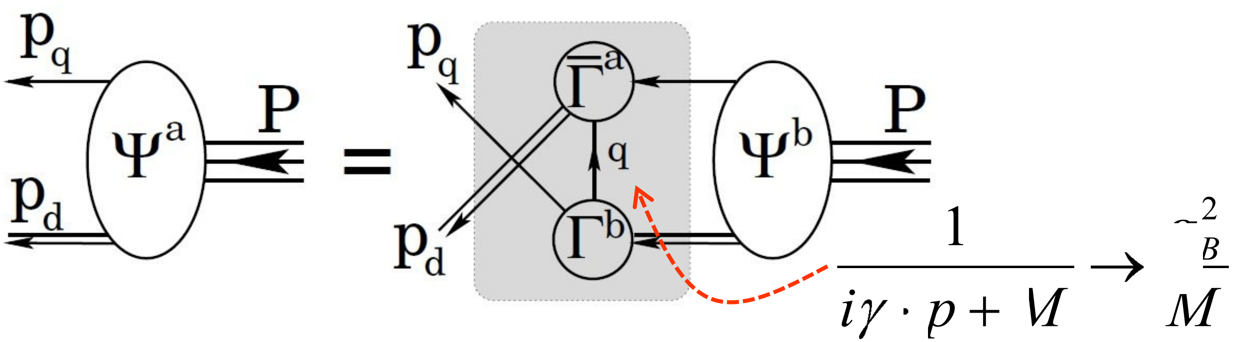


$$\frac{1}{i\gamma \cdot p + M} \rightarrow \frac{\sim^{2}_{B}}{M}$$

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- From the referee's report:
  In these calculations one could argue that the [static truncation] *is the weakest* [approximation]*. From what I understand, it is not of relevance here since the aim is to understand the dynamics of the interactions between the* [different] *types of diquark correlations with the spectator quark and their different contributions to the baryon's masses … this study illustrates rather well what can be expected from more sophisticated models, whether within a Dyson-Schwinger or another approach. … I can recommend the publication of this paper without further changes.*

$\overline{C}_1^{iu}(\omega = \,\,), M^2 r_{uv}^2) - \,\,), M^2 r_{ir}^2), C_1^{iu}(\omega = \,\, C_1^{iu}(\omega$

Spectrum of Baryons



With the right glasses; i.e., those polished by experience with the DSEs, one can look at this equation and see that increasing the current-quark mass will boost the mass of the bound-state

- Faddeev equation for Δ-resonance

$$1 = 8\frac{g_\Delta^2}{M}\frac{E_{qq_1+}^2}{m_{qq_1+}^2}\int\frac{d^4\ell'}{(2\pi)^4}\int_0^1 d\alpha\,\frac{(m_{qq_1+}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M)}{[\ell'^2 + \sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta)]^2}$$

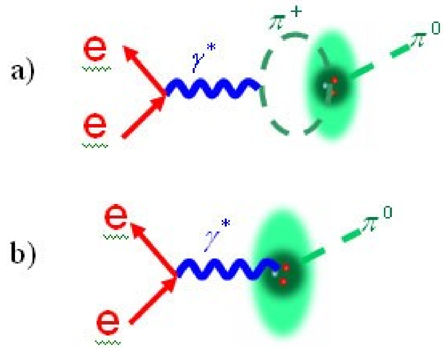
$$= \frac{g_\Delta^2}{M}\frac{E_{qq_1+}^2}{m_{qq_1+}^2}\frac{1}{2\pi^2}\int_0^1 d\alpha\,(m_{qq_1+}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M)\overline{\mathcal{C}}_1^{\mathrm{iu}}(\sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta))$$

- *One-dimensional eigenvalue problem, to which only the axial-vector diquark contributes*
- *Nucleon has scalar & axial-vector diquarks. It is a five-dimensional eigenvalue problem*

What's missing?

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Pion cloud



- Kernels constructed in the rainbow-ladder truncation do not contain any long-range interactions
  - These kernels are built only from dressed-quarks and -gluons
- But, QCD produces a very potent long-range interaction; namely that associated with the pion, without which no nuclei would be bound and we wouldn't be here
- The rainbow-ladder kernel produces what we describe as the hadron's dressed-quark core
- The contribution from pions is omitted, and can be added without "double counting"
- The pion contributions must be thoughtfully considered before any comparison can be made with the real world

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts, arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

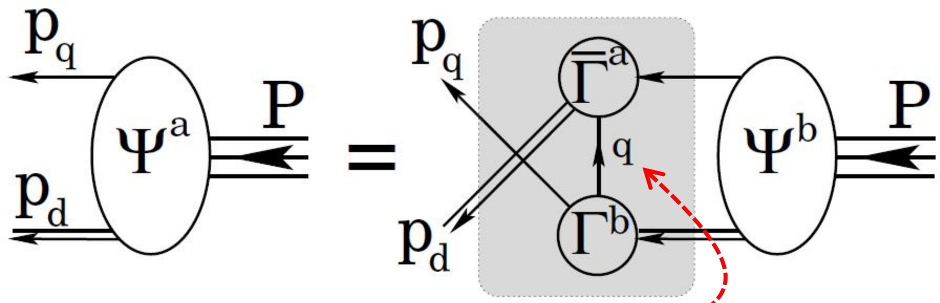
M.B. Hecht, M. Oettel, C.D. Roberts, S.M. Schmidt, P.C.Tandy and A.W. Thomas, Phys.Rev. C65 (2002) 055204

Pion cloud

Pion cloud typically reduces a hadron's mass

- The body of results described hitherto suggest that whilst corrections to our truncated DSE kernels may have a material impact on m_N and m_Δ separately, the modification of each is approximately the same, so that the mass-difference, δm, is largely unaffected by such corrections.
- This is consistent with analyses that considers the effect of pion loops, which are explicitly excluded in the rainbow-ladder truncation: whilst the individual masses are reduced by roughly 300MeV, the mass difference, δm, increases by only 50MeV.
- With the educated use of appropriate formulae, one finds that pion-loops yields a shift of (*−300*MeV) in m_N and (*−270*MeV) in m_Δ, from which one may infer that the uncorrected Faddeev equations should produce $m_N = 1.24$GeV and $m_\Delta = 1.50$GeV

Pion cloud



$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

All three spectrum parameters now fixed *(g_{SO}=0.24)*

- One can actually do better owing to the existence of the Excited Baryon Analysis Center (EBAC), which for five years has worked to understand and quantify the effects of a pseudoscalar meson cloud on baryons
- For the Δ-resonance, EBAC's calculations indicate that the dressed-quark core of this state should have

$$m_\Delta^{qqq} = 1.39\text{GeV}$$

- These observations indicate that the dressed-quark core Faddeev equations should yield

