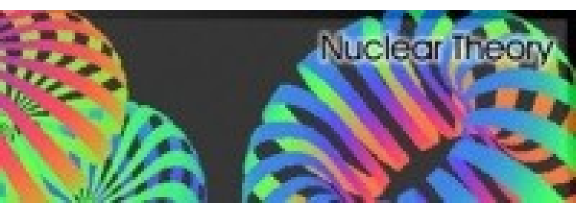
Emergence of DSEs in Real-World QCD

Craig Roberts



Physics Division

www.phy.anl.gov/theory/staff/cdr.html

Argonne NATIONAL LABORATORY

National Science Foundation WHERE DISCOVERIES BEGIN

Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

U.S. DEPARTMENT OF ENERGY

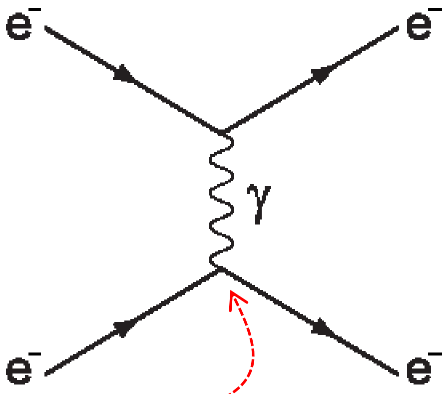
Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.
  - Higgs mechanism is (*almost*) irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.
  - Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator.
  - It is intimately connected with DCSB.

Relativistic quantum mechanics

- Dirac equation (1928):

  Pointlike, massive fermion interacting with electromagnet



$$(\gamma \cdot \partial - e\,\gamma \cdot A(x) + m)\,\Psi(x) = 0$$

$$\stackrel{\text{current}}{\Rightarrow} \quad i e\,\bar{u}(P')\gamma_\mu u(P)$$

$$\text{Gordon Identity}: \quad \bar{u}(p)\,(i\gamma \cdot p + m) = 0 = (i\gamma \cdot p + m)\,u(p)$$

$$\Rightarrow \quad i e\,\bar{u}(P')\gamma_\mu u(P)$$

$$= \quad e\,\bar{u}(P')\left[\frac{1}{2m}(P'+P)_\mu + i\frac{1}{2m}\sigma_{\mu\nu}(P'-P)_\mu\right]u(P)$$

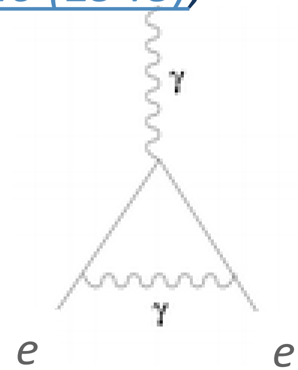
Electromagnetic interaction of a Point Fermion

$$i\,\bar{u}(P')\frac{e}{2m}g\,\frac{1}{2}\sigma_{\mu\nu}(P'-P)_\mu\,u(P)\,,\quad g \stackrel{\text{Dirac}}{=} 2$$

Spin Operator

Massive point-fermion
Anomalous magnetic moment

- Dirac's prediction held true for the electron until improvements in experimental techniques enabled the discovery of a small deviation: *H. M. Foley and P. Kusch, Phys. Rev. 73, 412 (1948)*.
  - Moment increased by a multiplicative factor: *1.001 19 ± 0.000 05*.
- This correction was explained by the first systematic computation using renormalized quantum electrodynamics (QED): *J.S. Schwinger, Phys. Rev. 73, 416 (1948)*,
  - vertex correction



0.001 16

$$\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right) \frac{q}{2m}$$

- The agreement with experiment established *quantum* electrodynamics as a valid tool.

Fermion electromagnetic current - General structure

$$iq\bar{u}(p_f)\left[\gamma_\mu F_1(k^2) + \frac{1}{2m}\sigma_{\mu\nu}k_\nu F_2(k^2)\right]u(p_i),$$

with $k = p_f - p_i$

- $F_1(k^2)$ – Dirac form factor; and $F_2(k^2)$ – Pauli form factor
  - Dirac equation:
    - $F_1(k^2) = 1$
    - $F_2(k^2) = 0$
  - Schwinger:
    - $F_1(k^2) = 1$
    - $F_2(k^2=0) = \alpha /[2 \pi]$

$$\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right) \frac{q}{2m}$$

Magnetic moment of a massless fermion?

- Plainly, can't simply take the limit $m \to 0$.
- Standard QED interaction, generated by minimal substitution:

$$\int d^4x\, iq\, \bar{\psi}(x)\gamma_\mu \psi(x) A_\mu(x)$$

- Magnetic moment is described by interaction term:

$$\int d^4x\, \tfrac{1}{2} q\, \bar{\psi}(x)\sigma_{\mu\nu}\psi(x)F_{\mu\nu}(x)$$

  – Invariant under local *U(1)* gauge transformations
  – but is not generated by minimal substitution in the action for a free Dirac field.
- Transformation properties under chiral rotations?
  – $\Psi(x) \to \exp(i\vartheta\gamma_5)\, \Psi(x)$

$$\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right) \frac{q}{2m}$$

Magnetic moment of a massless fermion?

- Standard QED interaction, generated by minimal substitution:

$$\int d^4x\, iq\, \bar{\psi}(x)\gamma_\mu \psi(x) A_\mu(x)$$

  - Unchanged under chiral rotation
  - Follows that QED without a fermion mass term is helicity conserving
- Magnetic moment interaction is described by interaction term:

$$\int d^4x\, \tfrac{1}{2} q\, \bar{\psi}(x)\sigma_{\mu\nu}\psi(x) F_{\mu\nu}(x)$$

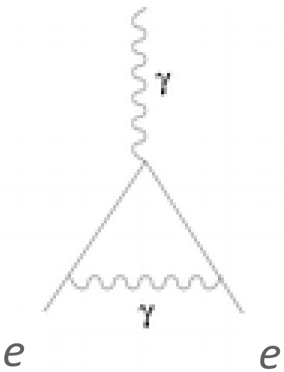
  - NOT invariant
  - picks up a phase-factor $exp(2i\vartheta\gamma_5)$
- *Magnetic moment interaction is forbidden in a theory with manifest chiral symmetry*

$$\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right)\frac{q}{2m}$$

Schwinger's result?

- One-loop calculation:

$$F_2(k^2) \; = \; \frac{\alpha}{2\pi}\int_0^1 dx \, \frac{m_e^2}{m_e^2 - k^2 x(1-x)}$$



- Plainly, one obtains Schwinger's result for $m_e^2 \neq 0$
- However,

$$F_2(k^2) = 0 \text{ when } m_e^2 = 0$$

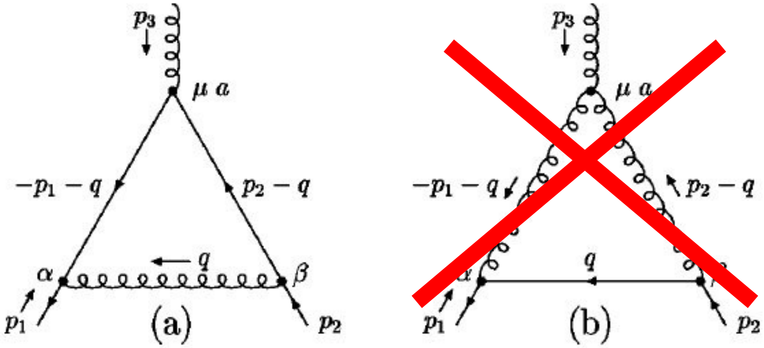
- There is no Gordon identity:

$m=0$

$$2m\bar{u}(p_f) i\gamma_\mu u(p_i) = \bar{u}(p_f)[2\ell_\mu + i\sigma_{\mu\nu}k_\nu]u(p_i).$$

So, no mixing
$\gamma_\mu \leftrightarrow \sigma_{\mu\nu}$

- Results are unchanged at every order in perturbation theory … owing to symmetry … *magnetic moment interaction is forbidden in a theory with manifest chiral symmetry*

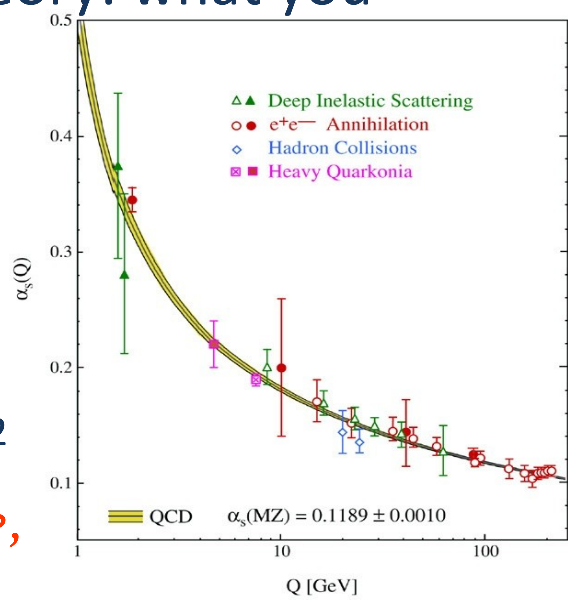


QCD and dressed-quark anomalous magnetic moments

- Schwinger's result for QED: $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- pQCD: two diagrams
  - (a) is QED-like
  - (b) is only possible in QCD – involves 3-gluon vertex
- Analyse (a) and (b)
  - (b) vanishes identically: the 3-gluon vertex does *not* contribute to a quark's anomalous chromomag. moment at leading-order
  - (a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "
    ~ (– ⅙) QED-result
- But, in QED and QCD, the *anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit*!

What happens in the real world?

- QED, by itself, is not an asymptotically free theory
  - Hence, cannot define a chiral limit & probably a trivial theory
    - As regularisation scale is removed, coupling must vanish
- Weak interaction
  - It's weak, so no surprises. Perturbation theory: what you see is what you get.
- Strong-interaction: QCD
  - Asymptotically free
    - Perturbation theory is valid and accurate tool at large-Q^2 & hence chiral limit is defined
  - Essentially nonperturbative for $Q^2 < 2\ \text{GeV}^2$
    - *Nature's only example of truly nonperturbative, fundamental theory*
    - *A-priori, no idea as to what such a theory can produce*

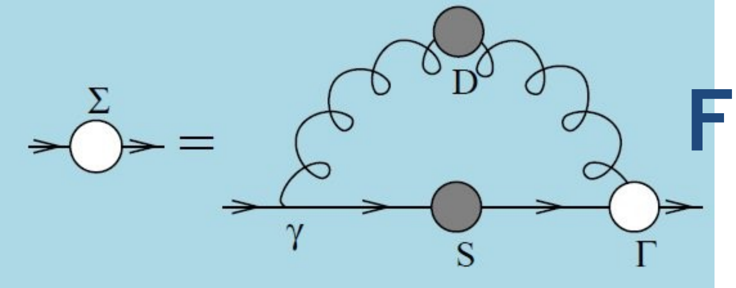


Craig Roberts: Emergence of DSEs in Real-World QCD 2B (89)

Dynamical Chiral Symmetry Breaking

- Strong-interaction: QCD
- Confinement
  - Empirical feature
  - Modern theory and lattice-QCD support conjecture
    - that light-quark confinement is real
    - associated with violation of reflection positivity; i.e., novel analytic structure for propagators and vertices
  - Still circumstantial, no proof yet of confinement
- On the other hand, *DCSB is a fact in QCD*
  - It is the most important mass generating mechanism for visible matter in the Universe.

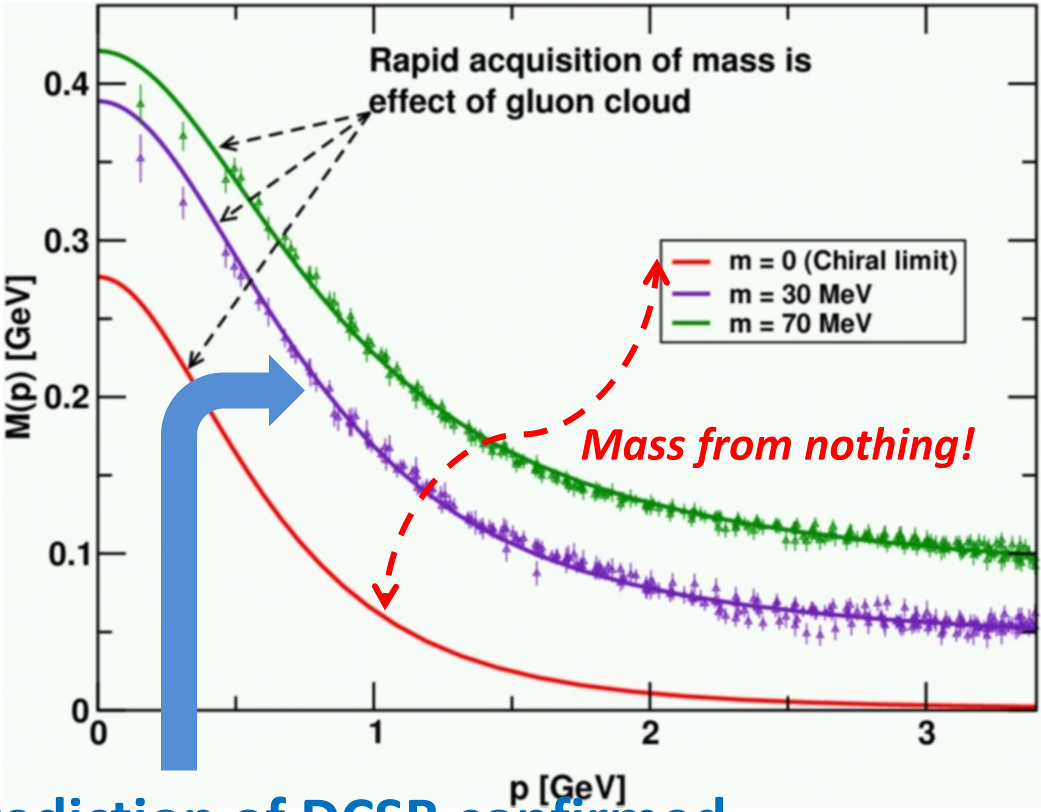
Responsible for approximately 98% of the proton’s mass.
Higgs mechanism is (*almost*) irrelevant to light-quarks.



Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



DSE prediction of DCSB confirmed

Craig Roberts: Emergence of DSEs in Real-World QCD 2B (89)

Strong-interaction: QCD

Dressed-quark-gluon vertex

- Gluons and quarks acquire momentum-dependent masses
  - characterised by an infrared mass-scale $m \approx 2\text{-}4\ \Lambda_{QCD}$
- Significant body of work, stretching back to 1980, which shows that, in the presence of DCSB, the dressed-fermion-photon vertex is materially altered from the bare form: γ_μ.
  - Obvious, because with

$A(p^2) \neq 1$ and $B(p^2) \neq constant$,

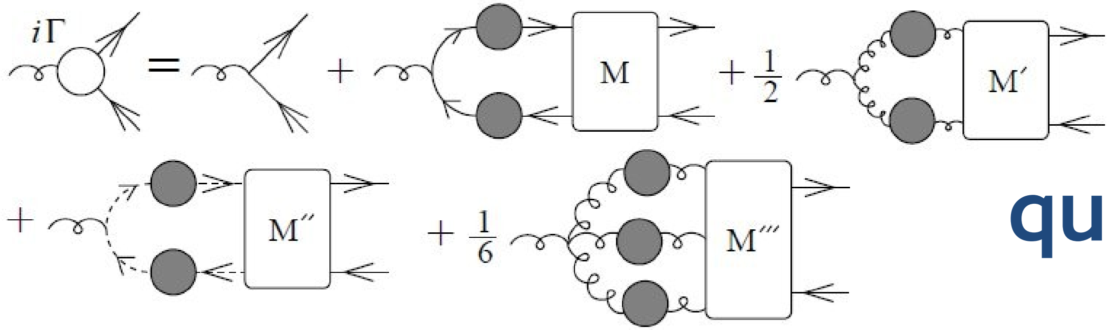
the bare vertex cannot satisfy the Ward-Takahashi identity; viz.,

$$i P_\mu \gamma_\mu \neq S^{-1}(k + P/2) - S^{-1}(k - P/2)$$

- Number of contributors is too numerous to list completely (300 citations to 1st J.S. Ball paper), but prominent contributions by:

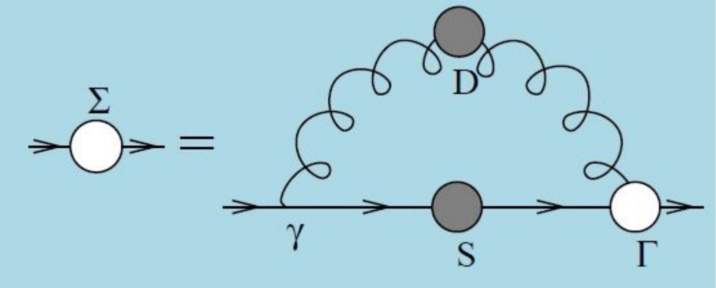
J.S. Ball, C.J. Burden, C.D. Roberts, R. Delbourgo, A.G. Williams, H.J. Munczek, M.R. Pennington, A. Bashir, A. Kizilersu, L. Chang, Y.-X. Liu …

Craig Roberts: Emergence of DSEs in Real-World QCD 2B (89)



Dressed-quark-gluon vertex

- Single most important feature
  - Perturbative vertex is helicity-conserving:
    - Cannot cause spin-flip transitions
  - *However,* DCSB introduces nonperturbatively generated structures that very strongly break helicity conservation
  - These contributions
    - Are large when the dressed-quark mass-function is large
      - Therefore vanish in the ultraviolet; i.e., on the perturbative domain
  - Exact form of the contributions is still the subject of debate *but* their *existence* is model-independent - *a fact*.



Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$* – dressed-gluon propagator**
- ***$\Gamma_\nu(q,p)$* – dressed-quark-gluon vertex**
- Until 2009, all studies of other hadron phenomena used the leading-order term in a symmetry-preserving truncation scheme; viz.,
  - ***$D_{\mu\nu}(k)$* =** dressed, as described previously
  - ***$\Gamma_\nu(q,p) = \gamma_\mu$***
    - **…** plainly, key nonperturbative effects are missed and cannot be recovered through any step-by-step improvement procedure

Bender, Roberts & von Smekal
Phys.Lett. B380 (1996) 7-12

Dynamical chiral symmetry breaking and the fermion--gauge-boson vertex, A. Bashir, R. Bermudez, L. Chang and C. D. Roberts, arXiv:1112.4847 [nucl-th], Phys. Rev. C85 (2012) 045205 [7 pages]

Gap Equation General Form

$$S_f(p)^{-1} = Z_2 \left(i\gamma \cdot p + m_f^{\rm bm}\right) + \Sigma_f(p)\,,$$

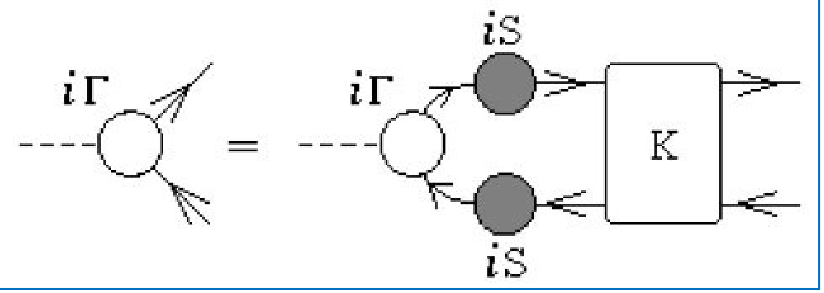
$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
  - ***good deal of information available***
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***
  - ***Information accumulating***

If kernels of Bethe-Salpeter and gap equations don't match, one won't even get right charge for the pion.

- **Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex**

What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation
Bound-State DSE

$$[\Gamma_\pi^j(k;P)]_{tu} = \int_q^\Lambda [S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)]_{sr} K_{tu}^{rs}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**

Blocked progress for more than 60 years.

Bethe-Salpeter Equation General Form

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$

$$- \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)$$

$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

- **Equivalent exact bound-state equation *but* in this form**

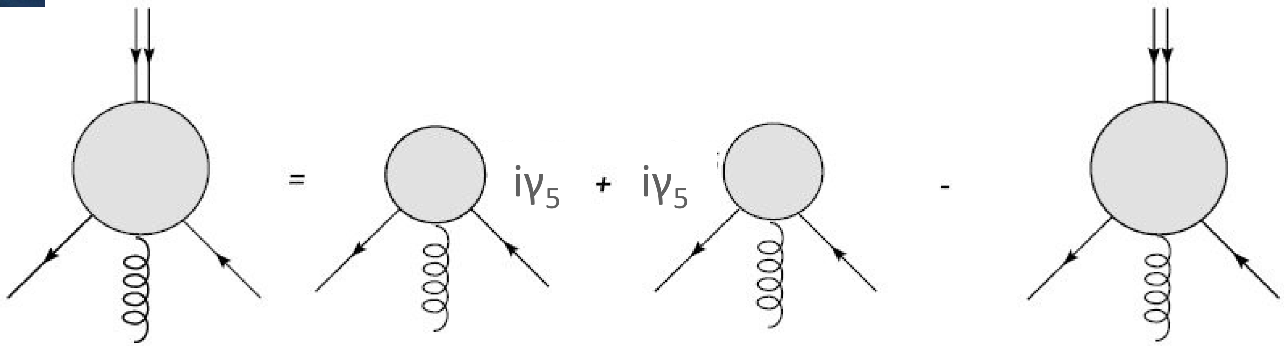
  K(q,k;P) → Λ(q,k;P)

  which is *completely determined by dressed-quark self-energy*

- **Enables derivation of a Ward-Takahashi identity for Λ(q,k;P)**

Ward-Takahashi Identity Bethe-Salpeter Kernel

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601



$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta) + m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity
- This enables the identification and elucidation of a wide range of novel consequences of DCSB

L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

➢ **DCSB** ⇒ **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term →

•Vanishes if no DCSB
•Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2) - F(q^2)}{p^2 - q^2}$$

Anom. chrom. mag. mom. contribution to vertex →

•Similar properties to BC term
•Strength commensurate with lattice-QCD
Skullerud, Bowman, Kizilersu *et al.* hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta\, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

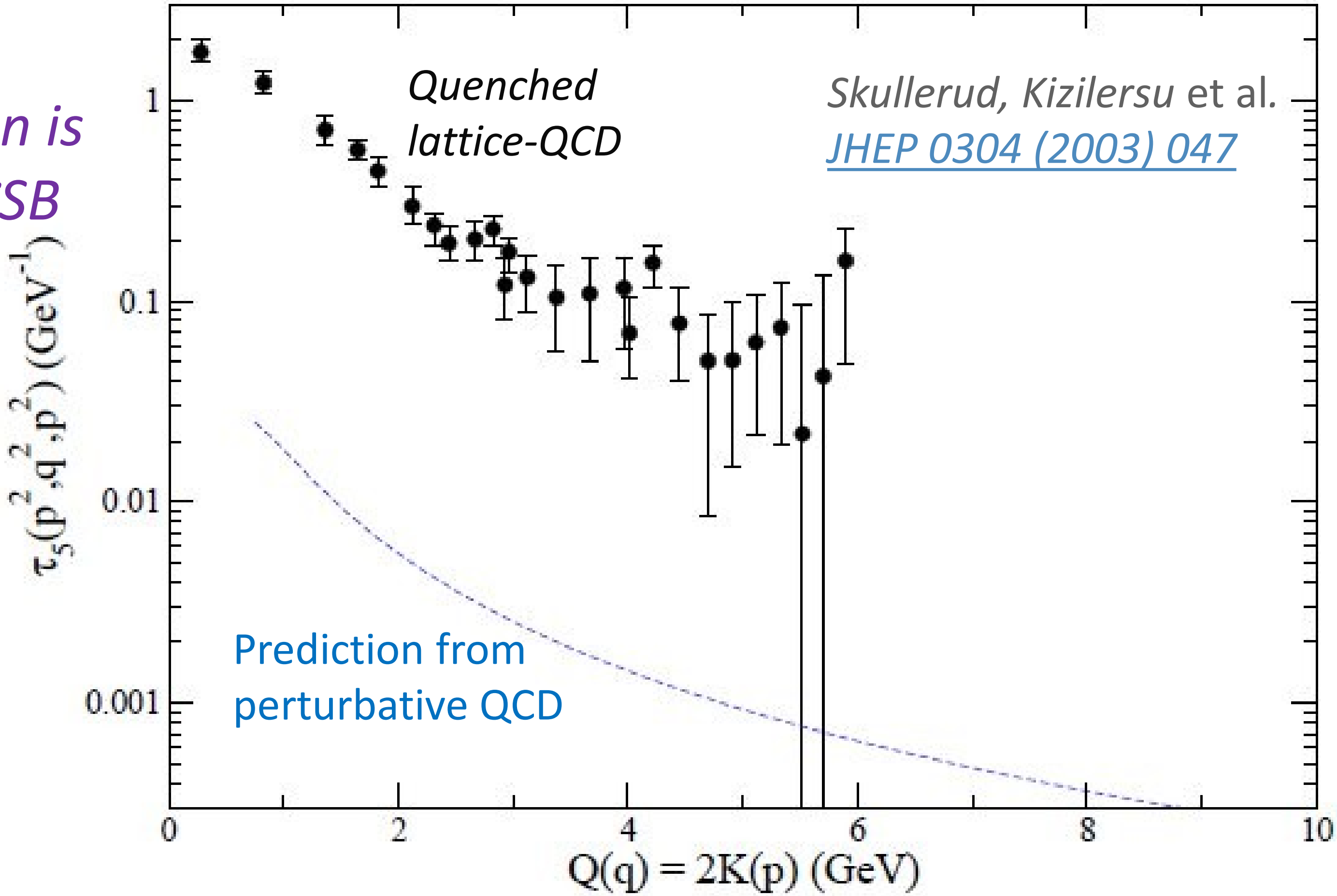
$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$,
Simplifies numerical analysis;

$M_E = \{s | s > 0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Dressed-quark anomalous chromomagnetic moment

- Lattice-QCD
  - *m = 115* MeV
- Nonperturbative result is *two orders-of-magnitude* larger than the perturbative computation
  - *This level of magnification is typical of DCSB*
  - *cf.*

Quark mass function:
$M(p^2=0)$= 400MeV
$M(p^2=10GeV^2)$=4 MeV



L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

- **DCSB** ⇒ **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term → $\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$

•Vanishes if no DCSB

•Appearance driven by STI

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom. chrom. mag. mom. contribution to vertex → $\Gamma^5_\mu(p,q) = \eta\,\sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$

•Similar properties to BC term

•Strength commensurate with lattice-QCD

Skullerud, Bowman, Kizilersu *et al.* hep-ph/0303176

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$, Simplifies numerical analysis;

$M_E = \{s | s>0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Role and importance is novel discovery

•Essential to recover pQCD

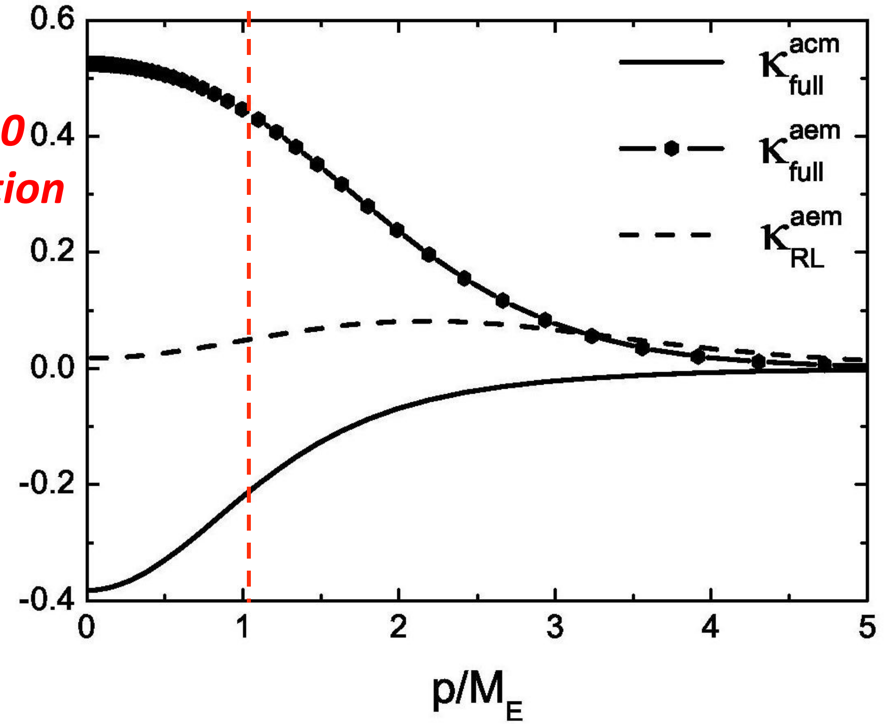
•Constructive interference with Γ^5

L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

- ➢Formulated and solved general Bethe-Salpeter equation
- ➢Obtained dressed electromagnetic vertex
- ➢Confined quarks don't have a mass-shell
  - ○ Can't unambiguously define magnetic moments
  - ○ But can define *magnetic moment distribution*

Factor of 10 magnification



- ➢ AEM is opposite in sign but of roughly equal magnitude as ACM

	M^E	κ^{ACM}	κ^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

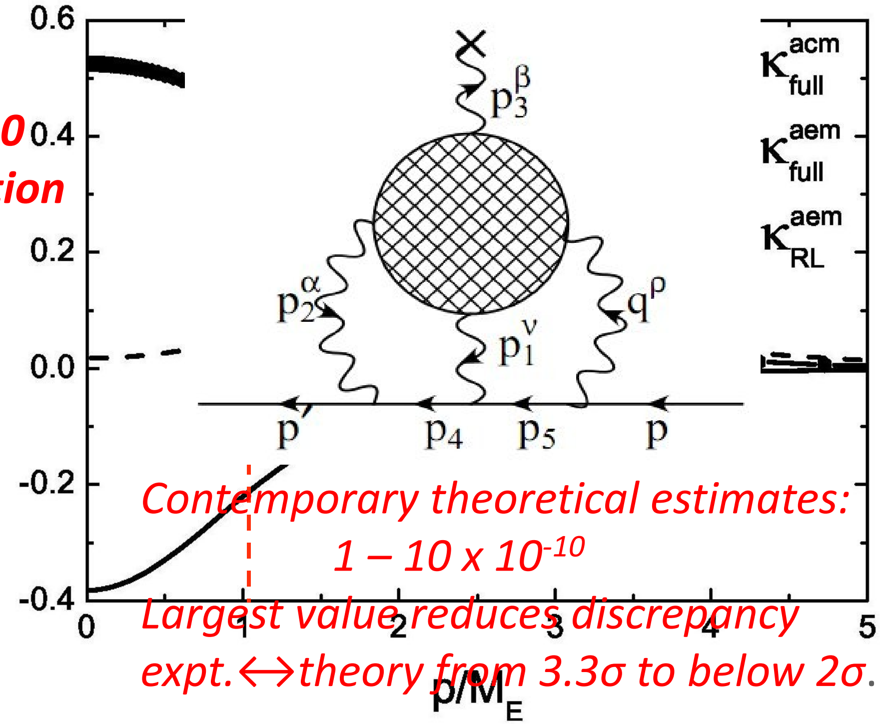
L. Chang, Y. –X. Liu and C.D. Roberts
arXiv:1009.3458 [nucl-th]
Phys. Rev. Lett. **106** (2011) 072001

Dressed-quark anomalous magnetic moments

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don't have a mass-shell
  - Can't unambiguously define magnetic moments
  - But can define

magnetic moment distribution

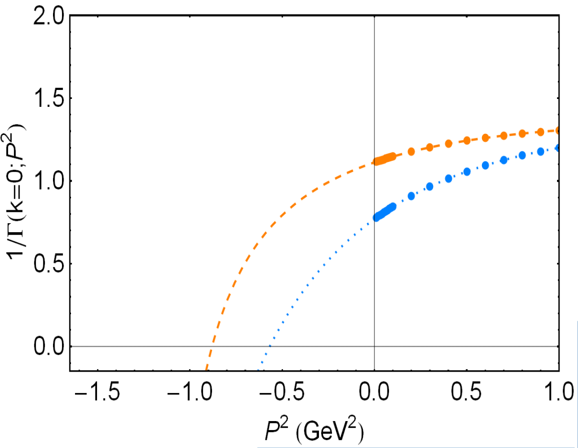
Factor of 10 magnification



Contemporary theoretical estimates: 1 – 10 x 10^{-10}

Largest value reduces discrepancy expt.↔theory from 3.3σ to below 2σ.

- Potentially important for elastic and transition form factors, etc.
- Significantly, also quite possibly for muon *g-2* – via Box diagram, which is not constrained by extant data.



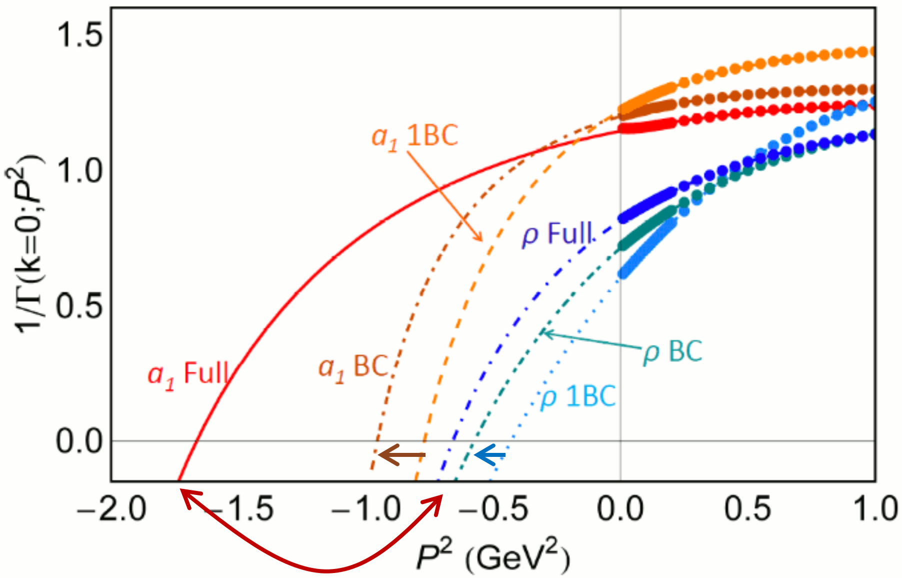
Location of zero marks $-m^2_{meson}$

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885		
ρ	770	644	764		
Mass splitting	455	115	121		

- Splitting known experimentally for more than 35 years
- Hitherto, no explanation
- Systematic symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.
  - Never better than ~ ¼ of splitting
- Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed:

Full impact of $M(p^2)$ cannot be realised!



Solves problem of a_1 - ρ mass splitting

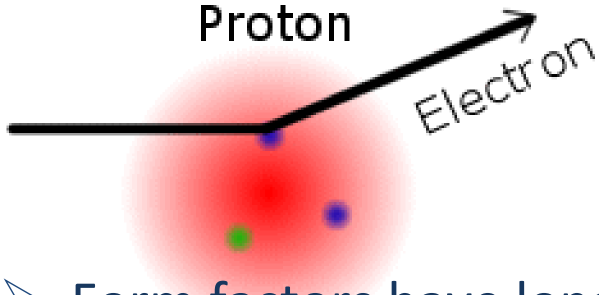
Lei Chang & C.D. Roberts,
arXiv:1104.4821 [nucl-th]
Tracing massess of ground-state light-quark mesons

$M(p^2)$ magnifies spin orbit splitting here, precisely as in σ-π comparison

- Fully nonperturbative BSE kernel that incorporates and expresses DCSB: establishes unambiguously that a_1 & ρ are parity-partner bound-states of dressed light valence-quarks.