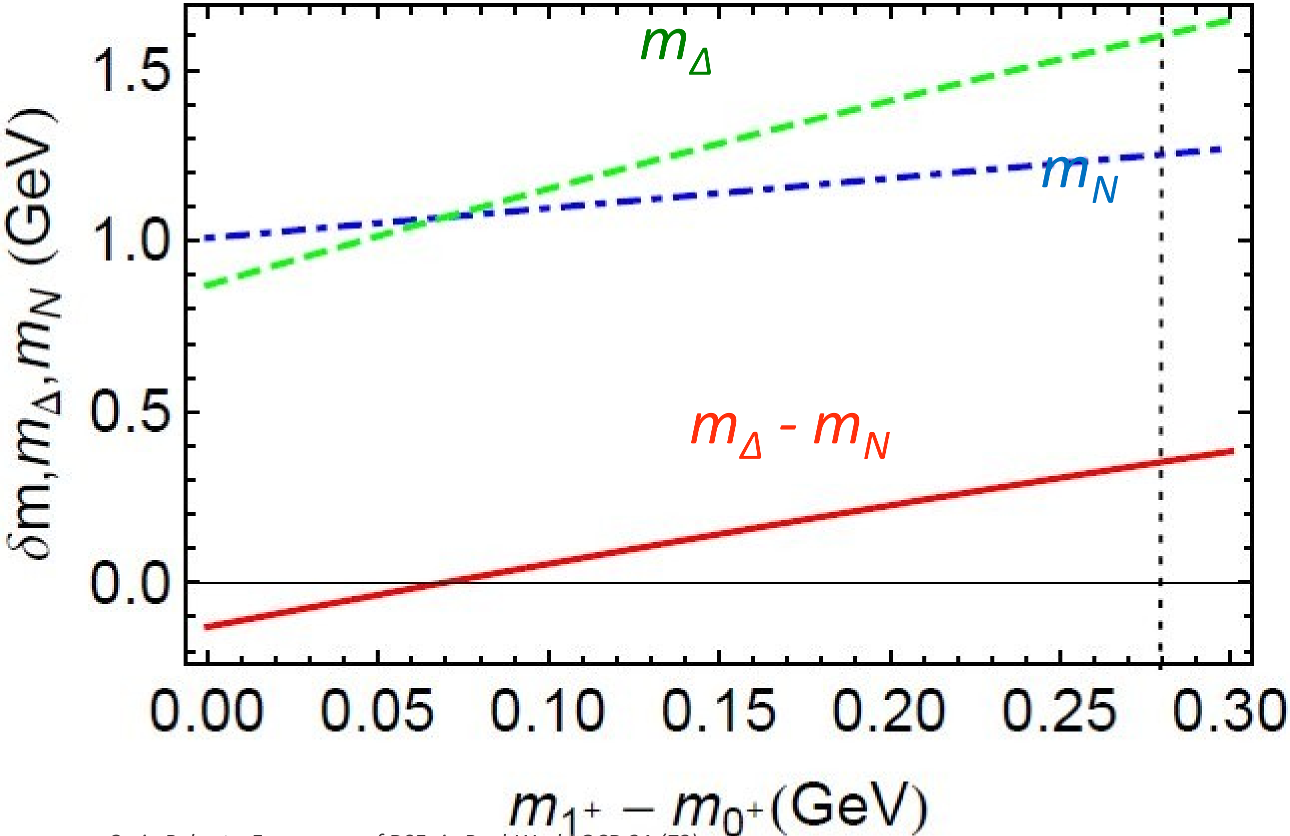
$$m_N = 1.14\text{GeV},\ m_\Delta = 1.39\text{GeV},\ \delta m = 0.25\text{GeV}$$

which requires $g_N = 1.18$, $g_\Delta = 1.56$

Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët
and Craig D. Roberts, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

Baryons & diquarks

- From apparently simple material, one arrives at a powerful elucidative tool, which provides numerous insights into baryon structure; e.g.,
  - *There is a causal connection between m_Δ - m_N & m_{1+}- m_{0+}*



Physical splitting grows rapidly with increasing diquark mass difference

Baryons & diquarks

- Provided numerous insights into baryon structure; e.g.,
  - $m_N \approx 3\,M$ & $m_\Delta \approx M+m_{1+}$

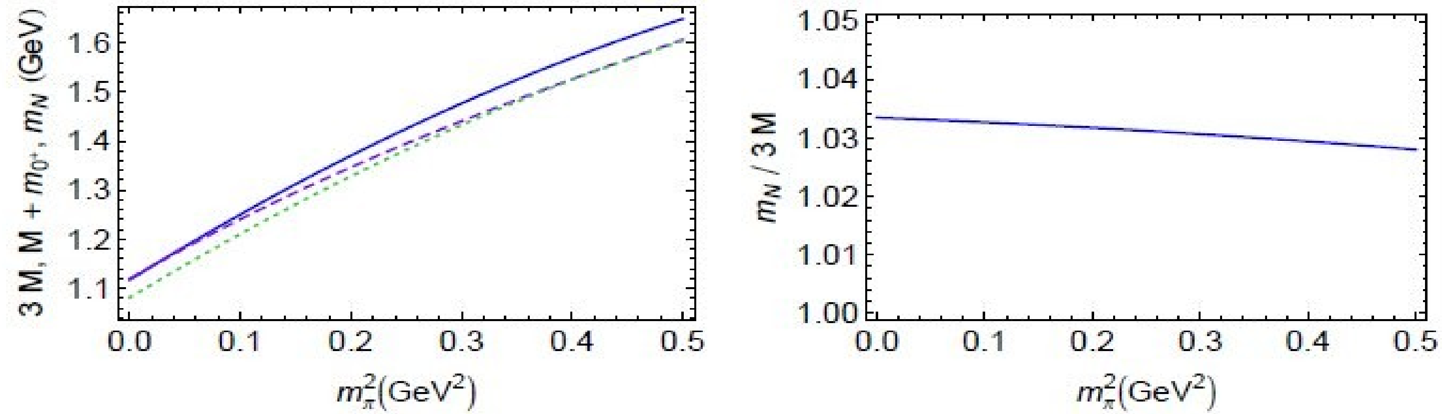


Fig. 3 *Left panel* – Evolution with current-quark mass of the: nucleon mass, m_N (*solid curve*); the sum $[M + m_{qq_{0+}}]$ (*dashed curve*); and $3\,M$ (*dotted curve*). *Right panel* – Evolution with current-quark mass of the ratio $m_N/[3M]$, which varies by less-than 1% on the domain depicted.

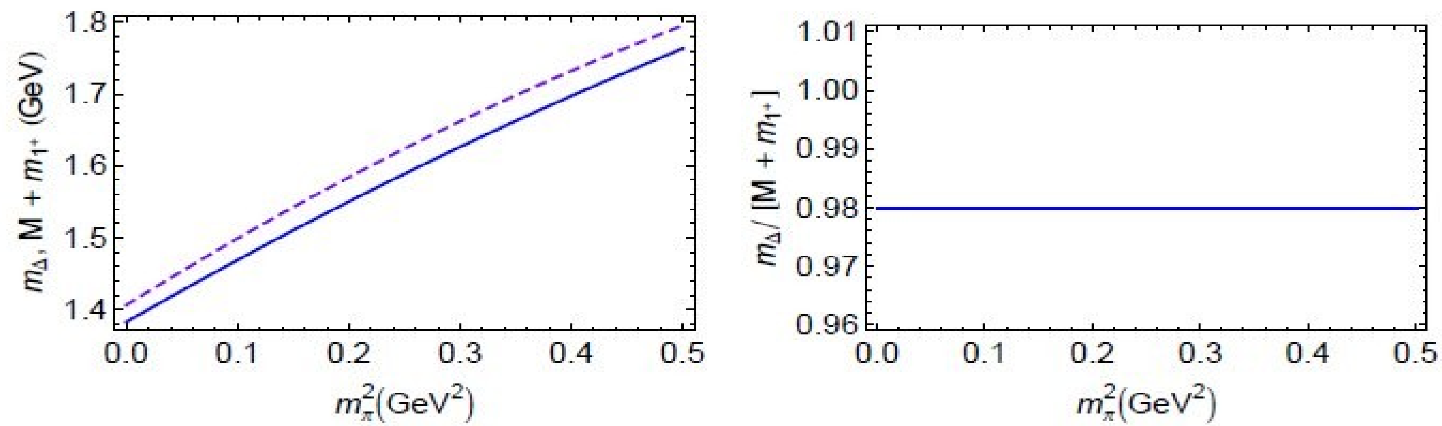


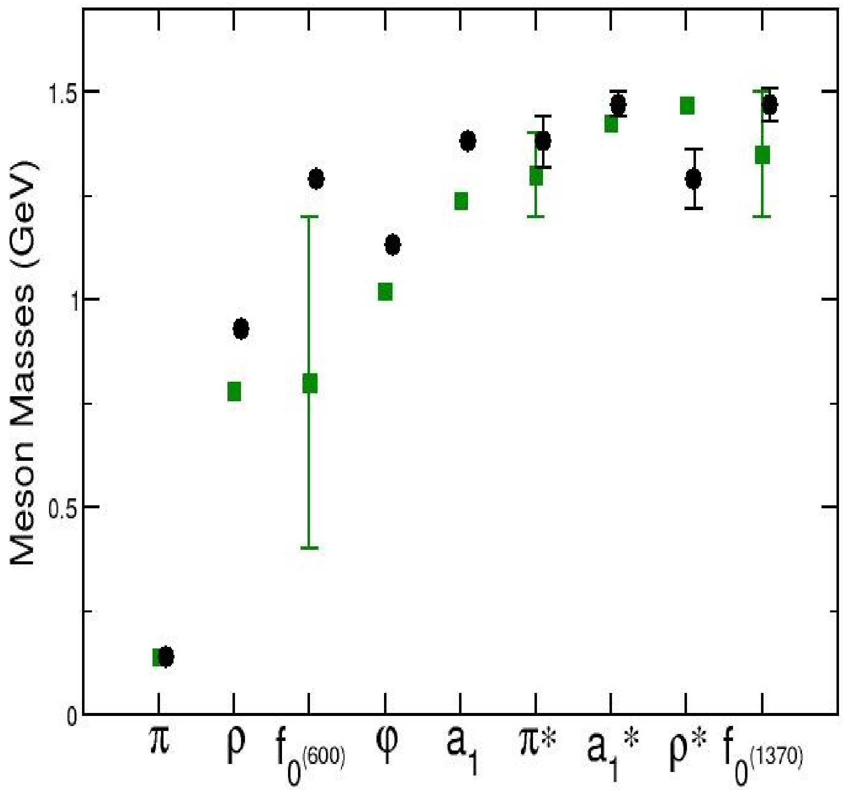
Fig. 4 *Left panel* – Evolution with current-quark mass of the: Δ mass, m_Δ (*solid curve*); and $[M + m_{qq_{1+}}]$ (dashed curve). *Right panel* – Evolution with current-quark mass of the ratio $m_\Delta/[M + m_{qq_{1+}}]$, which does not vary noticeably on the domain depicted.

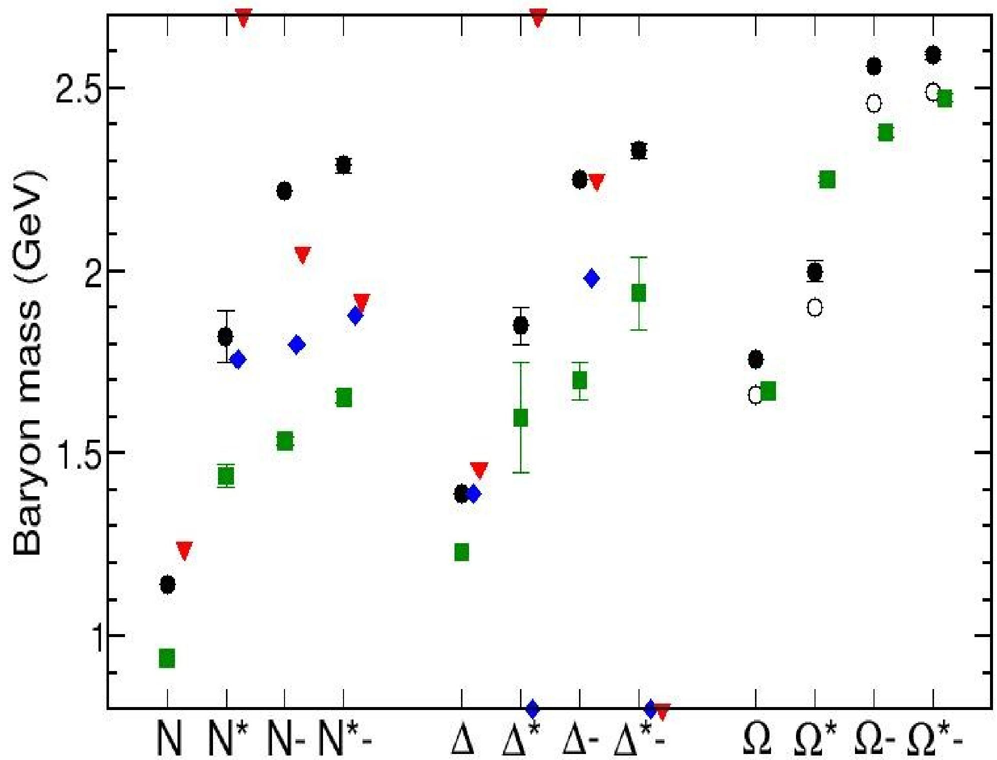
Craig Roberts: Emergence of DSEs in Real-World QCD 3A (72)

Hadron Spectrum

Legend:
- ■ Particle Data Group
- ● H.L.L. Roberts *et al.*
- ◆ EBAC
- ▼ Jülich

- Symmetry-preserving unification of the computation of meson & baryon masses
- rms-rel.err./deg-of-freedom = 13%
- PDG values (almost) uniformly overestimated in both cases
  - room for the pseudoscalar meson cloud?!





Masses of ground and excited-state hadrons
H.L.L. Roberts *et al.*, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

Baryon Spectrum

Table 4 *Row-1*: Dressed-quark-core masses for nucleon and Δ, their first radial excitations (denoted by "$*$"), and the parity-partners of these states, computed with $g_N = 1.18$, $g_\Delta = 1.56$, and the parameter values in Eq. (25) and Table 1. The errors on the masses of the radial excitations indicate the effect of shifting the location of the zero according to Eq. (30). *Row-2*: Bare-masses inferred from a coupled-channels analysis at the Excited Baryon Analysis Center (EBAC) [65]. EBAC's method does not provide a bare nucleon mass. *Row-3*: Bare masses inferred from the coupled-channels analysis described in Ref. [67], which describes the Roper resonance as dynamically-generated. In both these rows, "..." indicates states not found in the analysis. A visual comparison of these results is presented in Fig. 7.

	m_N	m_{N^*}	$m_{N\frac{1}{2}^-}$	$m_{N^*\frac{1}{2}^-}$	m_Δ	m_{Δ^*}	$m_{\Delta\frac{3}{2}^-}$	$m_{\Delta^*\frac{3}{2}^-}$
PDG label	N	$N(1440)\ P_{11}$	$N(1535)\ S_{11}$	$N(1650)\ S_{11}$	$\Delta(1232)\ P_{33}$	$\Delta(1600)\ P_{33}$	$\Delta(1700)\ D_{33}$	$\Delta(1940)\ D_{33}$
This work	1.14	1.82±0.07	2.22	2.29 ± 0.02	1.39	1.85 ± 0.05	2.25	2.33 ± 0.02
EBAC		1.76	1.80	1.88	1.39	...	1.98	...
Jülich	1.24	none	2.05	1.92	1.46	...	2.25	...