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1020	1280
ρ	770	644	764	800	840
Mass splitting	455	115	121	220	440

Looking deeper

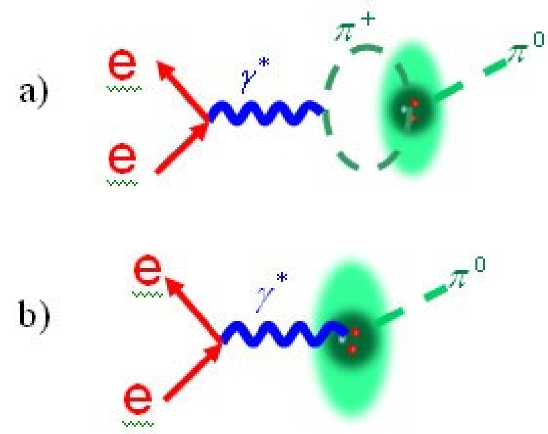


Form Factors
Elastic Scattering

- Form factors have long been recognised as a basic tool for elucidating bound-state properties.
- They are of particular value in hadron physics because they provide information on structure as a function of Q^2, the squared momentum-transfer:
  - Small-Q^2 is the nonperturbative domain
  - Large-Q^2 is the perturbative domain
  - Nonperturbative methods in hadron physics must explain the behaviour from $Q^2=0$ through the transition domain, whereupon the behaviour is currently being measured
- Experimental and theoretical studies of hadron electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.
- *Despite this, many urgent questions remain unanswered*

Some questions

- How can we use experiment to chart the long-range behaviour of the β-function in QCD?
  - Given the low mass of the pion and its strong coupling to protons and neutrons, how can we disentangle spectral features produced by final-state interactions from the intrinsic properties of hadrons?



  - At which momentum-transfer does the transition from nonperturbative -QCD to perturbative- QCD take place?
  - …

Contemporary evaluation of current status

1. J. Arrington, C. D. Roberts and J. M. Zanotti
   "Nucleon electromagnetic form factors,"
   J. Phys. G **34,** S23 (2007); [arXiv:nucl-th/0611050]
2. C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
   "Nucleon electromagnetic form factors,"
   Prog. Part. Nucl. Phys. **59,** 694 (2007); [arXiv:hep-ph/0612014].

- *However, the experimental and theoretical status are changing quickly, so aspects of these reviews are already out-of-date*
- So, practitioners must keep abreast through meetings and workshops, of which there are many.
  - An expanded edition of "1." is supposed to be in preparation for Rev. Mod. Phys.

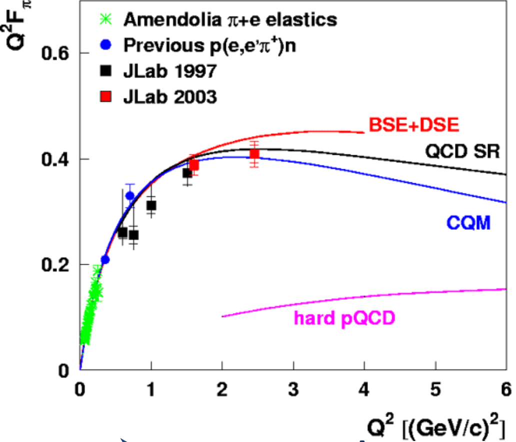


Illustration: Pion form factor

- Many theorists have pretended that computing the pion form factor is easy
- Problems:
  - *Those theorists have no understanding of DCSB*
  - There are no pion targets and hence it is difficult to obtain an unambiguous measurement of the pion form factor
- Notwithstanding these difficulties, the DSEs provide the best existing tool, because so many exact results are proved for the pion
- A quantitative *prediction* was obtained by combining
  - Dressed-rainbow gap equation
  - Dressed-ladder Bethe-Salpeter equation
  - Dressed impulse approximation for the form factor

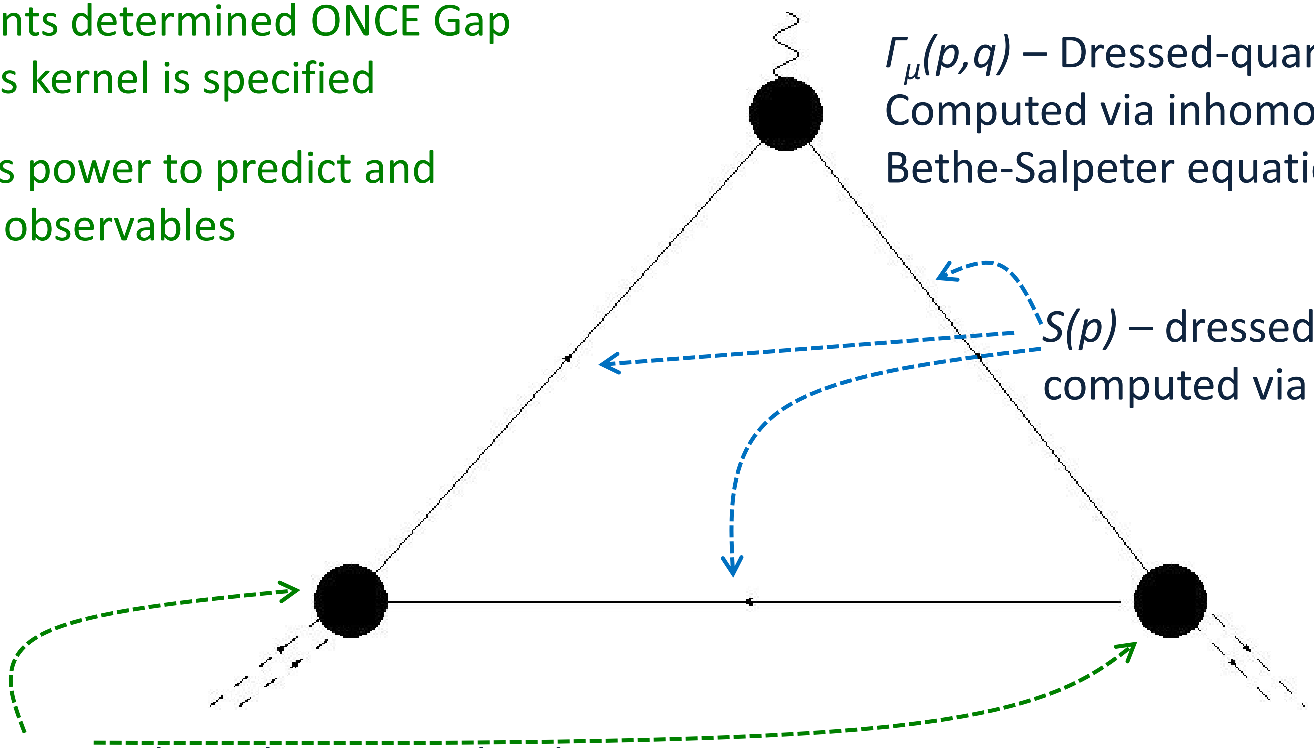
Electromagnetic pion form factor

Leading-order in a nonperturbative, symmetry-preserving truncation scheme

Valid formulation of the DSEs preserves all symmetry relations between the elements

All elements determined ONCE Gap Equation's kernel is specified

Enormous power to predict and correlate observables



$\Gamma_\mu(p,q)$ – Dressed-quark-photon vertex: Computed via inhomogeneous Bethe-Salpeter equation

$S(p)$ – dressed-quark propagator: computed via the Gap Equation

$\Gamma_\pi(k;P)$ – Pion Bethe-Salpeter amplitude: computed via the Bethe-Salpeter equation

Electromagnetic pion form factor

Leading-order in a nonperturbative, symmetry-preserving truncation scheme

Valid formulation of the DSEs preserves all symmetry relations between the elements

All elements determined ONCE Gap Equation's kernel is specified

Enormous power to predict and correlate observables

$\Gamma_\mu(p,q)$ – Dressed-quark-photon vertex: Computed via Bethe-Salpeter equation

$$(p_2+p_1)_\mu F_\pi^{\rm em}(Q^2) = 2N_c \int \frac{d^4t}{(2\pi)^4} {\rm tr_D}\left[i\Gamma_\pi(t;-p_2) S(t+p_2) i\Gamma_\mu(t+p_2,t+p_1) S(t+p_1)\, i\Gamma_\pi(t;p_1)\, S(t)\right]$$

$\Gamma_\pi(k;P)$ – Pion Bethe-Salpeter amplitude: computed via the Bethe-Salpeter equation

$S(p)$ – dressed-quark propagator: computed via the Gap Equation

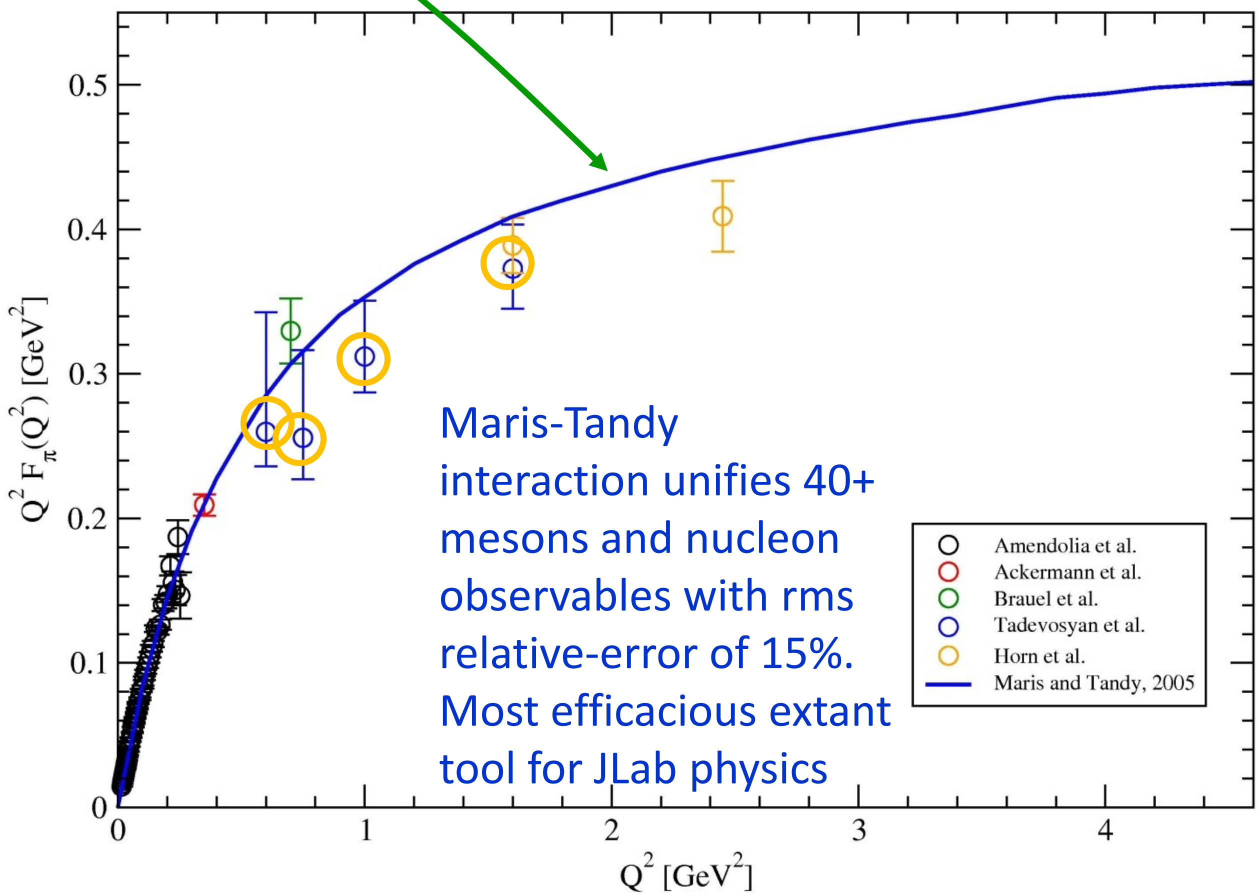
After solving gap and Bethe-Salpeter equations, one four-dimensional integral remains to be done.

Result is successful ***prediction*** of $F_\pi(Q^2)$ by Maris and Tandy, Phys.Rev. C **62** (2000) 055204, nucl-th/0005015

Result is successful *prediction* of $F_\pi(Q^2)$ by Maris and Tandy, Phys.Rev. C **62** (2000) 055204, nucl-th/0005015

Electromagnetic pion form factor

- Prediction published in 1999. Numerical technique improved subsequently, producing no material changes
- Data from Jlab published in 2001
- DSE Computation has one parameter, $m_G \approx 0.8$GeV, and unifies $F_\pi(Q^2)$ with *numerous* other observables



*Maris, Roberts and Tandy, nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

Corrected an error, which had prevented progress for 18years

- Pion's Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k;P) \right]$$

Pseudovector components necessarily nonzero. Cannot be ignored!

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

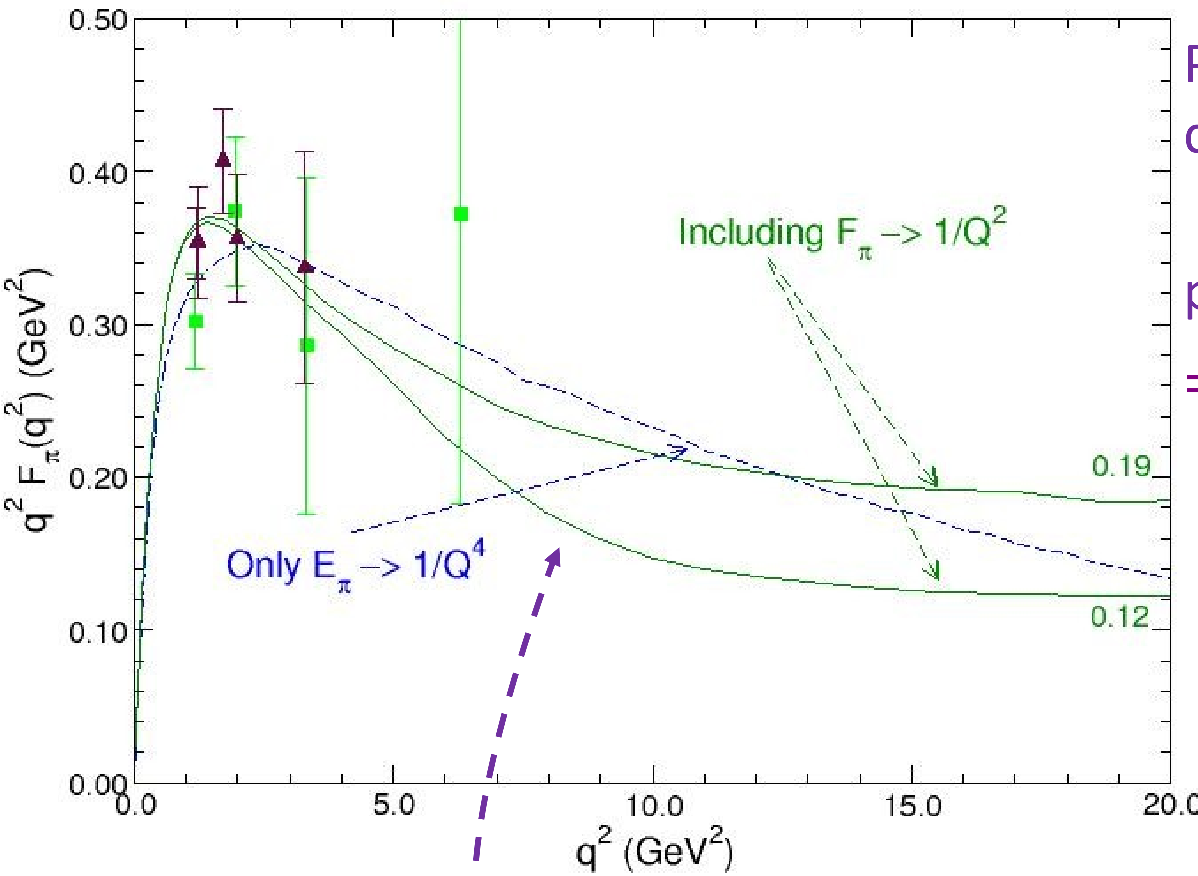
- Axial-vector Ward-Takahashi identity entails

Exact in Chiral QCD

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

*Maris and Roberts, nucl-th/9804062, Phys.Rev. C **58** (1998) 3659-3665*

Pion's GT relation Implications for observables?



Pseudovector components dominate in ultraviolet:

$(½Q)^2 = 2$ GeV^2

pQCD point for $M(p^2)$

⇒ pQCD at $Q^2 = 8$ GeV^2

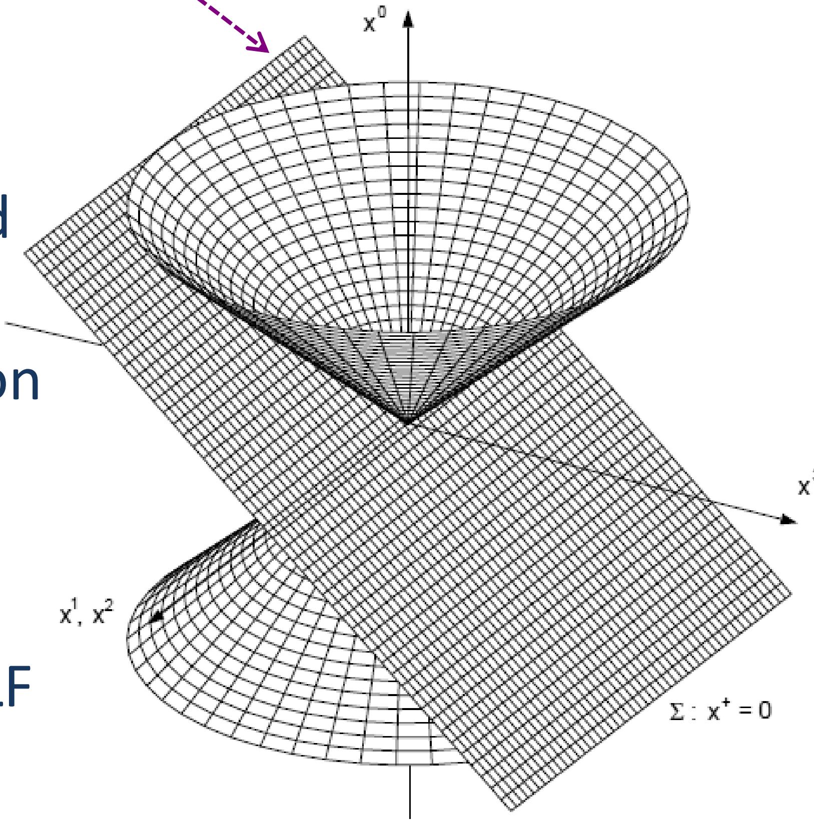

I'm not sure that these shoes really go with these trousers... what do you all think ?
During an interview, it's important to be succinct and avoid too many digressions or side issues.