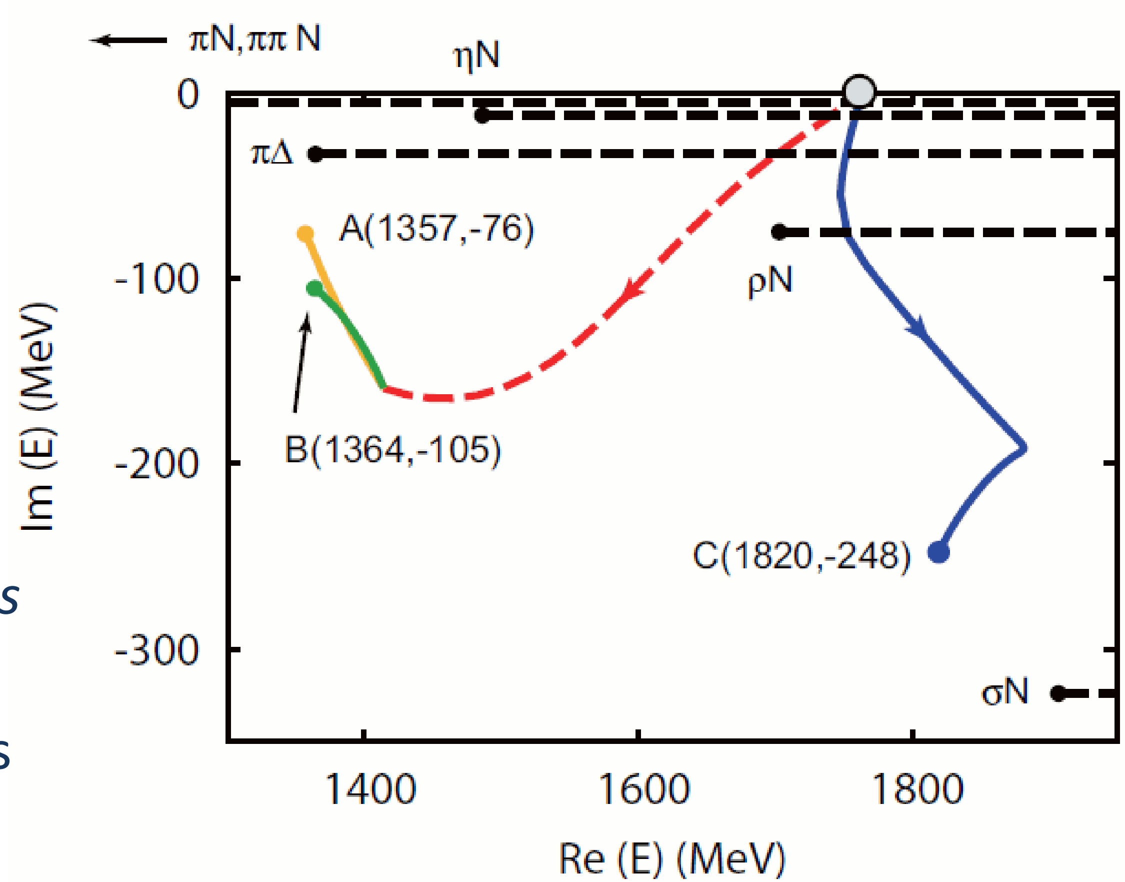
- In connection with EBAC's analysis, dressed-quark Faddeev-equation predictions for bare-masses agree within rms-relative-error of 14%.
  - *Notably, EBAC finds a dressed-quark-core for the Roper resonance, at a mass which agrees with Faddeev Eq. prediction.*

Roper Resonance

- Consider the *N(1440)*P_{11}, $J^P = (1/2)^+$ "Roper resonance," whose discovery was reported in 1964 – part of Roper's PhD thesis
- In important respects the Roper appears to be a copy of the proton. However, its (Breit-Wigner) mass is 50% greater.
- Features of the Roper have long presented a problem within the context of constituent-quark models formulated in terms of color-spin potentials, which typically produce a mass of $2\ M_N$ and the following level ordering:
  - ground state, $J^P = (1/2)^+$ with radial quantum number $n = 0$ and angular momentum $l = 0$;
  - first excited state, $J^P = (1/2)^-$ with $(n, l) = (0, 1)$;
  - second excited state, $J^P = (1/2)^+$, with $(n, l) = (1, 0)$; *etc.*
- The difficulty is that the lightest $l = 1$ *baryon appears to be the* *N(1535)*S_{11}, which is heavier than the Roper!

EBAC & the Roper resonance

- EBAC examined the dynamical origins of the two poles associated with the Roper resonance are examined.
- Both of them, together with the next higher resonance in the P_{11} partial wave were found to have the same originating bare state
- Coupling to the meson-baryon continuum induces multiple observed *resonances* from the same bare state.
- All PDG identified resonances consist of a core state and meson-baryon components.



EBAC
& the Roper resonance

- *Nuclear Physics: Exploring the Heart of Matter*
  Decadal Report, issued 2012, by
  the National Academy of Sciences
  - In a recent breakthrough, theorists at the Excited Baryon Analysis Center (EBAC) at Jefferson Lab

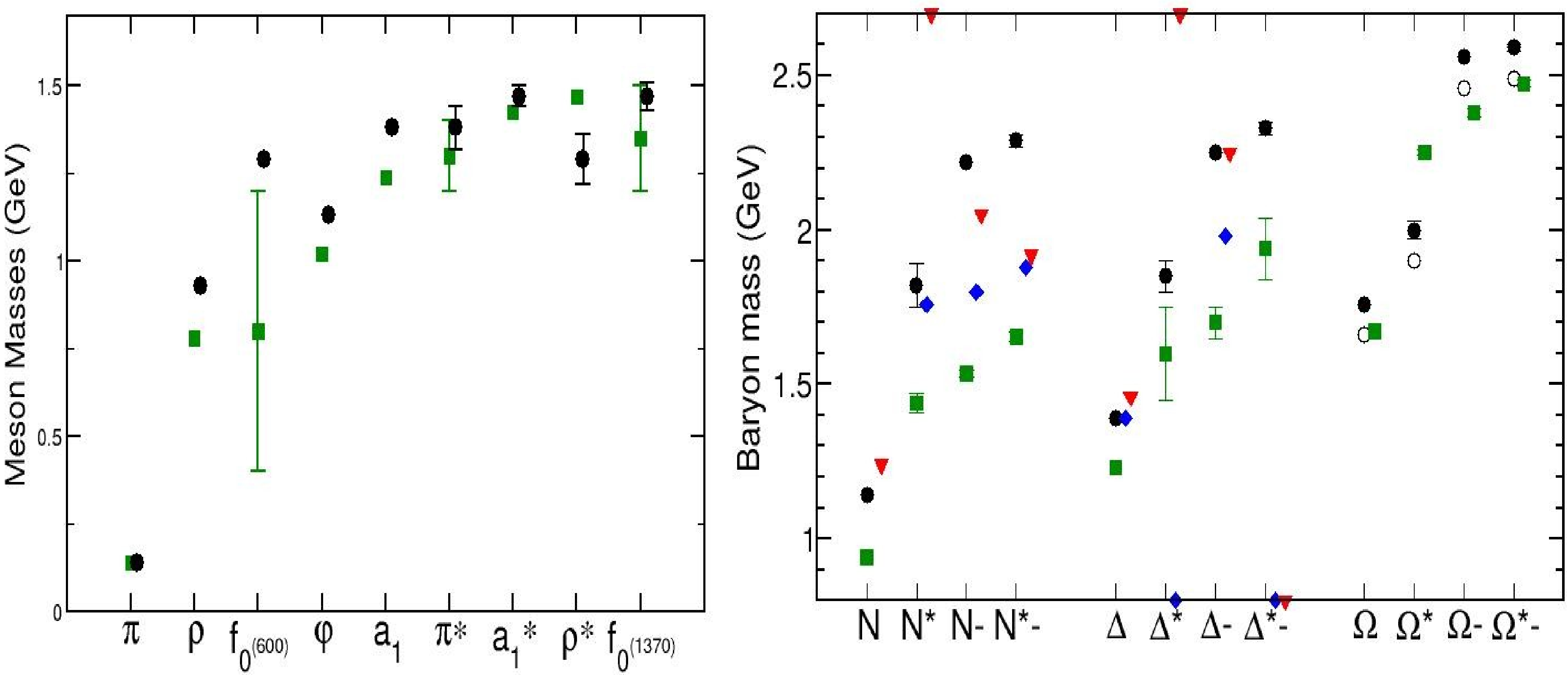
    – led by T.-S. H. Lee, Argonne –

    demonstrated that the Roper resonance is the proton's first radial excitation, with its lower-than-expected mass coming from a quark core shielded by a dense cloud of pions and other mesons.
  - This breakthrough was enabled by both new analysis tools and new high quality data from the CLAS-Collaboration.