A side issue

Light-front Frame

- Hamiltonian formulation of quantum field theory.
  - Fields are specified on a particular initial surface:

    Light front $x^+ = x^0 + x^3 = 0$
- Using LF quantisation:
  - ✓ quantum mechanics-like wave functions can be defined;
  - ✓ quantum-mechanics-like expectation values can be defined and evaluated
  - ✓ Parton distributions are correlation functions at equal LF-time x^+ ; namely, within the initial surface $x^+ = 0$ and can thus be expressed directly in terms of ground state LF wavefunctions



Very much not the case in equal time quantisation: $x^0=0$.

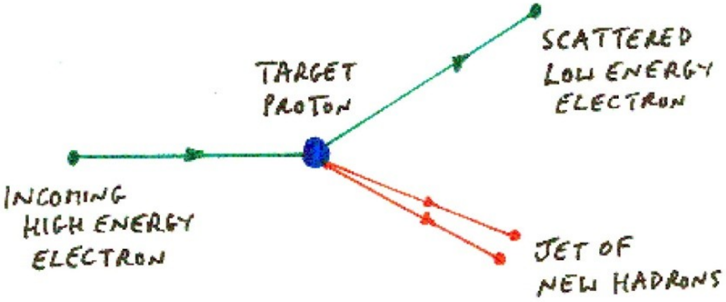
Light-front Frame

- These features owe to particle no. conservation in LF frame:
  - ✓ zero-energy particle-antiparticle production impossible because $p^+ > 0$ for all partons. Hence state with additional particle-antiparticle pair has higher energy
- Thus, in LF frame, parton distributions have a very simple physical interpretation
  - as single particle momentum densities, where $x_{Bj} = x_{LF}$ measures the fraction of the hadron's momentum carried by the parton
- It follows that the Light-Front Frame is the natural choice for theoretical analysis of
  - Deep inelastic scattering
  - Asymptotic behaviour of pQCD scattering amplitudes

In many cases, planar diagrams are all that need be evaluated. Others are eliminated by the $p^+ > 0$ constraint

Full Poincaré covariance

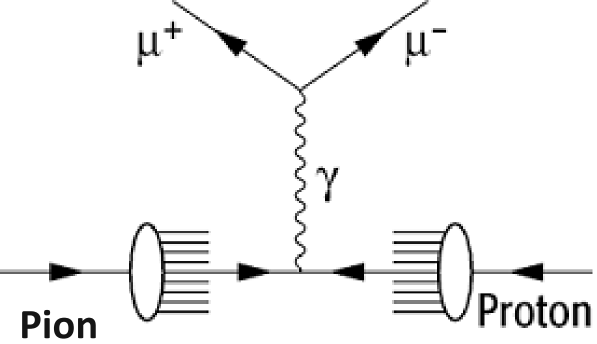
- Light front frame is special, with many positive features
- However, not Poincaré-covariant; e.g.,
  - Rotational invariance is lost
  - Very difficult to preserve Ward-Takahashi identities in any concrete calculation: different interaction terms in different components of the same current, J_+ cf. J_-
  - $P^+ > 0$ constraint has hitherto made it impossible to unravel mechanism of DCSB within LF formalism
- LF formalism is practically useless as nonperturbative tool in QCD
- DSEs are a Poincaré-covariant approach to quantum field theory
  - Truncations can be controlled. Omitted diagrams change anomalous dimension but not asymptotic power laws
  - Proved existence of DCSB in QCD
  - Can be used to compute light-front parton distributions



Deep inelastic scattering

- Quark discovery experiment at SLAC
  (1966-1978, Nobel Prize in 1990)
- Completely different to elastic scattering
  - *Blow the target to pieces instead of keeping only those events where it remains intact.*
- Cross-section is interpreted as a measurement of the momentum-fraction probability distribution for quarks and gluons within the target hadron: $q(x)$, $g(x)$ ← *Probability that a quark/gluon within the target will carry a fraction x of the bound-state's light-front momentum*

Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



Empirical status of the Pion's valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

  $\pi\, p \rightarrow \mu^{+}\, \mu^{-}\, X$

- Three experiments: CERN (1983 & 1985) and FNAL (1989). No more recent experiments because theory couldn't even explain these!
- Problem

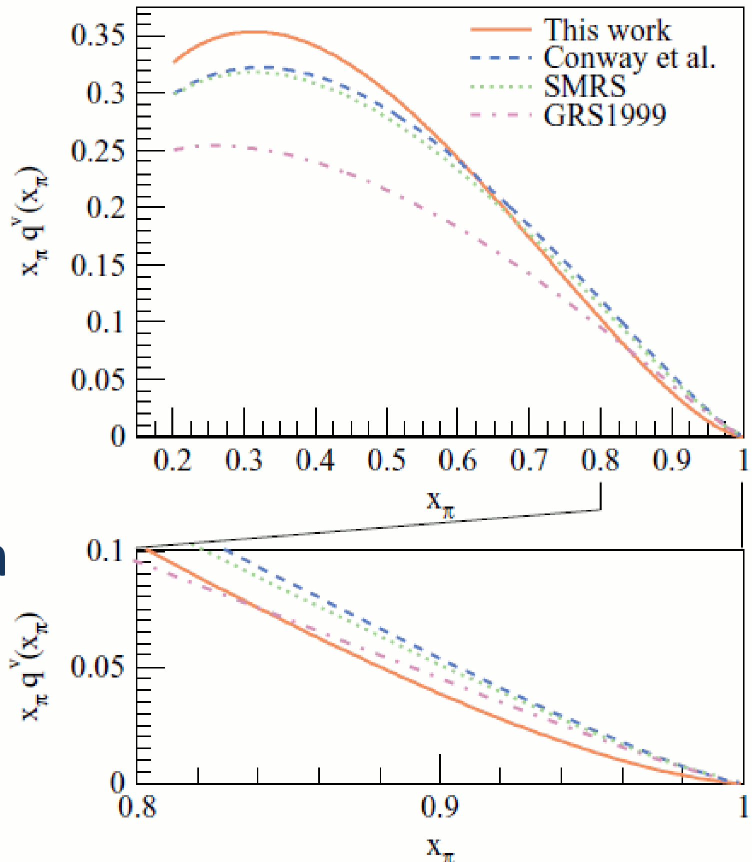
  Conway *et al*. Phys. Rev. D **39**, 92 (1989)

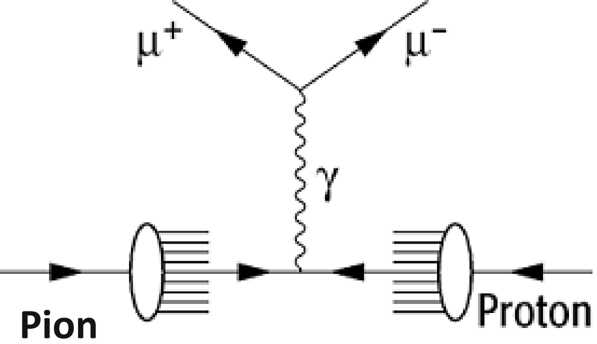
  Wijesooriya *et al*. Phys.Rev. C **72** (2005) 065203

  Behaviour at large-*x* inconsistent with pQCD; viz,

  expt. $(1-x)^{1+\varepsilon}$

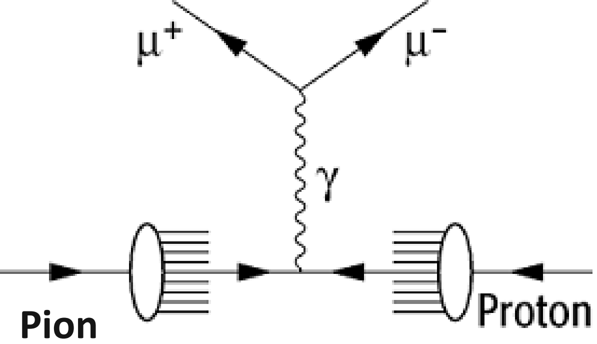
  cf. QCD $(1-x)^{2+\gamma}$





Models of the Pion's valence-quark distributions

- *$(1-x)^\beta$ with β=0 (i.e., a* constant – any fraction is equally probable!)
  - AdS/QCD models using light-front holography
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- *$(1-x)^\beta$ with β=1*
  - *Nambu*–Jona-Lasinio NJL models with a hard cutoff
  - Duality arguments produced by some theorists
- *$(1-x)^\beta$ with 0<β<2*
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- *$(1-x)^\beta$ with 1<β<2*
  - Instanton-based models, all of which have incorrect large-k^2 behaviour

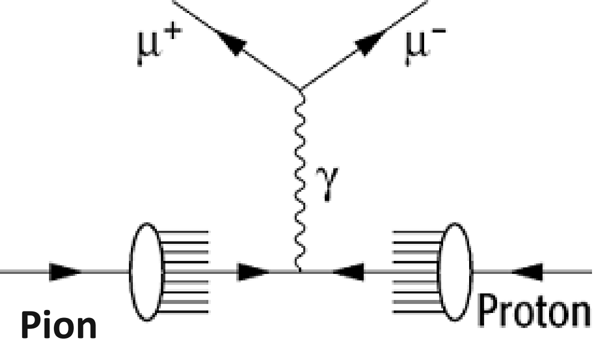


Models of the Pion's valence-quark distributions

- *$(1-x)^\beta$ with $\beta=0$ (i.e., a* constant – any fraction is equally probable!)

Completely unsatisfactory. Impossible to suggest that there's even qualitative agreement!

- *$(1-x)^\beta$ with $1<\beta<2$*
  - Instanton-based models



DSE prediction of the Pion's valence-quark distributions

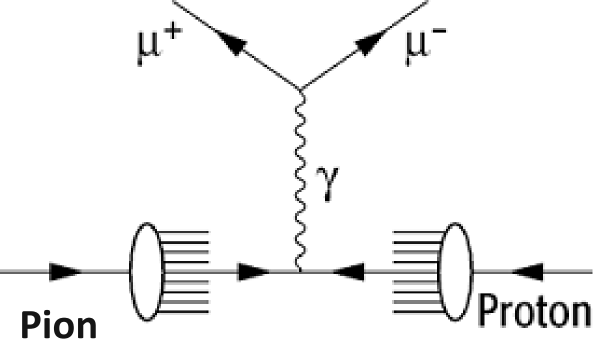
- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 \gg m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

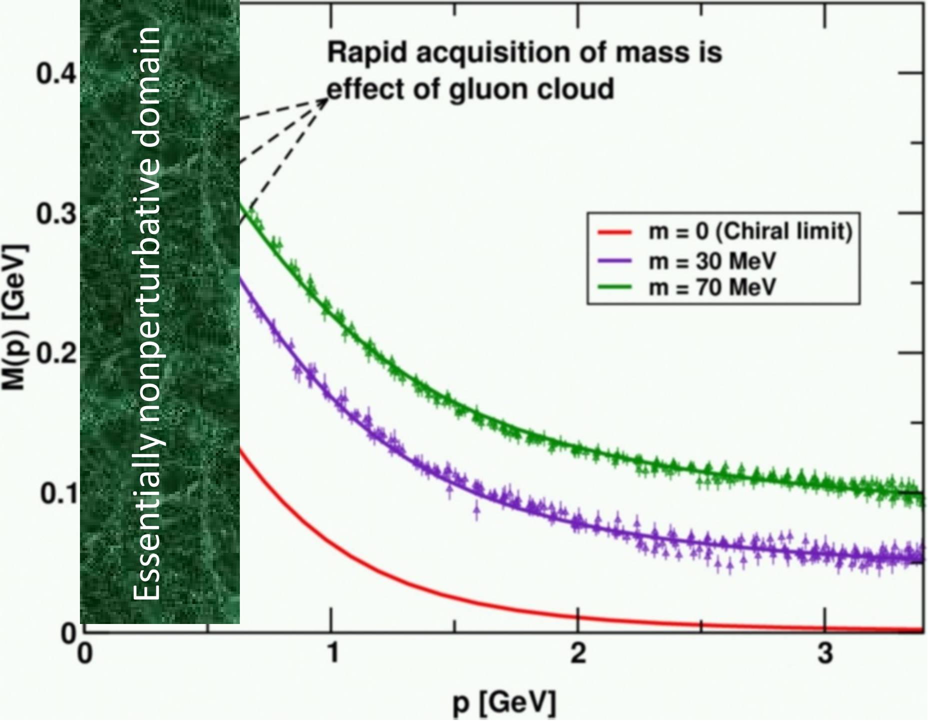
- In QCD, β=1 and hence

$$^{QCD}\, u_\pi(x;Q_0) \sim (1-x)^2$$



DSE prediction of the Pion's valence-quark distributions

- *Completely unambigous!*
- *Direct connection between experiment and theory, empowering both as tools of discovery.*



"Model Scale"

- At what scale Q_0 should the prediction be valid?
- Hitherto, PDF analyses within models have used the resolving scale Q_0 as a parameter, to be chosen by requiring agreement between the model and low-moments of the PDF that are determined empirically.
- Modern DSE studies have exposed a natural value for the model scale; viz.,

$$Q_0 \approx m_G \approx 0.6 \text{ GeV}$$

which is the location of the inflexion point in the chiral-limit dressed-quark mass function

Phys. Rev. C 63, 025213 (2001) [8 pages]

Valence-quark distributions in the pion

Abstract | References | Citing Articles (24)

M. B. Hecht, C. D. Roberts, and S. M. Schmidt
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843

Received 24 August 2000; published 23 January 2001

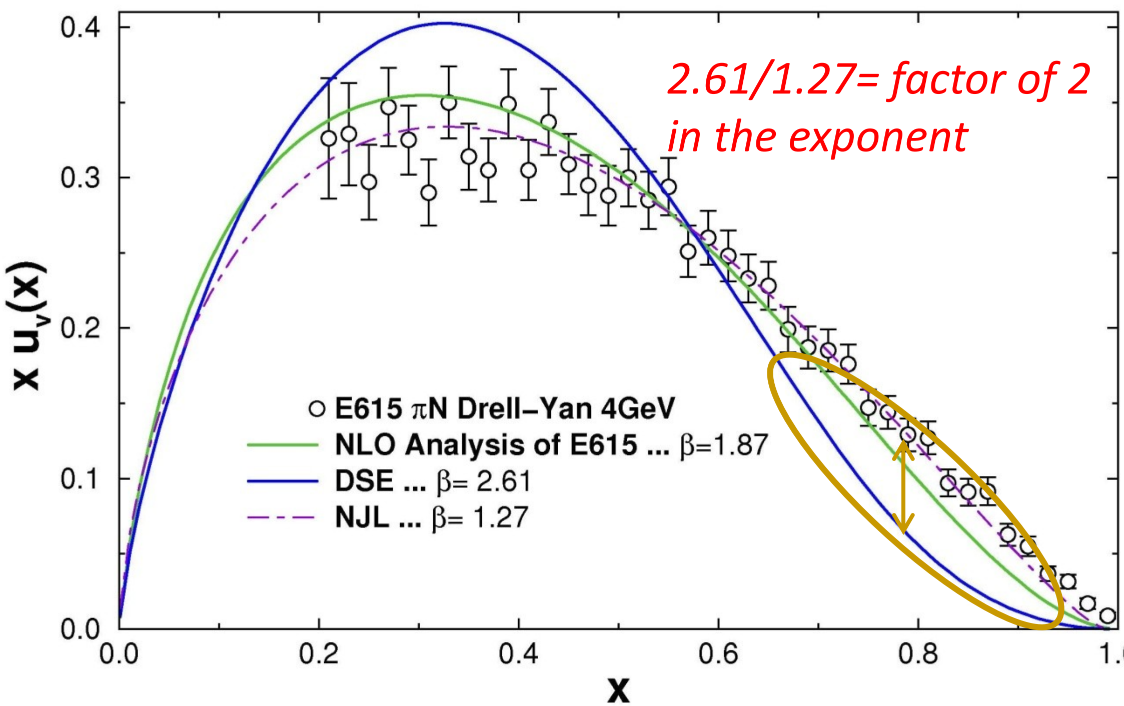
We calculate the pion's valence-quark momentum-fraction probability distribution using a Dyson-Schwinger equation model. Valence quarks with an active mass of 0.30 GeV carry 71% of the pion's momentum at a resolving scale q_0=0.54 GeV=1/(0.37 fm). The shape of the calculated distribution is characteristic of a strongly bound system and, evolved from q_0 to q=2 GeV, it yields first, second, and third moments in agreement with lattice and phenomenological estimates, and valence-quarks carrying 49% of the pion's momentum. However, pointwise there is a discrepancy between our calculated distribution and that hitherto inferred from parametrizations of extant pion-nucleon Drell-Yan data.