Hadron Spectrum

Legend:
- Particle Data Group
- H.L.L. Roberts *et al.*
- EBAC
- Jülich

Now and for the foreseeable future, QCD-based theory will provide only dressed-quark core masses;
EBAC or EBAC-like tools necessary for mesons and baryons



Recapitulation

- One method by which to validate QCD is computation of its hadron spectrum and subsequent comparison with modern experiment. Indeed, this is an integral part of the international effort in nuclear physics.
- For example, the N^* programme and the search for hybrid and exotic mesons together address the questions:
  - which hadron states and resonances are produced by QCD?
  - how are they constituted?
- This intense effort in hadron spectroscopy is a motivation to extend the research just described and treat ground- and excited-state hadrons with *s*-quark content. (New experiments planned in Japan)
- Key elements in a successful spectrum computation are:
  - symmetries and the pattern by which they are broken;
  - the mass-scale associated with confinement and DCSB;
  - and full knowledge of the physical content of bound-state kernels.

All this is provided by the DSE approach.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson, Few Body Syst. (2012) DOI: 10.1007/s00601-012-0466-3

Spectrum of Hadrons with Strangeness

- Solve gap equation for *u* & *s*-quarks

Table 1 Computed dressed-quark properties, required as input for the Bethe-Salpeter and Faddeev equations, and computed values for in-hadron condensates [52; 53; 54]. All results obtained with $\alpha_{IR} = 0.93\pi$ and (in GeV) $\Lambda_{ir} = 0.24$, $\Lambda_{uv} = 0.905$. N.B. These parameters take the values determined in the spectrum calculation of Ref. [6]; and we assume isospin symmetry throughout. (All dimensioned quantities are listed in GeV.)

m_u	m_s	m_s/m_u	M_0	M_u	M_s	M_s/M_u	$\kappa_0^{1/3}$	$\kappa_\pi^{1/3}$	$\kappa_K^{1/3}$
0.007	0.17	24.3	0.36	0.37	0.53	1.43	0.241	0.243	0.246

- Input ratio m_s/m_u *= 24* is consistent with modern estimates
- Output ratio M_s/M_u *= 1.43* shows dramatic impact of DCSB, even on the *s*-quark
- κ = in-hadron condensate rises slowly with mass of hadron

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Mesons with Strangeness

- Solve Bethe-Salpeter equations for **mesons** and diquarks

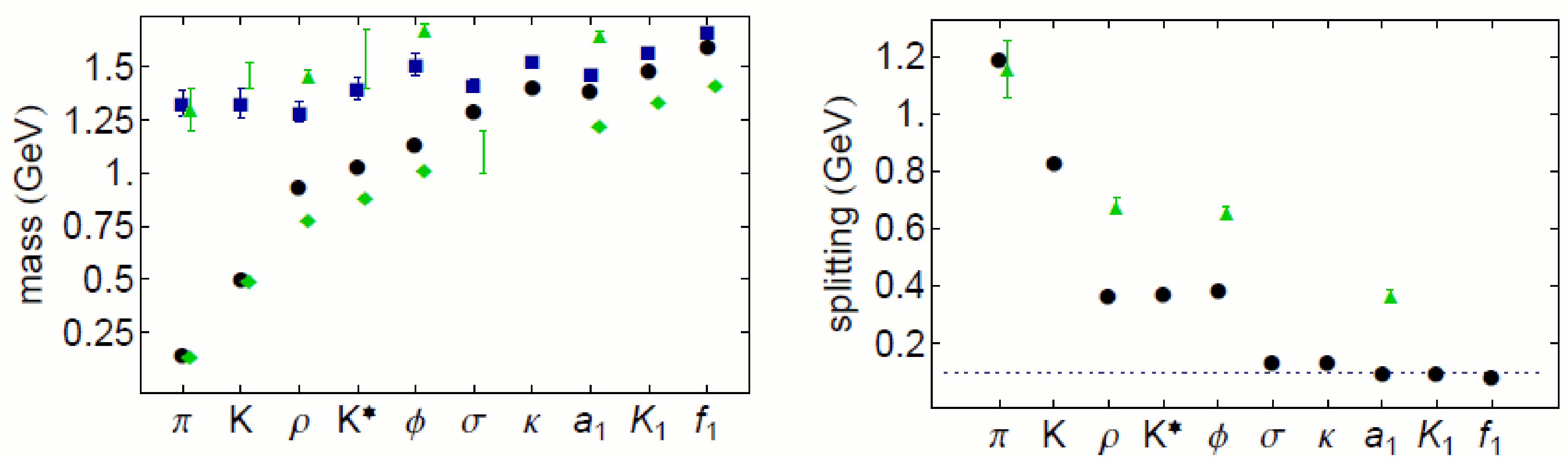


Fig. 2 Left panel: Pictorial representation of Table 2. *Circles* – computed ground-state masses; *squares* – computed masses of radial excitations; *diamonds* – empirical ground-state masses in Row 2; and *triangles* – empirical radial excitation masses in Row 4. Right panel: *Circles* – computed splitting between the first radial excitation and ground state in each channel; and *triangles* – empirical splittings, where they are known. The *dashed line* marks a splitting of 0.1 GeV.

[arXiv:1204.2553 [nucl-th]](https://arxiv.org/abs/1204.2553), *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Mesons with Strangeness

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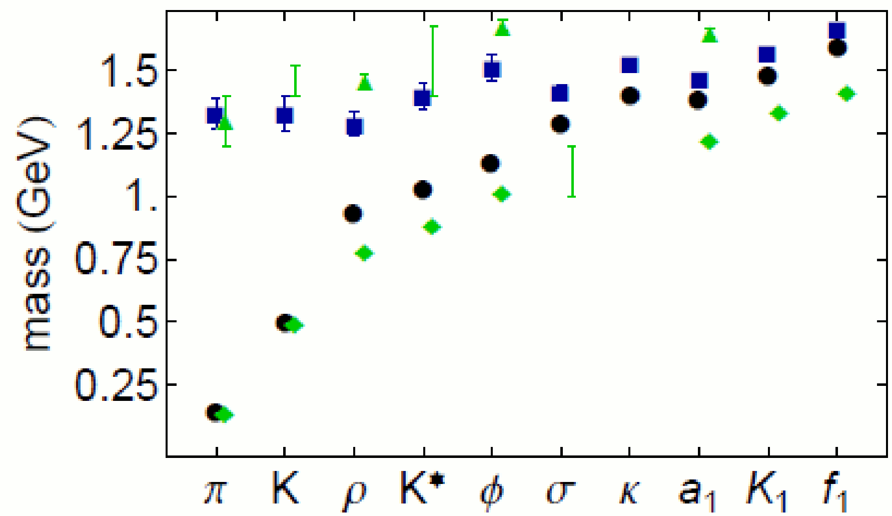
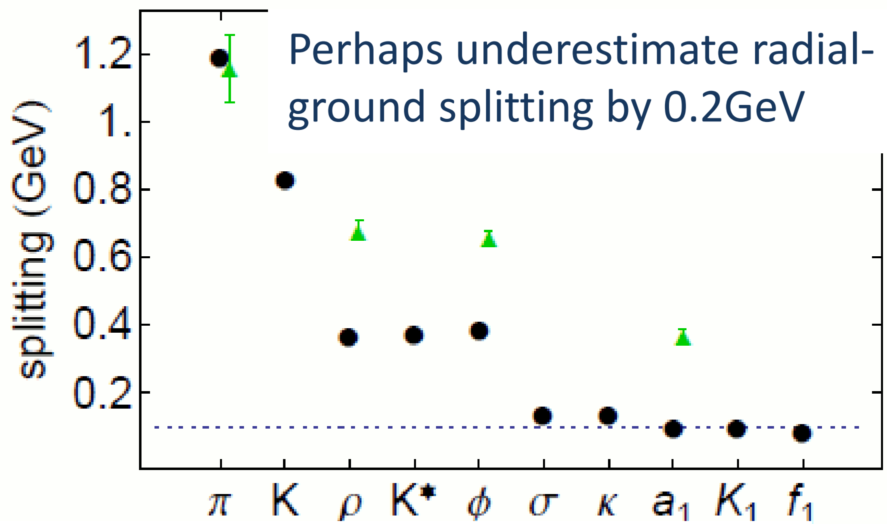