URL: http://link.aps.org/doi/10.1103/PhysRevC.63.025213

QCD-based calculation

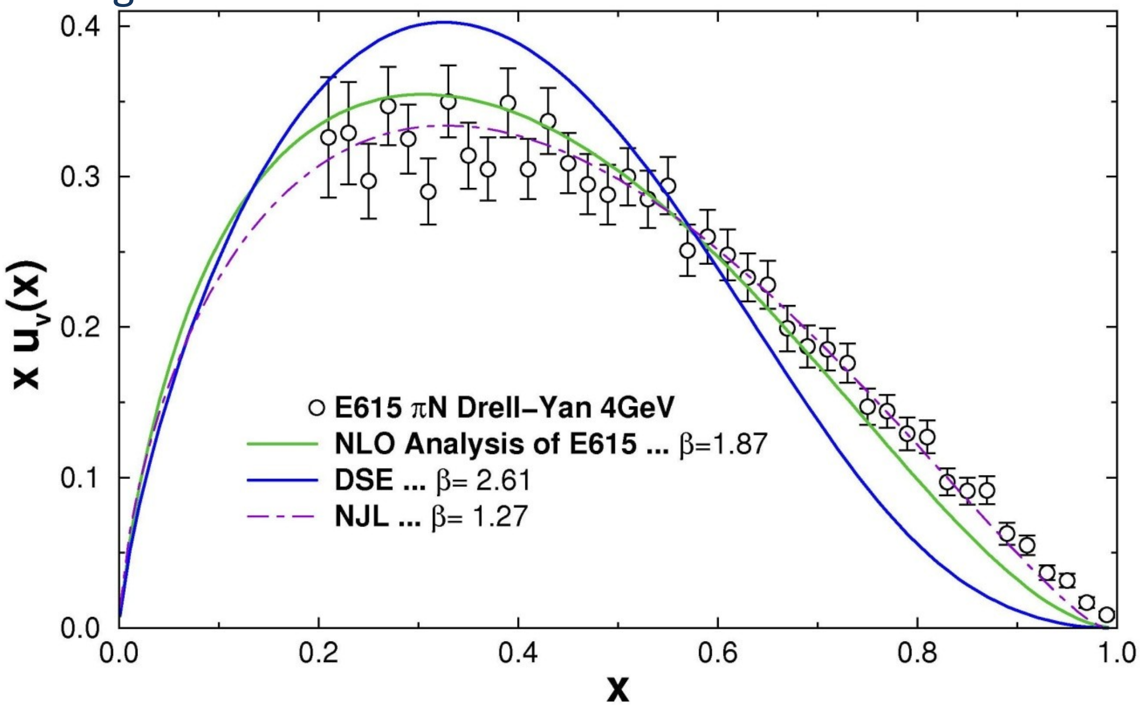
Computation of $q_v^\pi(x)$

- As detailed in preceding transparencies, before the first DSE computation, which used the running dressed-quark mass described previously, numerous authors applied versions of the Nambu–Jona-Lasinio model, etc., and were content to vary parameters and Q_0 in order to reproduce the data, arguing therefrom that the inferences from pQCD were wrong
- After the first DSE computation, real physicists (i.e., *experimentalists*) again became interested in the process because
  - DSEs agreed with pQCD but disagreed with the data and models
- Disagreement on the "valence domain," which is uniquely sensitive to $M(p^2)$



Reanalysis of $q_v^\pi(x)$

- After the first DSE computation, the "Conway *et al.*" data were reanalysed, this time at next-to-leading-order (Wijesooriya *et al.* Phys.Rev. C **72** (2005) 065203)
- The new analysis produced a much larger exponent than initially obtained; viz., β=*1.87*, but now it disagreed equally with NJL-model results and the DSE prediction
  - ✓ NB. Within pQCD, one can readily understand why adding a higher-order correction leads to a suppression of $q_v^\pi(x)$ at large-*x*.
- New experiments were proposed … for accelerators that do not yet exist but the situation remained otherwise unchanged
- Until the publication of *Distribution Functions of the Nucleon and Pion in the Valence Region*, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044

Reanalysis of $q_v^{\pi}(x)$

PRL **105**, 252003 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion

Matthias Aicher,[1] Andreas Schäfer,[1] and Werner Vogelsang[2]
[1]*Institute for Theoretical Physics, University of Regensburg, D-93040 Regensburg, Germany*
[2]*Institute for Theoretical Physics, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*
(Received 15 September 2010; published 16 December 2010)

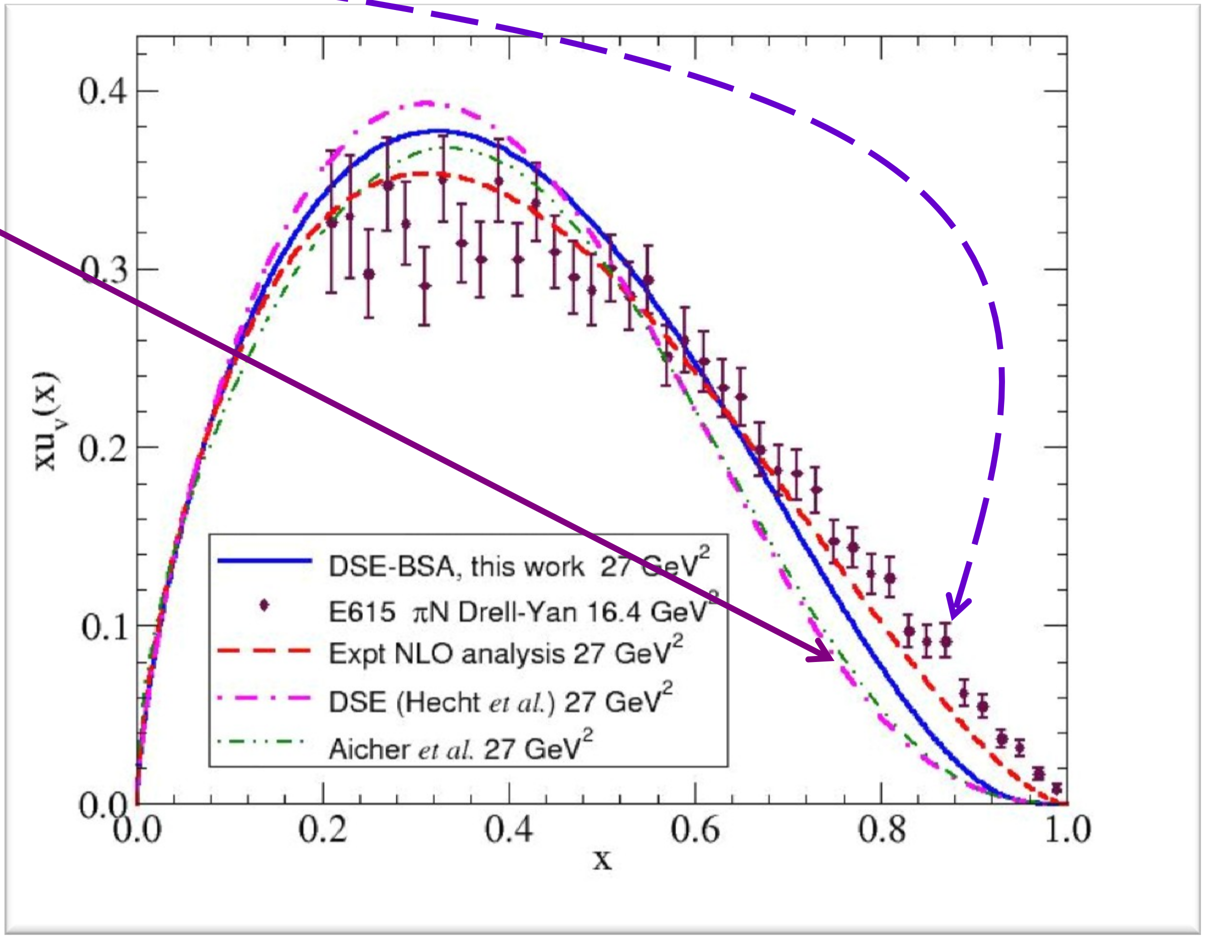
- This article emphasised and explained the importance of the persistent discrepancy between the DSE result and experiment as a challenge to QCD
- It prompted another reanalysis of the data, which accounted for a long-overlooked effect: viz., “soft-gluon resummation,”
  - *Compared to previous analyses, we include next-to-leading-logarithmic threshold resummation effects in the calculation of the Drell-Yan cross section. As a result of these, we find a considerably softer valence distribution at high momentum fractions x than obtained in previous next-to-leading-order analyses, in line with expectations based on perturbative-QCD counting rules or Dyson-Schwinger equations.*

Aicher, Schäfer, Vogelsang, “Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion,” Phys. Rev. Lett. ***105*** *(2010) 252003*

Trang, Bashir, Roberts & Tandy, "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

Current status of $q_v^\pi(x)$

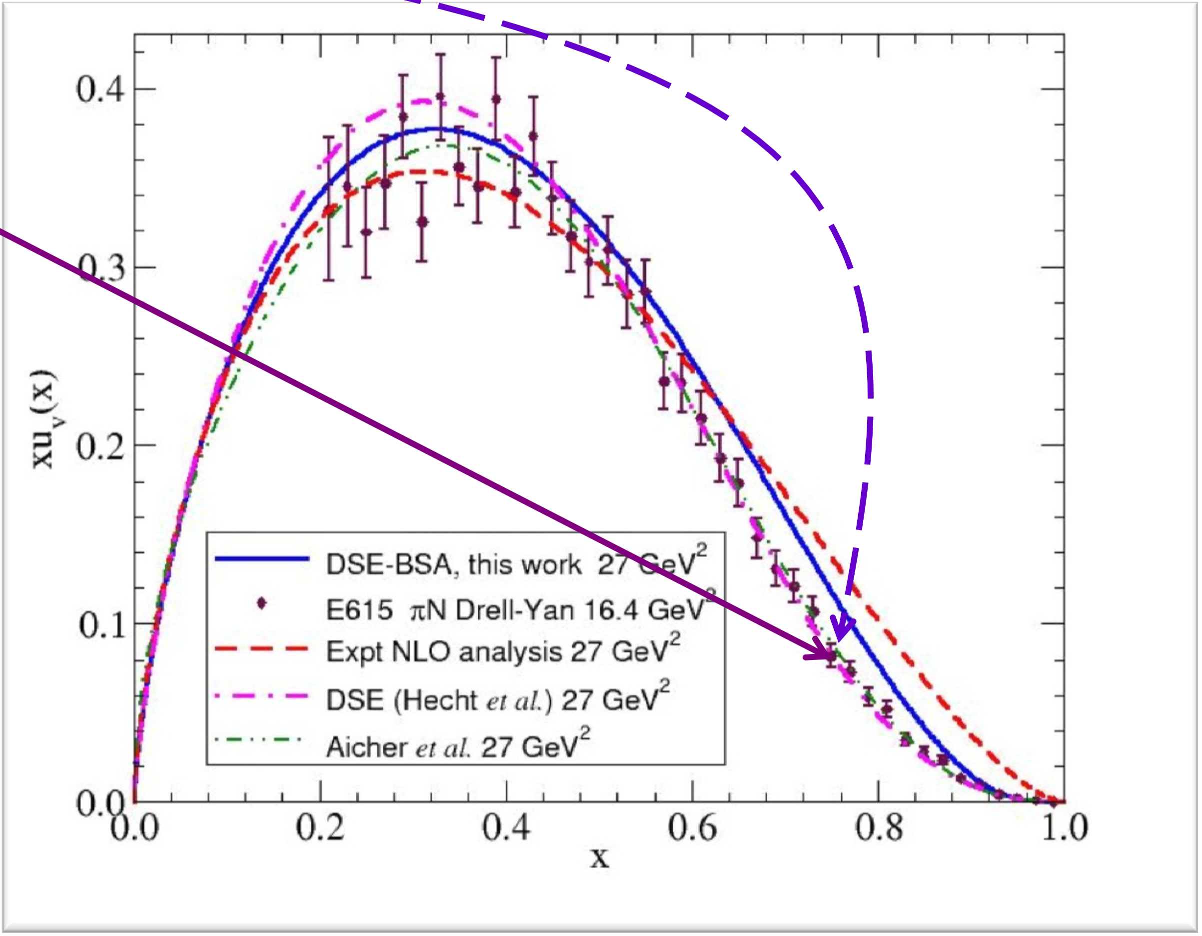
- Data as reported byE615
- DSE prediction (2001)



Trang, Bashir, Roberts & Tandy, "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

Current status of $q_v^\pi(x)$

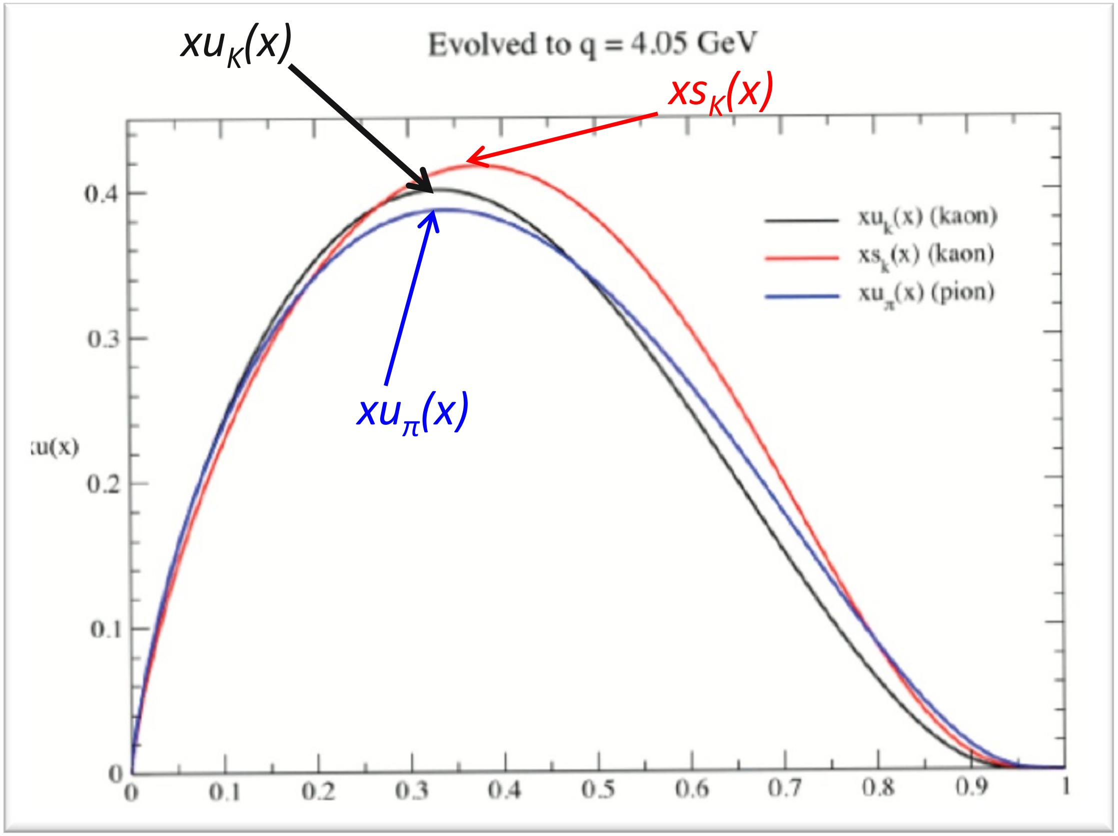
- Data after inclusion of soft-gluon resummation
- DSE prediction and modern representation of the data are *indistinguishable* on the valence-quark domain
- Emphasises the value of using a single internally-consistent, well-constrained framework to correlate and unify the description of hadron observables



Trang, Bashir, Roberts & Tandy, "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

$q_v^{\pi}(x)$ & $q_v^{K}(x)$

- $m_s \approx 24\, m_u$ & $M_s \approx 1.25\, M_u$
  Expect the *s*-quark to carry more of the kaon's momentum than the *u*-quark, so that $xs_K(x)$ peaks at larger value of x than $xu_K(x)$
- Expectation confirmed in computations, with *s*-quark distribution peaking at 15% larger value of x
- *Even though deep inelastic scattering is a high-Q^2 process, constituent-like mass-scale explains the shift*

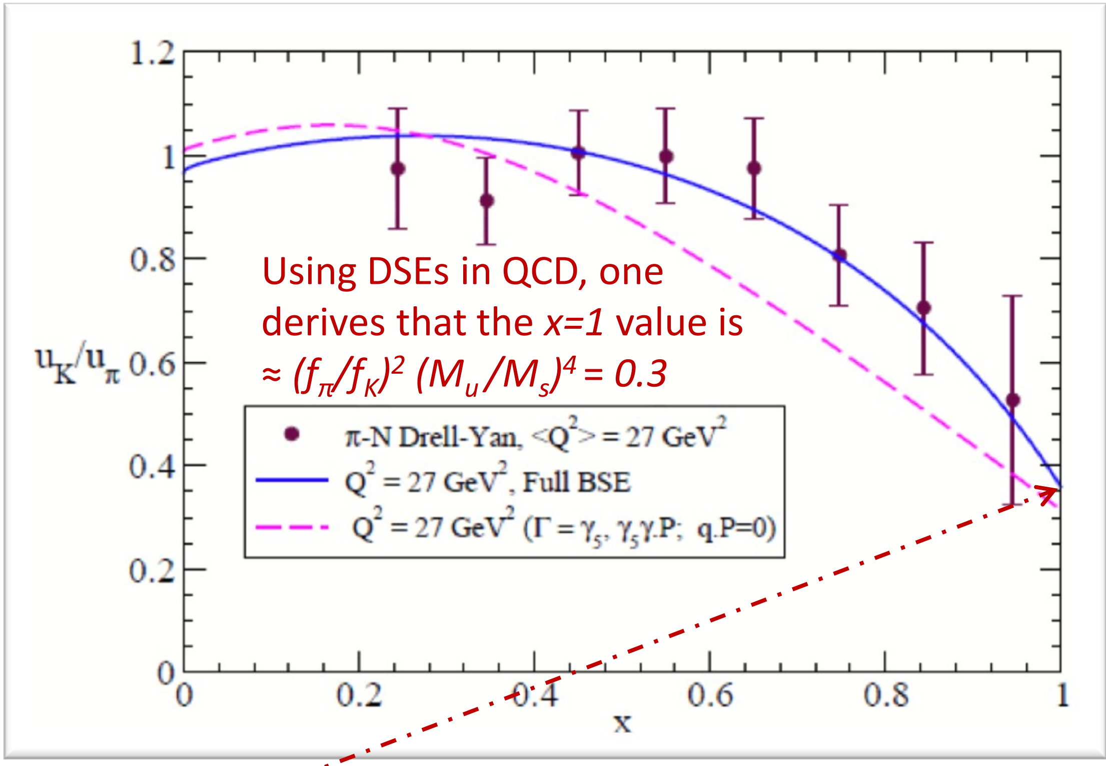


Trang, Bashir, Roberts & Tandy, "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

$u_K(x)/u_\pi(x)$

- Drell-Yan experiments at CERN (1980 & 1983) provide the only extant measurement of this ratio
- DSE result in complete accord with the measurement
- New Drell-Yan experiments are capable of validating this comparison
- It should be done so that complete understanding can be claimed

Value of ratio at x=0 will approach "1" under evolution to higher resolving scales. This is a feature of perturbative dynamics



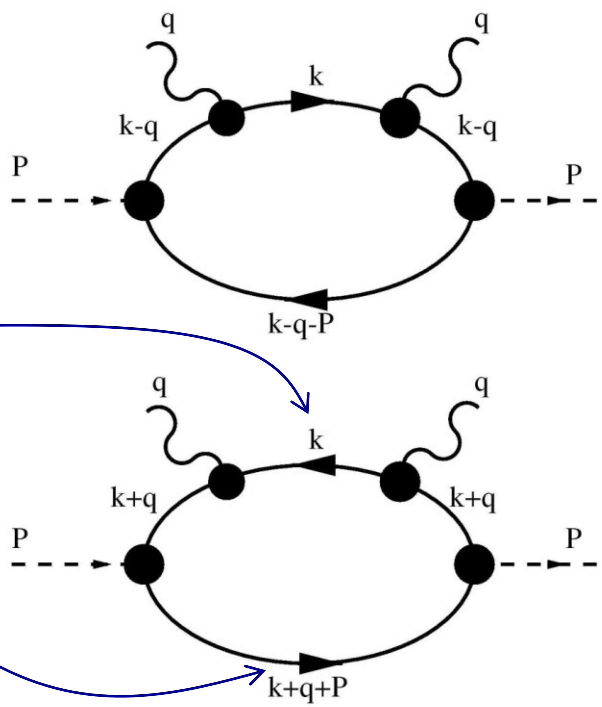
Value of ratio at x=1 is a fixed point of the evolution equations
Hence, it's a very strong test of nonperturbative dynamics

Reconstructing PDF from moments

- Suppose one cannot readily compute the PDF integral,
  - perhaps because one has employed a Euclidean metric, such as is typical of *all nonperturbative* studies with QCD connection
- Preceding computations employed a *dirty trick* to proceed from Euclidean space to the light-front; viz.,
  - Spectator pole approximation:

    $S_{dressed}(p) \to 1/(i\,\gamma\cdot p + M)$

    for internal lines
- Can one otherwise determine the PDF, without resorting to artifices?