Perhaps underestimate radial-ground splitting by 0.2GeV

Fig. 2 comput empiric excitat dashed ... res – les – adial The

- ✓ Computed values for ground-states are greater than the empirical masses, where they are known.
- ✓ Typical of DCSB-corrected kernels that omit resonant contributions; i.e., do not contain effects that may phenomenologically be associated with a meson cloud.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Diquarks with Strangeness

- Solve Bethe-Salpeter equations for mesons and **diquarks**

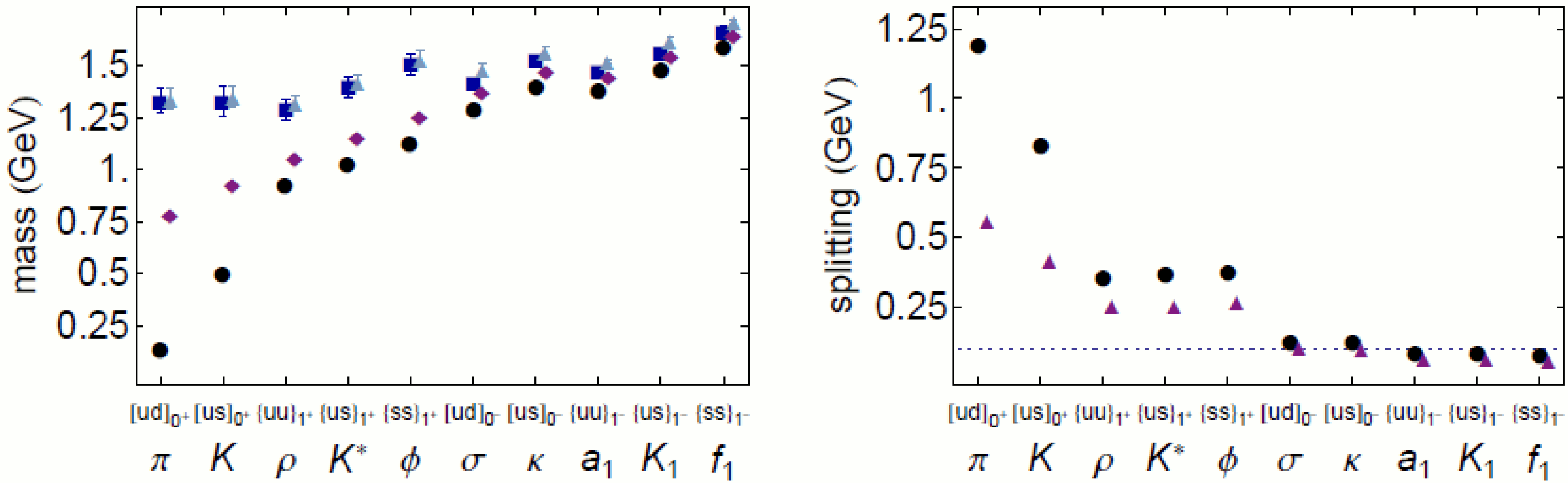
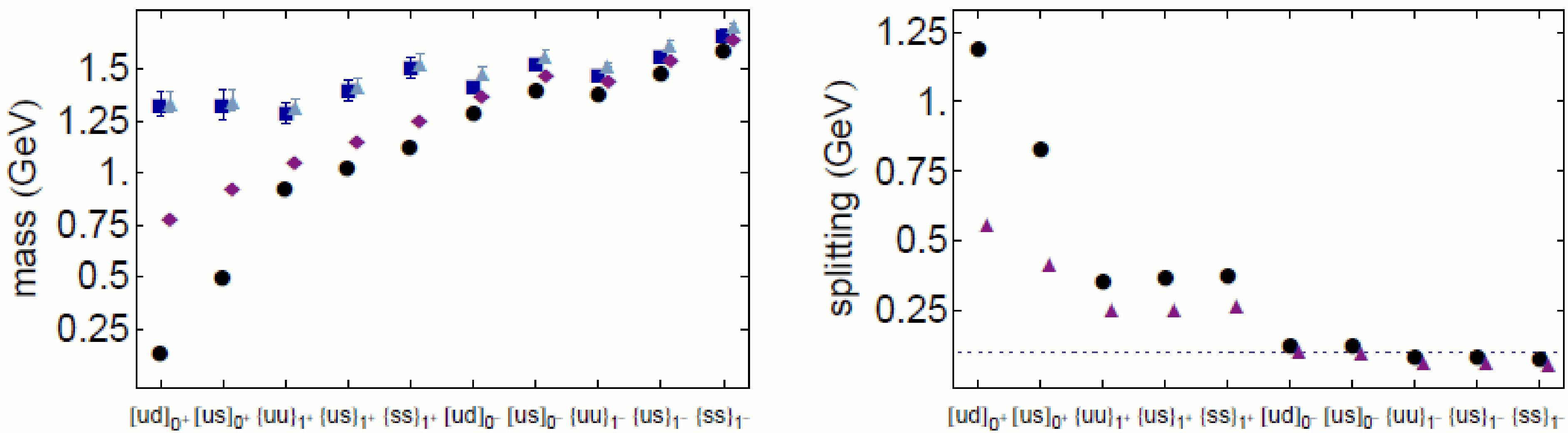


Fig. 3 Left panel: Pictorial representation of Table 4. *Diamonds* – ground-state diquark masses in Row 1; *circles* – ground-state meson masses in Row 2; *triangles* – masses of diquark first radial excitations in Row 3; and *squares* – masses of meson radial excitations in Row 4. Right panel: *Diamonds* – for diquarks, computed splittings between first radial excitation and ground state; and *circles* – for mesons, computed splitting between the first radial excitation and ground state in each channel. The *dashed line* marks a splitting of 0.1 GeV.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Diquarks with Strangeness

- Solve Bethe-Salpeter equations for mesons and **diquarks**



1 f_1

Fig. 3 … Row 1;
circles - … Row 3;
and squ… …puted
splitting … …tween
the first … V.

- ✓ Level ordering of diquark correlations is same as that for mesons.
- ✓ In all diquark channels, except scalar, mass of diquark's partner meson is a fair guide to the diquark's mass:
  - Meson mass bounds the diquark's mass from below;
  - Splitting always less than 0.13GeV and decreases with increasing meson mass
- ✓ Scalar channel "special" owing to DCSB

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Bethe-Salpeter amplitudes

- Bethe-Salpeter amplitudes are couplings in Faddeev Equation

Table 3 The structure of meson Bethe-Salpeter amplitudes is described in Sect. 2.2.1 and App. B. Here we list the canonically normalised amplitude associated with each of the BSE eigenstates in Table 2. Only pseudoscalar mesons involve two independent amplitudes when a vector×vector contact interaction is treated systematically in rainbow-ladder truncation.

		m_π	m_K	m_ρ	m_{K^*}	m_ϕ	m_σ	m_κ	m_{a_1}	m_{K_1}	m_{f_1}
n=0	$E_{q\bar q}$	3.60	3.86	1.53	1.62	1.74	0.47	0.47	0.31	0.31	0.31
	$F_{q\bar q}$	0.48	0.60								
n=1	$E_{q\bar q}$	0.83	0.76	0.72	0.70	0.66	0.34	0.35	0.28	0.28	0.28
	$F_{q\bar q}$	0.05	1.18								

- Magnitudes for diquarks follow precisely the meson pattern

Table 5 The structure of diquark Bethe-Salpeter amplitudes is described in Sect. 2.2.2 and App. B. Here we list all canonically normalised amplitudes that are relevant to the baryons we consider. Only scalar diquarks involve two independent amplitudes.