Reconstructing PDF from moments

- Rainbow-ladder truncation – general expression for PDF moments:

π Bethe-Salpeter amplitude

Dressed-quark propagator

$$(n\cdot P)^{m+1}\langle x^m\rangle = \frac{3}{2i}\int\frac{d^4k}{(2\pi)^4}(n\cdot k)^m \,\mathrm{tr}\Big[\bar{\Gamma}_\pi(k-P/2)S(k)$$

Dressed-quark-photon vertex

$$\times\; n_\mu\Gamma_\mu(k,k)S(k)\Gamma_\pi(k-P/2)S(k-p)\Big]$$

$n^2=0$, $n.P= -m_\pi$

- Consider vector-vector interaction with exchange *$(1/k^2)^n$, $n=0$* then

$\langle x^m\rangle = 1/(m+1)$

- To which distribution does this correspond?

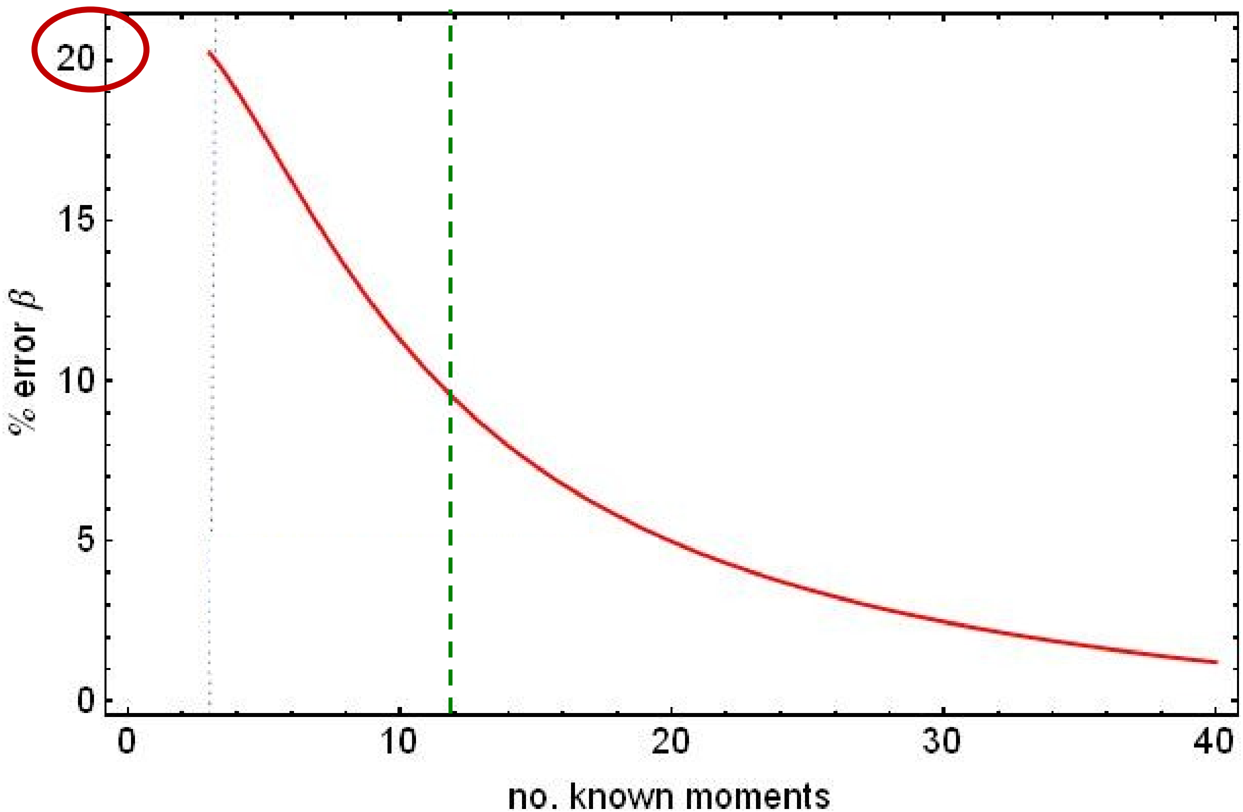
  Solve *$\int_0^1 dx\, x^m\, u_\pi(x) = 1/(m+1)$* for *$u_\pi(x)$*

  Answer *$u_\pi(x)=1$* can be verified by direct substitution

- Many numerical techniques available for more interesting interactions

Reconstructing the Distribution Function

- Suppose one has "N" nontrivial moments of the quark distribution function & assume $u_\pi(x) \sim x^\alpha (1-x)^\beta$
- Then, how accurately can one obtain the large-*x* exponent, β?
  - Available moments from lattice-QCD … not better than 20%
  - 12 moments needed for 10% accuracy
- Lower bound … For a more complicated functional form, one needs more moments.



With 40 nontrivial moments, obtain β=2.03 from *1/k²* input

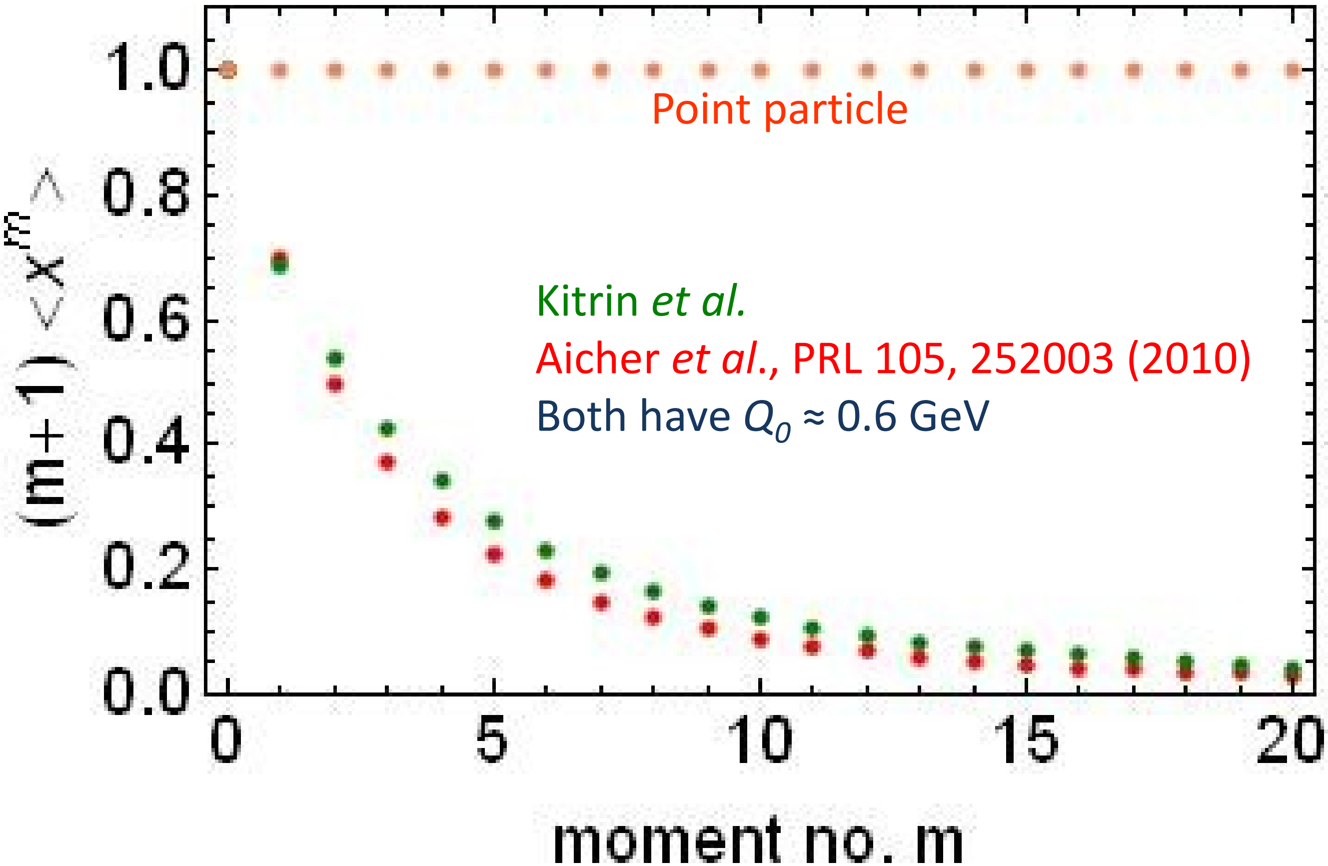
Khitrin, Roberts & Tandy, in progress;
Si-xue Qin, Lei Chang, Yu-xin Liu, Craig Roberts and David Wilson, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** 042202(R) (2011)

Euclidean Space Moments of the Distribution Function

- Best rainbow-ladder interaction available for QCD:

  $|\pi_{\text{bound-state}}>$

- Adjusted with one parameter to reflect inclusion of sea-quarks via pion cloud: $Z_D = 0.87$
- Origin in comparison with ChPT; viz., dressed-quark core produces 80% of ≈ r_π^2 and chiral-logs produce ≈ 20%



Used extensively in pQCD & by high-energy physicists pretending that nonpert. phenomena can be analysed using a simplistic convolution hybrid of pert. & nonperturbative QCD

Pion's valence-quark Distribution Amplitude

- Exact expression in QCD for the pion's valence-quark distribution amplitude

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, \underbrace{S(k)\Gamma_\pi(k;P)S(k-P)}$$

Pion's Bethe-Salpeter wave function

- Expression is Poincaré invariant but a probability interpretation is only valid in the light-front frame because only therein does one have particle-number conservation.

Whenever a nonrelativistic limit is realistic, this would correspond to the Schroedinger wave function.

- Probability that a valence-quark or antiquark carries a fraction

  $x=k_+ / P_+$

  of the pion's light-front momentum $\{ n^2=0, n.P = -m_\pi \}$

Pion's valence-quark Distribution Amplitude

- Moments method is also ideal for $\varphi_\pi(x)$:

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5\gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$

entails

$$(n\cdot P)^{m+1} \int_0^1 dx\, x^m\, \varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} (n\cdot k)^m \gamma_5\gamma\cdot n\, \chi_\pi(k;P)$$

Pion's Bethe-Salpeter wave function

- Contact interaction

$(1/k^2)^\nu$, $\nu=0$

Straightforward exercise to show

$\int_0^1 dx\, x^m\, \varphi_\pi(x) = f_\pi\, 1/(1+m)$, hence $\varphi_\pi(x)= f_\pi\, \Theta(x)\Theta(1-x)$

Work now underway with sophisticated rainbow-ladder interaction: *Khitrin, Cloët, Roberts & Tandy*

Pion's valence-quark Distribution Amplitude

- The distribution amplitude $\varphi_\pi(x)$ is actually dependent on the momentum-scale at which a particular interaction takes place; viz., $\varphi_\pi(x)= \varphi_\pi(x,Q)$
- One may show in general that $\varphi_\pi(x)$ has an expansion in terms of Gegenbauer polynomials:

$$\varphi_\pi(x;Q) = 6\,x\,(1-x)\left[1 + \sum_{n=2,4,6,\ldots}^{\infty} a_n(Q)\,C_n^{3/2}(1-2x)\right]$$

  Only even terms contribute because the neutral pion is an eigenstate of charge conjugation, so $\varphi_\pi(x)=\varphi_\pi(1-x)$
- Evolution, analogous to that of the parton distribution functions, is encoded in the coefficients $a_n(Q)$

Pion's valence-quark Distribution Amplitude

- Evolution, analogous to that of the parton distribution functions, is encoded in the coefficients $a_n(Q)$
- At leading-order:

  $C_2(R)=4/3$

  $C_2(G)=3$

$$\frac{a_n(Q)}{a_n(Q_0)} = \left(\frac{\alpha(Q)}{\alpha(Q_0)}\right)^{\frac{\gamma_n^0}{2\beta_0}}$$

$$\gamma_n^0 = -2C_2(R)\left[3+\frac{2}{(n+1)(n+2)}-4\sum_{k=1}^{n+1}\frac{1}{k}\right]$$

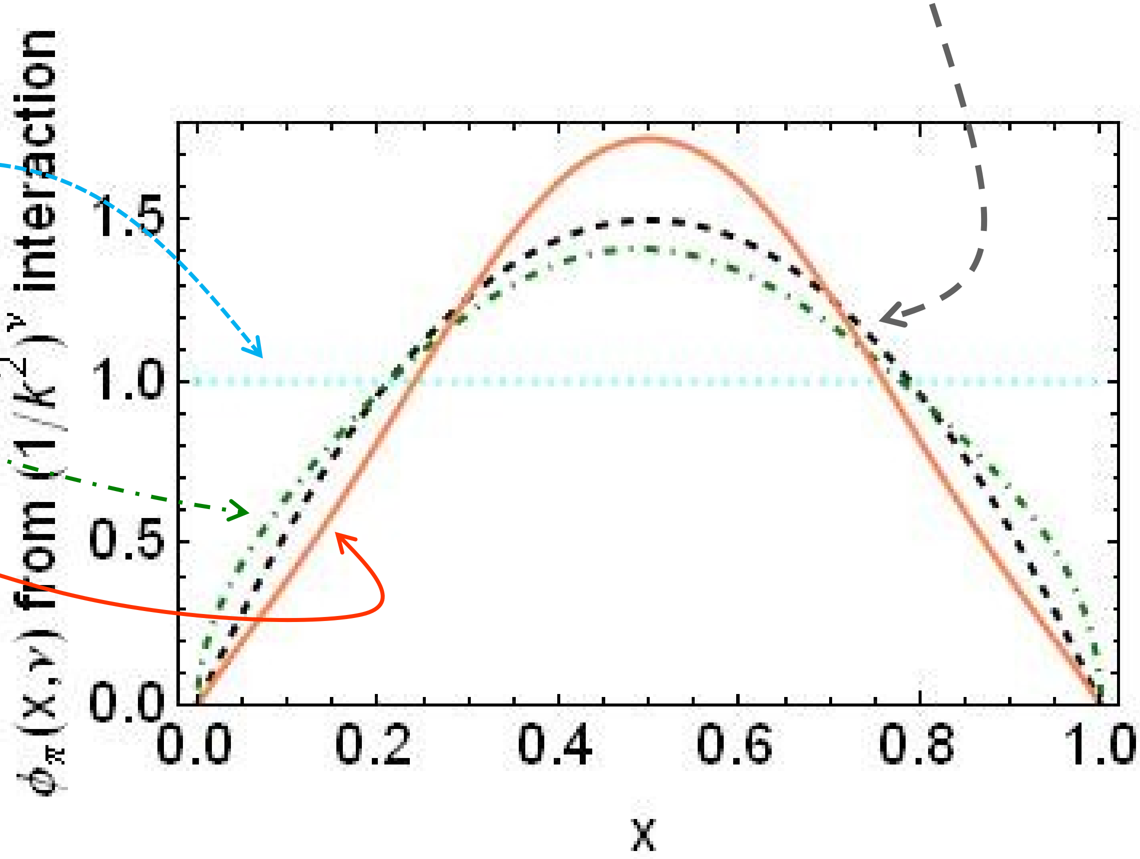
$$\beta_0 = \frac{11}{3}C_2(G)-\frac{2}{3}N_f$$

- Easy to see that $\gamma_n^0 > 0$, so that the $a_n(Q) < a_n(Q_0)$ for $Q > Q_0$. Thus, for all n, $a_n(Q \to \text{infinity}) \to 0$.
- Hence, $\varphi_\pi(x,Q \to \text{infinity}) = 6\, x\, (1-x)$ … "the asymptotic distribution" … the limiting pQCD distribution

Pion's valence-quark Distribution Amplitude

- Using simple parametrisations of solutions to the gap and Bethe-Salpeter equations, rapid and semiquantitatively reliable estimates can be made for $\varphi_\pi(x)$
  - $(1/k^2)^{\nu=0}$
  - $(1/k^2)^{\nu=½}$
  - $(1/k^2)^{\nu=1}$
- Again, unambiguous and direct mapping between behaviour of interaction and behaviour of distribution amplitude

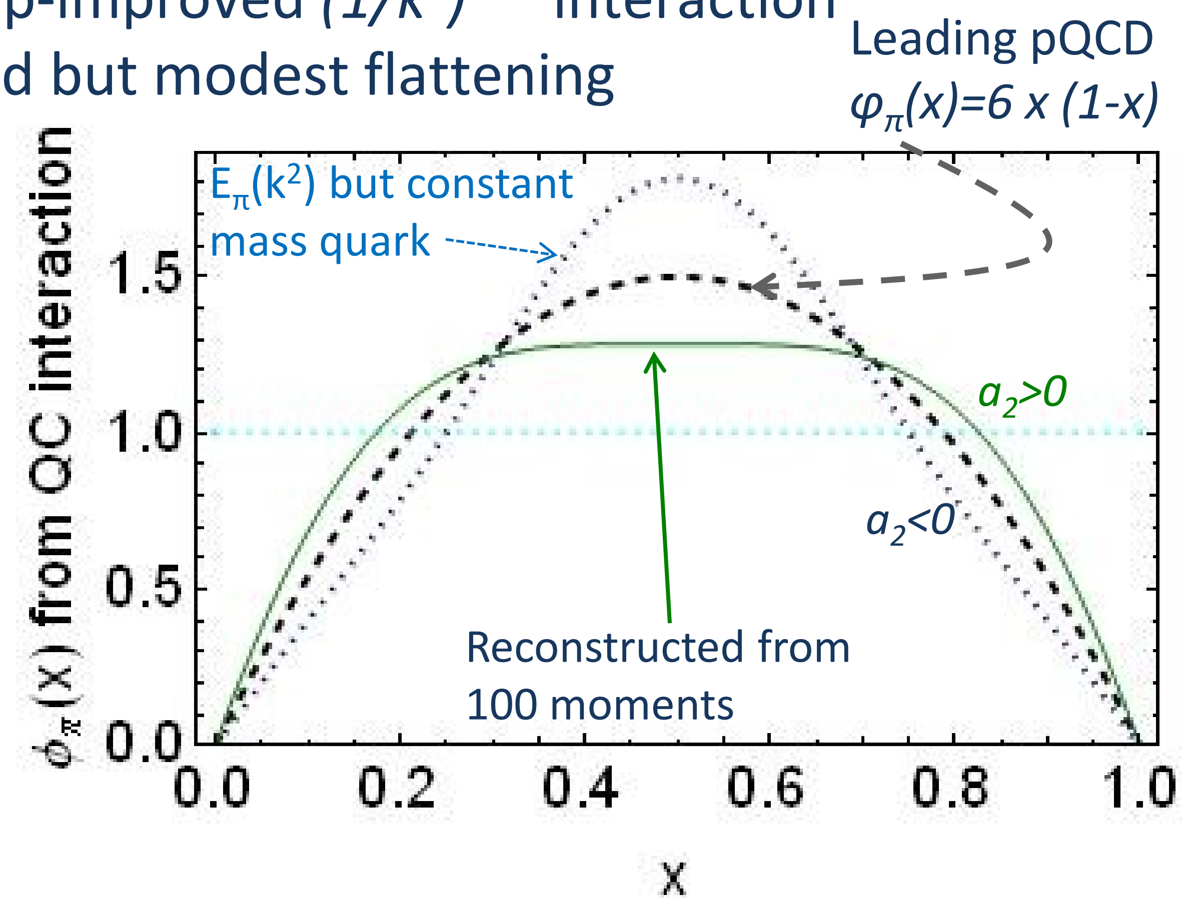
Leading pQCD $\varphi_\pi(x)=6\,x\,(1-x)$



Chang, Cloët, Roberts, Schmidt & Tandy, in progress;
Si-xue Qin, Lei Chang, Yu-xin Liu, Craig Roberts and David Wilson,
arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** 042202(R) (2011)

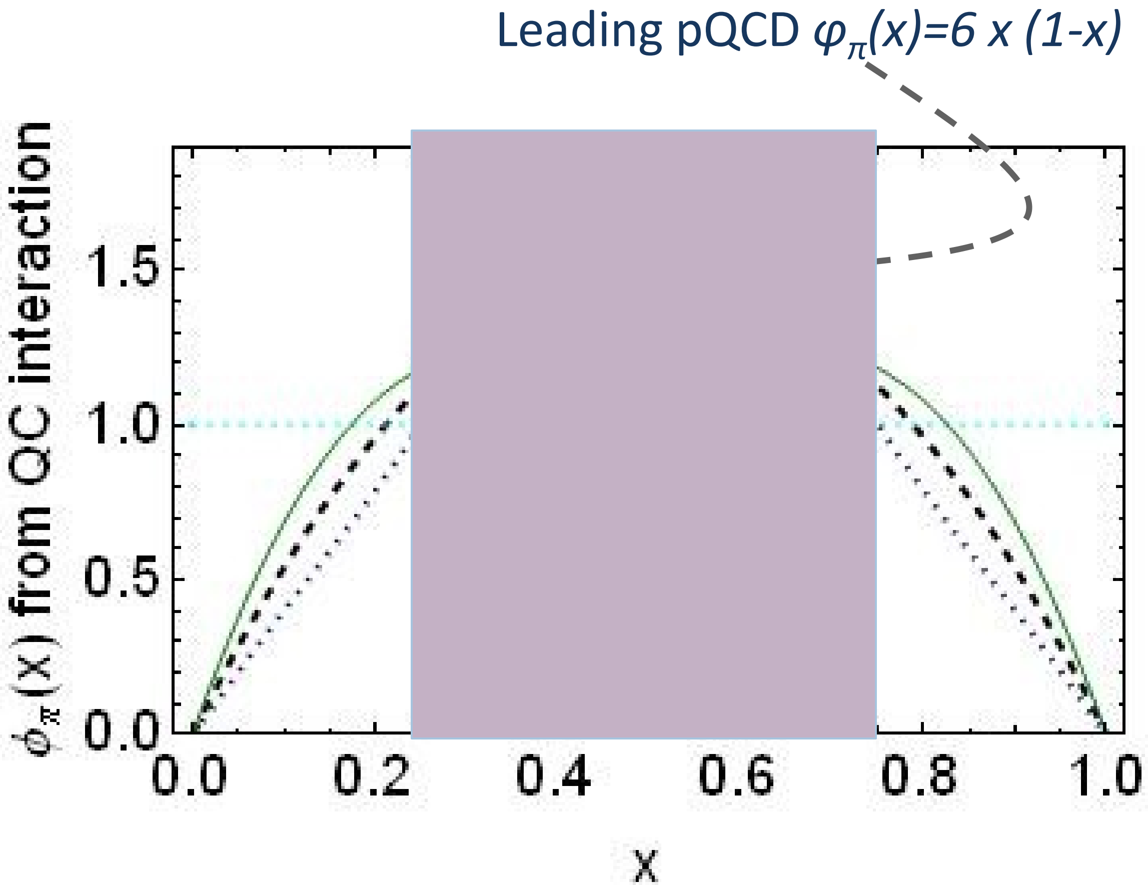
Pion's valence-quark Distribution Amplitude

- Preliminary results: rainbow-ladder QCD analyses of renormalisation-group-improved $(1/k^2)^{\nu=1}$ interaction – humped disfavoured but modest flattening
- Such behaviour is only obtained with
  (1) Running mass in dressed-quark propagators
  (2) Pointwise expression of Goldstone's theorem



Pion's valence-quark Distribution Amplitude

- $x \approx 0$ & $x \approx 1$ correspond to maximum relative momentum within bound-state
  - *expose pQCD physics*
- $x \approx ½$ corresponds to minimum possible relative momentum
  - *behaviour of distribution around midpoint is strongly influence by DCSB*



- Preliminary results, rainbow-ladder QCD analyses of $(1/k^2)^{\nu=1}$ interaction humped disfavoured but modest flattening

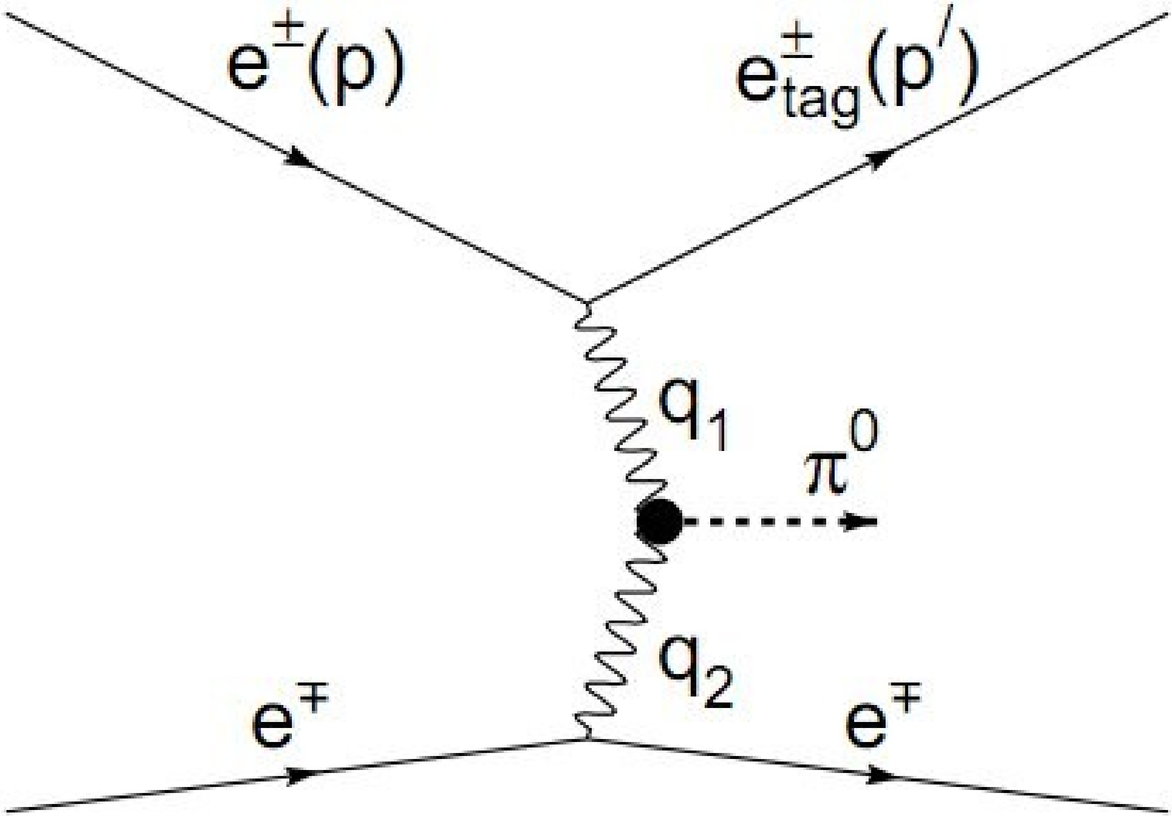
Pion's valence-quark Distribution Amplitude

- $x \approx 0$ & $x \approx 1$ correspond to maximum relative momentum within bound-state

Leading pQCD $\varphi_\pi(x)=6\,x\,(1-x)$

These computations are the first to offer the possibility of directly exposing DCSB – pointwise – in the light-front frame.

humped disfavoured but modest flattening



Abelian anomaly and neutral pion production

Why $\gamma^* \gamma \rightarrow \pi^0$?

- The process $\gamma^* \gamma \rightarrow \pi^0$ *is fascinating*
  - To explain this transition form factor within the standard model on the full domain of momentum transfer, one must combine
    - an explanation of the essentially nonperturbative Abelian anomaly
    - with the features of perturbative QCD.
  - Using a *single internally-consistent* framework!
- The case for attempting this has received a significant boost with the publication of data from the BaBar Collaboration (Phys.Rev. D80 (2009) 052002) because:
  - They agree with earlier experiments on their common domain of squared-momentum transfer (CELLO: Z.Phys. C49 (1991) 401-410; CLEO: Phys.Rev. D57 (1998) 33-54)
  - But the BaBar data are unexpectedly far *above the prediction of perturbative* QCD at larger values of Q^2.

Why $\gamma^* \gamma \rightarrow \pi^0$?

➢ The

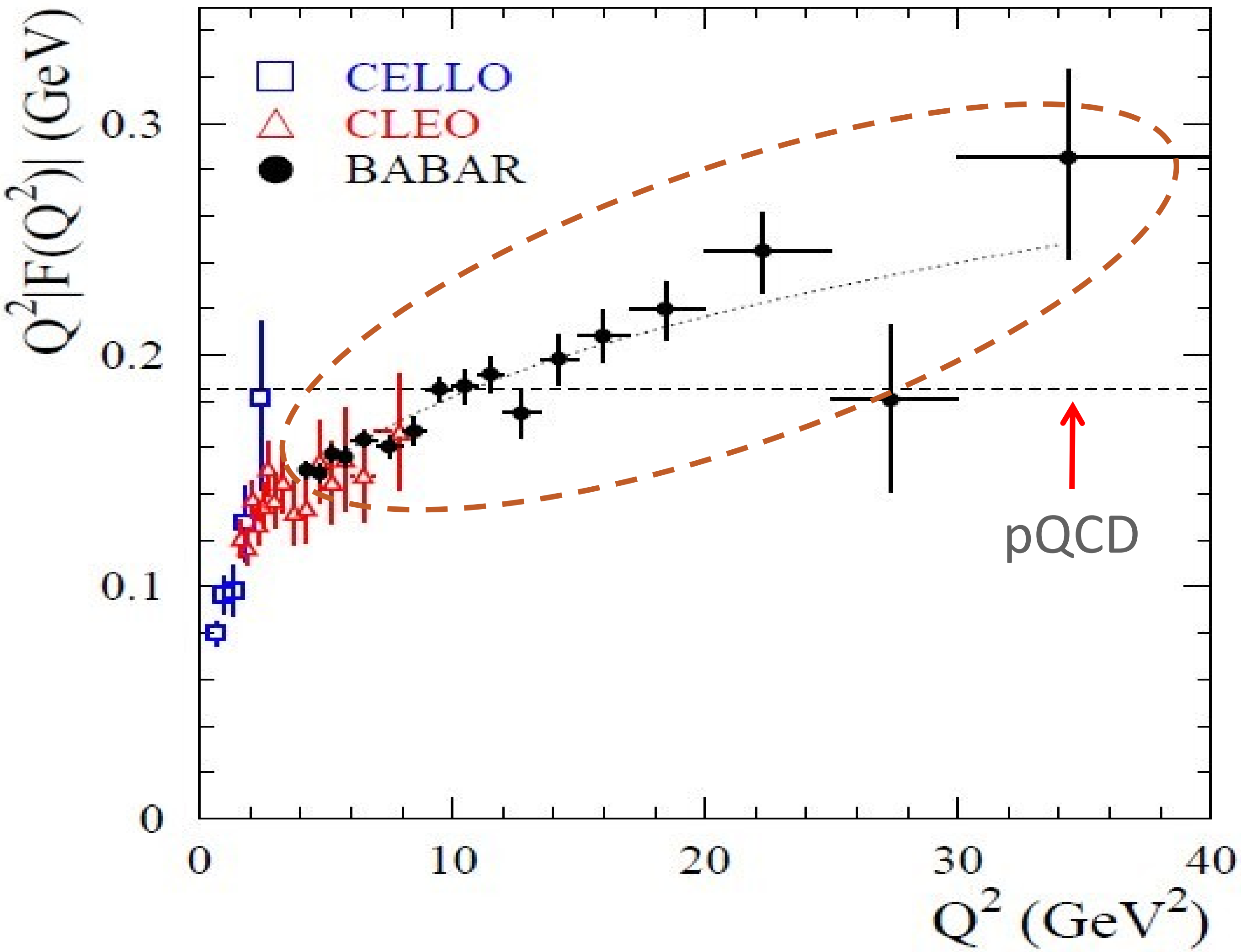
– T … n
t

– U

➢ The … th
the …
D80

– T
s … ;
C

– E

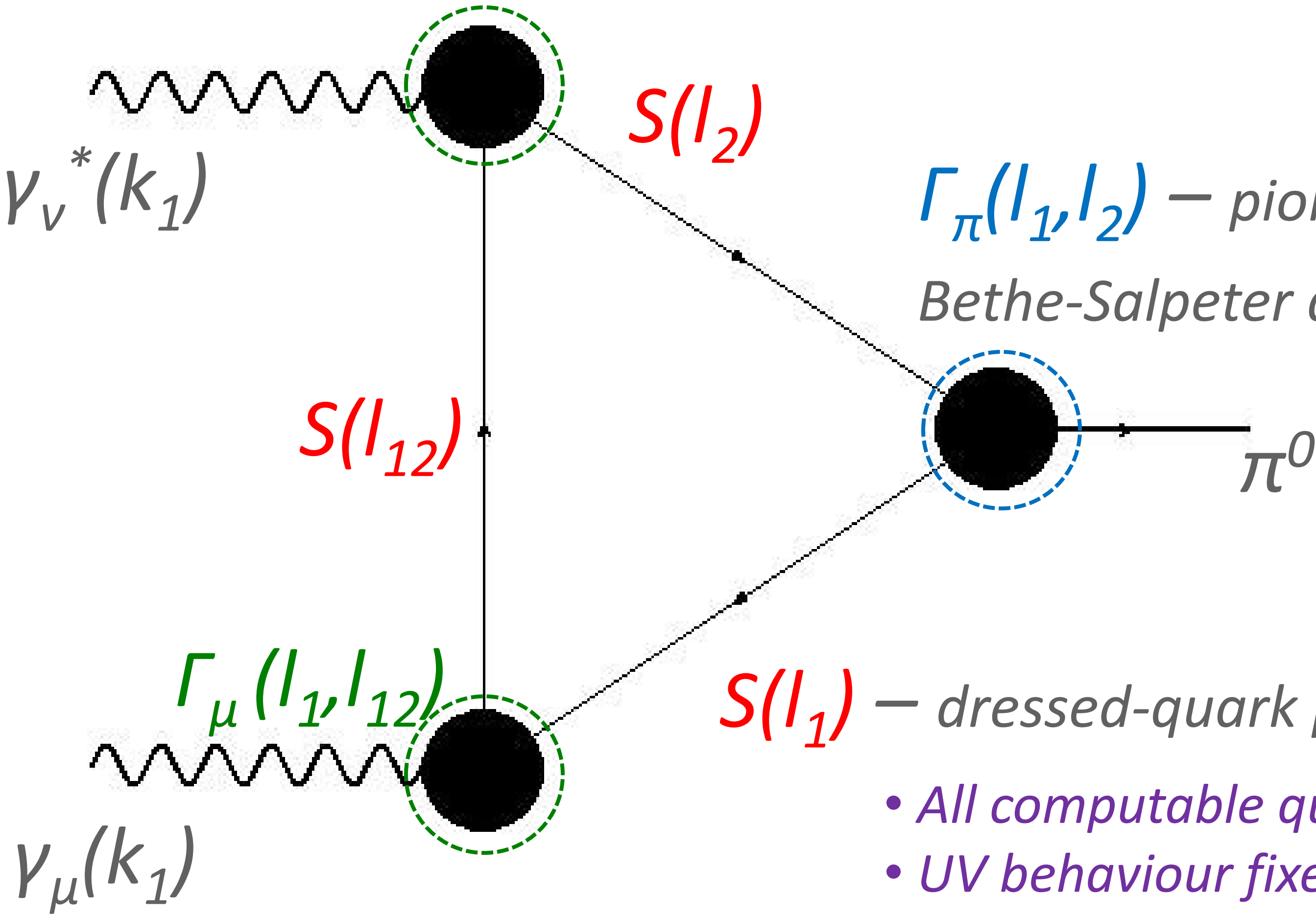


perturbative QCD at larger values of Q^2.