	$[u,d]_{0+}$	$[s,u]_{0+}$	$\{u,u\}_{1+}$	$\{s,u\}_{1+}$	$\{s,s\}_{1+}$	$[u,d]_{0-}$	$[s,u]_{0-}$	$\{u,u\}_{1-}$	$\{s,u\}_{1-}$	$\{s,s\}_{1-}$
E_{qq}	2.74	2.91	1.30	1.36	1.42	0.40	0.39	0.27	0.27	0.26
F_{qq}	0.31	0.40								

Owing to DCSB, FE couplings in $½^-$ channels are 25-times weaker than in $½^+$!

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness*, Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Baryons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

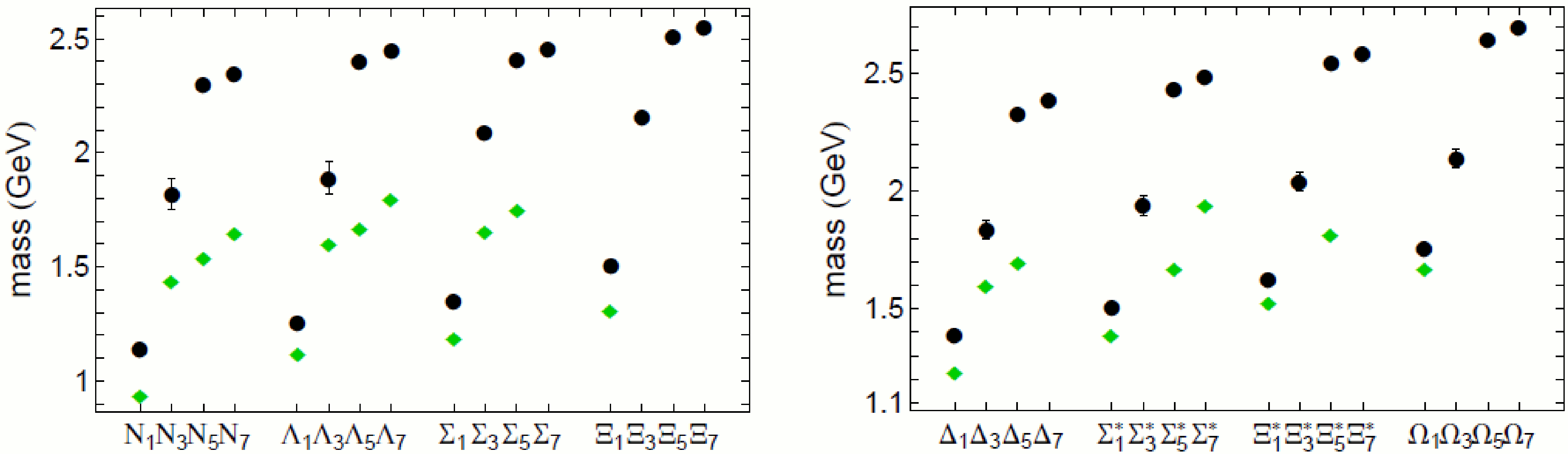


Fig. 4 Left panel: Pictorial representation of octet masses in Table 6. *Circles* – computed masses; and *diamonds* – empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., N_1 means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner's radial excitation. Right panel: Analogous plot for the decuplet masses in Table 6.

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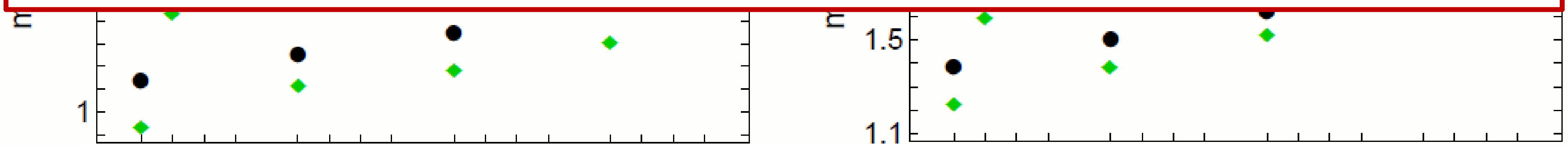
Spectrum of Baryons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

	$N_{940}P_{11}$	$N_{1440}P_{11}$	$N_{1535}S_{11}$	$N_{1650}S_{11}$	$\Delta_{1232}P_{33}$	$\Delta_{1700}D_{33}$
Table 6 (DSE)	1.14	$1.82_{0.07}$	2.30	$2.35_{0.01}$	1.39	2.33
M_B^0 Jülich])	1.24		2.05	1.92	1.46	2.25
M_B^0 EBAC])		1.76	1.80	1.88	1.39	1.98



- As with mesons, computed baryon masses lie uniformly above the empirical values.
- Success because our results are those for the baryons' dressed-quark cores, whereas empirical values include effects associated with meson-cloud, which typically produce sizable reductions.

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arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Structure of Baryons with Strangeness

- Baryon structure is flavour-blind

Table 7 Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression s^i or a^i is a diquark enumeration label associated with Eq. (31), except for [2, 3] and [6, 8], which are the $I = 0$ combinations in Eq. (49).

Diquark content

		s^1	s^2	$s^{[2,3]}$	a_1^4	a_2^4	a_1^5	a_2^5	a_1^6	a_2^6	$a_1^{[6,8]}$	$a_2^{[6,8]}$	a_1^9	a_2^9	$P_{J=0}$
$(P=+, n=0)$	N	0.88			-0.38	0.27	-0.06	0.04							78%
	Λ	0.67		-0.27							-0.45	-0.09			79%
	Σ		0.85		-0.45	0.26			0.12	0.02					72%
	Ξ		0.91		0.14	0.08							0.39	0.00	82%
$(P=+, n=1)$	N	-0.02			0.52	-0.37	-0.63	0.44							0%
	Λ	0.03		0.06							-0.78	0.63			0%
	Σ		0.00		-0.04	0.02			0.83	-0.55					0%
	Ξ		0.00		0.01	-1.00							-0.02	0.06	0%
$(P=-, n=0)$	N	0.71			-0.41	0.29	0.41	-0.29							50%
	Λ	0.64		0.44							-0.47	0.42			61%
	Σ		0.61		-0.47	0.23			0.55	-0.21					38%
	Ξ		0.76		-0.34	0.35							0.33	-0.28	58%
$(P=-, n=1)$	N	0.66			-0.41	0.29	0.45	-0.32							44%
	Λ	0.60		0.43							-0.48	0.47			55%
	Σ		0.57		-0.47	0.23			0.58	-0.24					33%
	Ξ		0.73		-0.34	0.37							0.33	-0.31	54%

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen, Chang, Roberts, Wan and Wilson & *Nucleon and Roper em elastic and transition form factors,* D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C85 (2012) 025205 [21 pages]

Structure of Baryons with Strangeness

- Baryon structure is flavour-blind

Table 7 Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression s^i or a^i is a diquark enumeration label associated with Eq. (31), except for [2, 3] and [6, 8], which are the $I = 0$ combinations in Eq. (49).

Diquark content

		s^1	s^2	$s^{[2,3]}$	a_1^4	a_2^4	a_1^5	a_2^5	a_1^6	a_2^6	$a_1^{[6,8]}$	$a_2^{[6,8]}$	a_1^9	a_2^9	$P_{J=0}$
$(P=+, n=0)$	N														78%
	Λ														79%
80%	Σ														72%
	Ξ														82%
$(P=+, n=1)$	N														0%
	Λ														0%
0%	Σ														0%
	Ξ														0%
$(P=-, n=0)$	N														50%
	Λ														61%
50%	Σ														38%
	Ξ														58%
$(P=-, n=1)$	N														44%
	Λ														55%
50%	Σ														33%
	Ξ														54%

- J_{qq}=0 content of J=½ baryons is almost independent of their flavour structure
- *Radial excitation of ground-state octet possess zero scalar diquark content!*
- *This is a consequence of DCSB*
  - *Ground-state $(1/2)^+$ possess unnaturally large scalar diquark content*
  - *Orthogonality forces radial excitations to possess (almost) none at all!*

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness*, Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Hadrons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

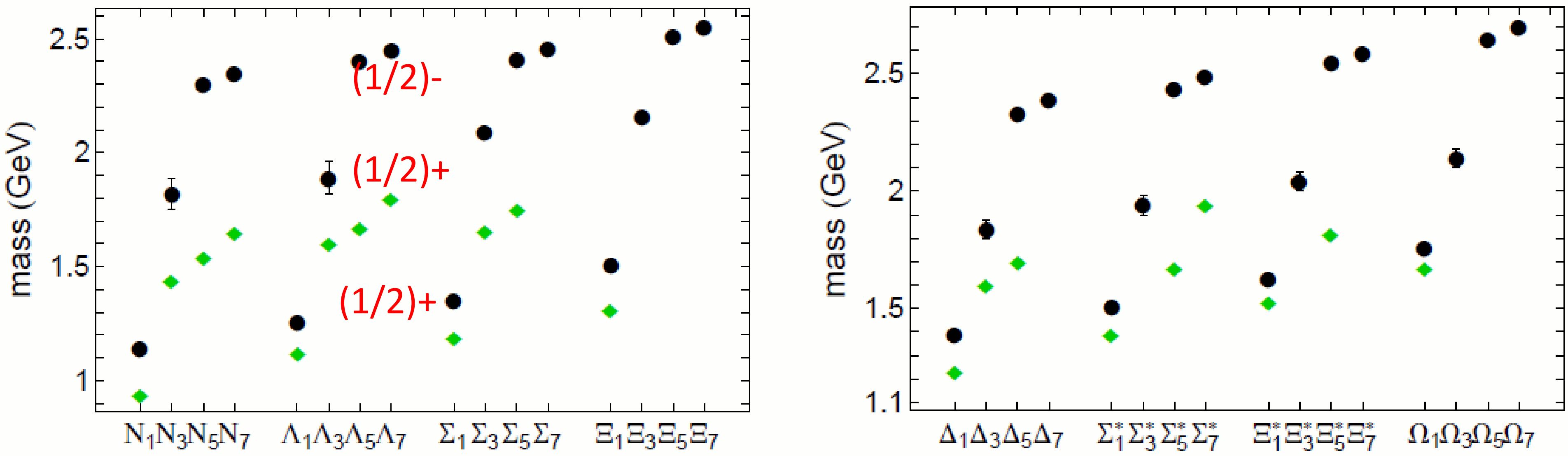

Fig. 4 Left panel: Pictorial representation of octet masses in Table 6. *Circles* – computed masses; and *diamonds* – empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., N_1 means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner's radial excitation. Right panel: Analogous plot for the decuplet masses in Table 6.

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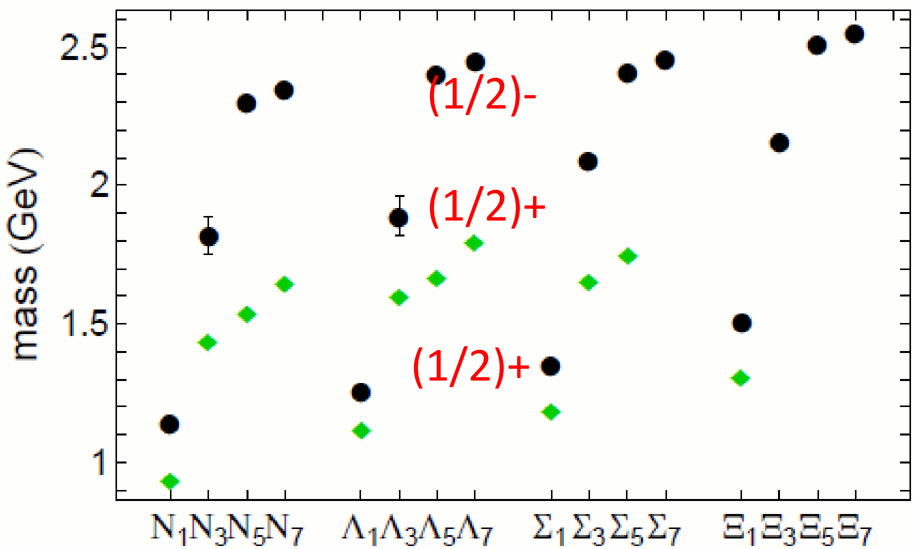
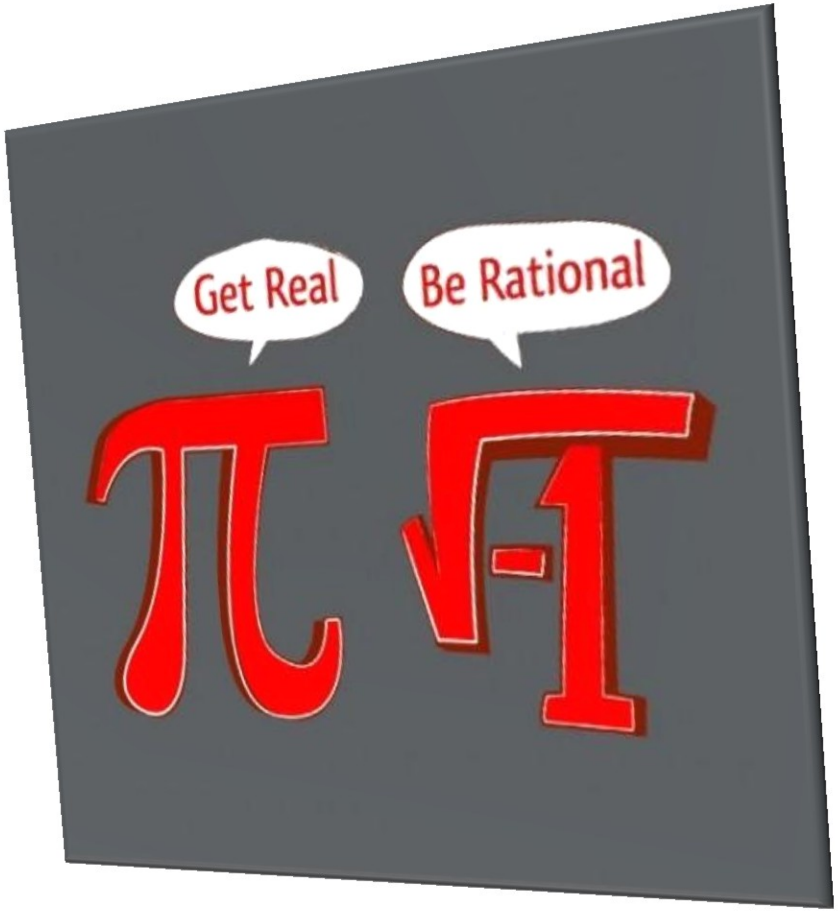


Fig. 4 Left panel: Pictorial representation of octet
diamonds – empirical masses. On the horizontal axis
its row in the table; e.g., N_1 means nucleon column, r
radial excitation, parity partner, parity partner's radial
masses in Table 6.

- This level ordering has long been a problem in CQMs with linear or HO confinement potentials
- *Correct ordering owes to DCSB*
  - *Positive parity diquarks have Faddeev equation couplings 25-times greater than negative parity diquarks*
- Explains why approaches within which DCSB cannot be realised (CQMs) or simulations whose parameters suppress DCSB will both have difficulty reproducing experimental ordering



Getting real