Transition Form Factor

$\gamma^*(k_1)\gamma(k_2) \rightarrow \pi^0$

$\Gamma_\nu(l_{12},l_2)$ – *dressed quark-photon vertex*



$\Gamma_\pi(l_1,l_2)$ – *pion Bethe-Salpeter amplitude*

$S(l_1)$ – *dressed-quark propagator*

- *All computable quantities*
- *UV behaviour fixed by pQCD*
- *IR Behaviour informed by DSE- and lattice-QCD*

Maris, Roberts and Tandy
nucl-th/9707003

π^0: Goldstone Mode & bound-state of strongly-dressed quarks

Critically!
Pseudovector components are necessarily nonzero. Cannot be ignored!

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

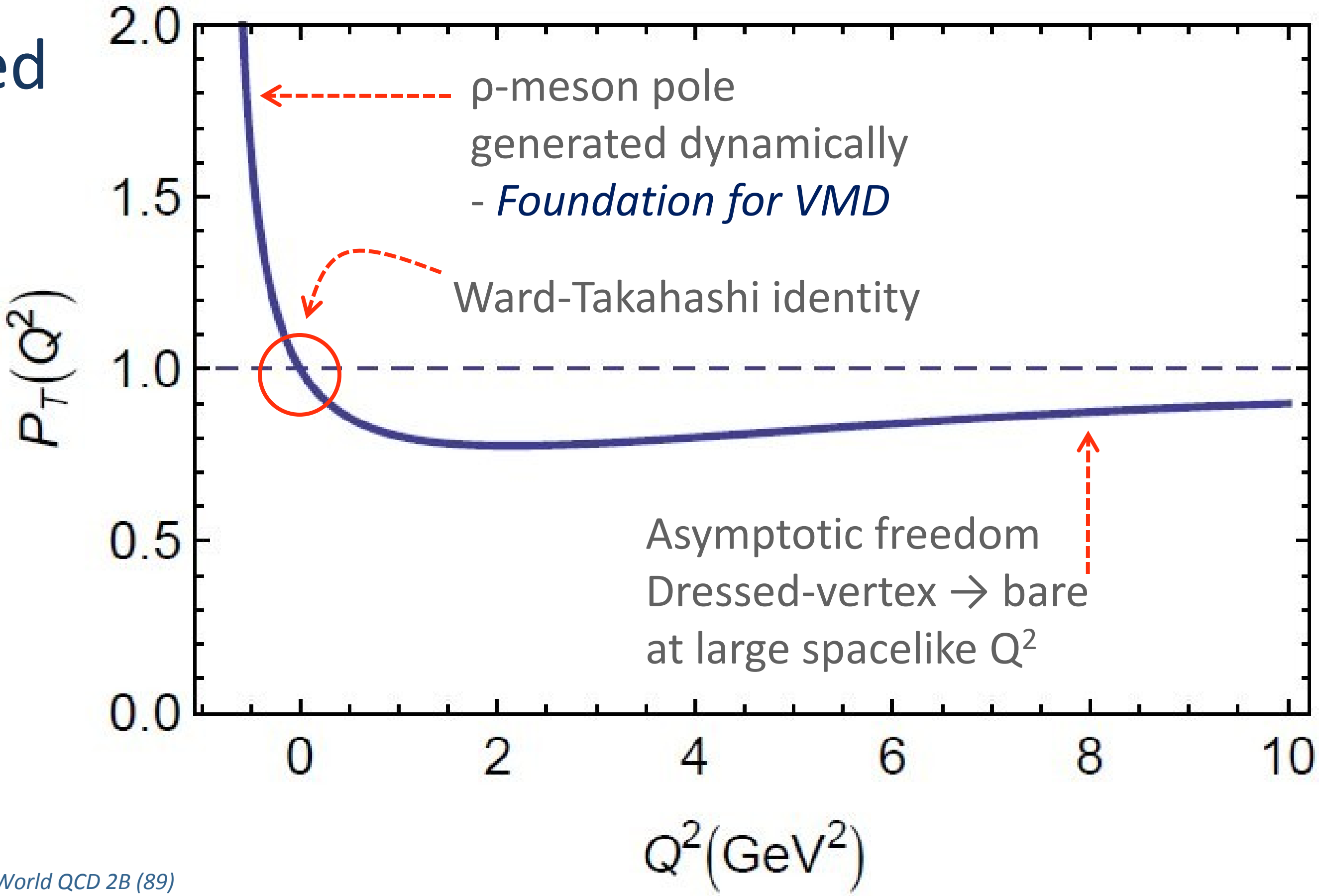
$$f_\pi E_\pi(k;P=0) = B(p^2)$$

Exact in Chiral QCD

Goldstones' theorem:
Solution of one-body problem solves the two-body problem

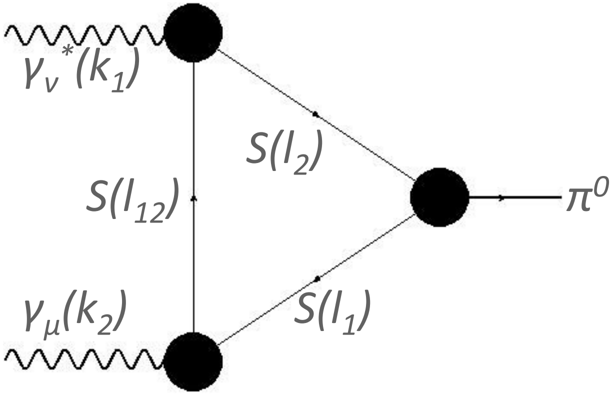
Dressed-quark-photon vertex

- Linear integral equation
  - Eight independent amplitudes
- Readily solved
- Leading amplitude



Transition Form Factor

$\gamma^*(k_1)\gamma(k_2) \to \pi^0$



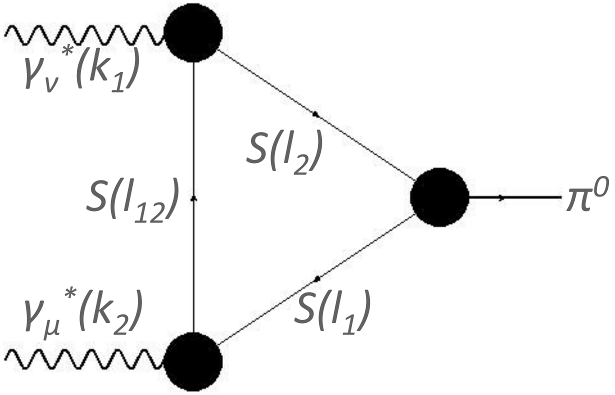
- *Calculation now straightforward*
- *However, before proceeding, consider slight modification*

$$\mathcal{T}_{\mu\nu}(k_1,k_2) = T_{\mu\nu}(k_1,k_2) + T_{\nu\mu}(k_2,k_1)\,,$$

$$\begin{aligned} T_{\mu\nu}(k_1,k_2) &= \frac{\alpha_{\rm em}}{\pi f_\pi}\,\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\,G(k_1^2, k_1\cdot k_2, k_2^2) \\ &= {\rm tr}\!\int\!\frac{d^4\ell}{(2\pi)^4}\,S(\ell_1)\,\Gamma_\pi(\ell_1,\ell_2)\,S(\ell_2)\,i\mathcal{Q}\Gamma_\mu(\ell_2,\ell_{12})\,S(\ell_{12})\,i\mathcal{Q}\Gamma_\nu(\ell_{12},\ell_1), \end{aligned}$$

with $\ell_1 = \ell - k_1$, $\ell_2 = \ell + k_2$, $\ell_{12} = \ell - k_1 + k_2$,
and $\mathcal{Q} = {\rm diag}[e_u, e_d] = e\,{\rm diag}[2/3, -1/3]$, $\alpha_{\rm em} = e^2/(4\pi)$.
The kinematic constraints are:

$$k_1^2 = Q^2\,,\;\; k_2^2 = 0\,,\;\; 2\,k_1\cdot k_2 = -(m_\pi^2 + Q^2)\,.$$



Transition Form Factor

$\gamma^*(k_1)\gamma^*(k_2) \rightarrow \pi^0$

- *Calculation now straightforward*
- *However, before proceeding, consider slight modification*

$$T_{\mu\nu}(k_1,k_2) = T_{\mu\nu}(k_1,k_2) + T_{\nu\mu}(k_2,k_1)\,,$$

$$T_{\mu\nu}(k_1,k_2) = \frac{\alpha_{\rm em}}{\pi f_\pi}\,\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\,G(k_1^2,k_1\cdot k_2,k_2^2)$$
$$= {\rm tr}\!\int\!\frac{d^4\ell}{(2\pi)^4}\,S(\ell_1)\,\Gamma_\pi(\ell_1,\ell_2)\,S(\ell_2)\,iQ\Gamma_\mu(\ell_2,\ell_{12})\,S(\ell_{12})\,iQ\Gamma_\nu(\ell_{12},\ell_1),$$

with $\ell_1 = \ell - k_1$, $\ell_2 = \ell + k_2$, $\ell_{12} = \ell - k_1 + k_2$,
and $Q = {\rm diag}[e_u, e_d] = e\,{\rm diag}[2/3, -1/3]$, $\alpha_{\rm em} = e^2/(4\pi)$.
The kinematic constraints are:

$$k_1^2 = Q^2\,,\ k_2^2 = Q^2,\ 2\,k_1\cdot k_2 = -(m_\pi^2 + 2Q^2)\,.$$

Only changes cf. $\gamma^*(k_1)\gamma(k_2) \rightarrow \pi^0$

Maris & Roberts, Phys.Rev. C58 (1998) 3659

Maris & Tandy, Phys.Rev. C65 (2002) 045211

Transition Form Factor

$\gamma^*(k_1)\gamma^*(k_2) \rightarrow \pi^0$

- Anomalous Ward-Takahashi Identity

chiral-limit: $G(0,0,0) = ½$

- Inviolable prediction
  - No computation believable if it fails this test
  - No computation believable if it doesn't confront this test.
- DSE prediction, model-independent:

$Q^2=0, G(0,0,0)=1/2$

Corrections from $m_\pi^2 \neq 0$, just 0.4%

Maris & Tandy, Phys.Rev. C65 (2002) 045211

Transition Form Factor
$\gamma^*(k_1)\gamma^*(k_2) \rightarrow \pi^0$

- pQCD prediction

$$\lim_{Q^2\to\infty} Q^2 G(Q^2, -Q^2 - m_\pi^2/2, Q^2) = \frac{2}{3} 4\pi^2 f_\pi^2$$

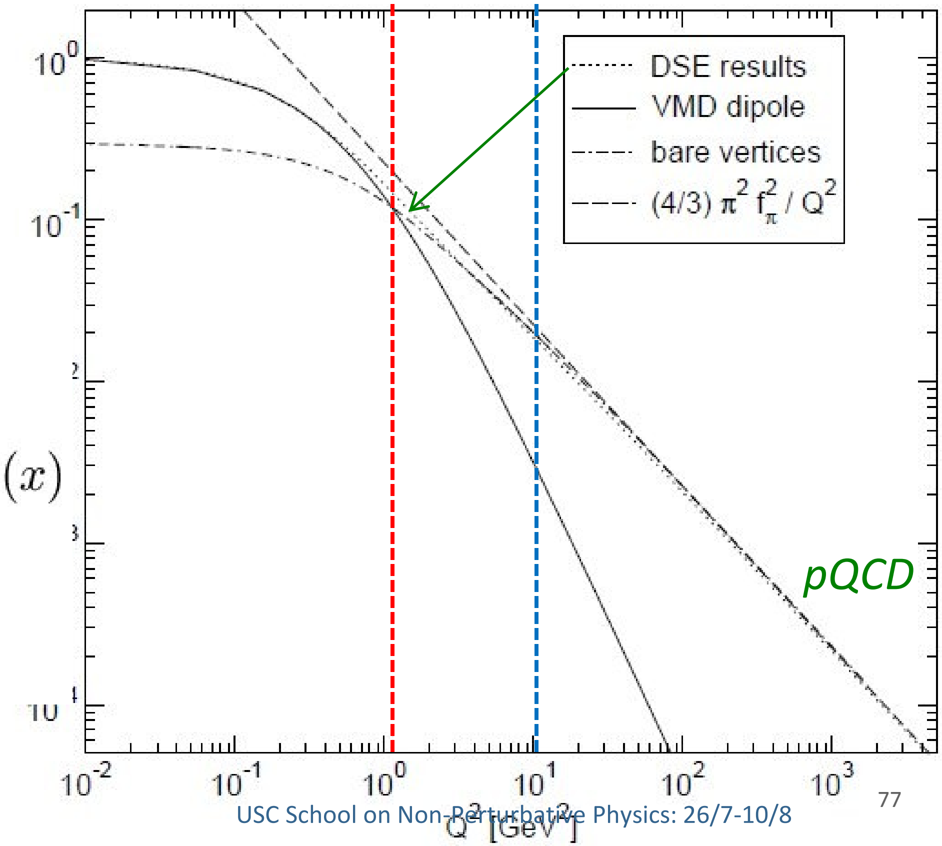
Obtained *if, and only if*, asymptotically, $\Gamma_\pi(k^2) \sim 1/k^2$

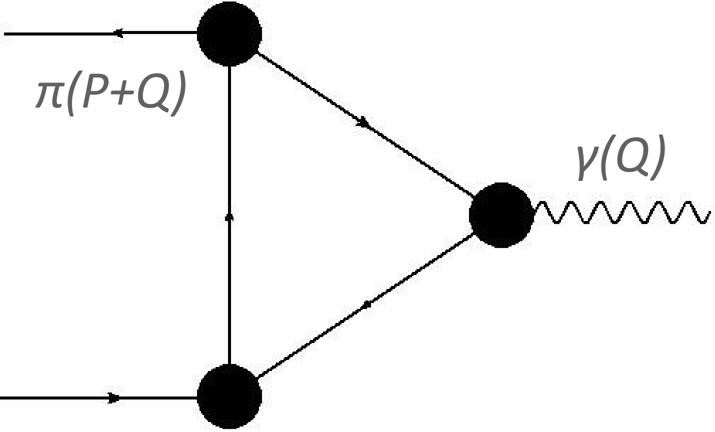
- Moreover, *absolutely no sensitivity* to $\varphi_\pi(x)$; viz., pion distribution amplitude

$$Q^2 G(Q^2) \stackrel{Q^2\to\infty}{\longrightarrow} 4\pi^2 f_\pi^2 J(w=0)$$

$$J(w) = \frac{2}{3}\int_0^1 dx \frac{1}{1 - w^2(2x-1)}\phi_\pi(x)$$

$$w = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2} \stackrel{k_1^2=Q^2=k_2^2}{=} 0\,.$$

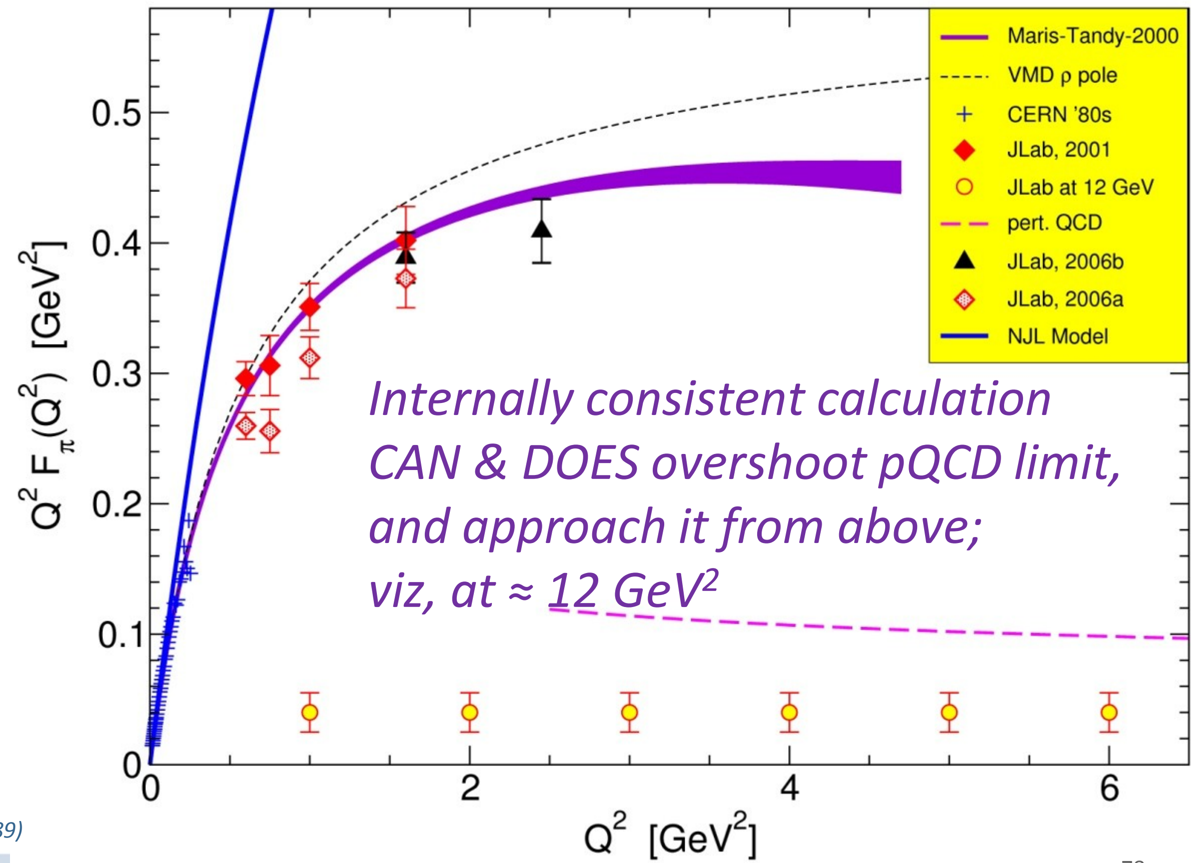


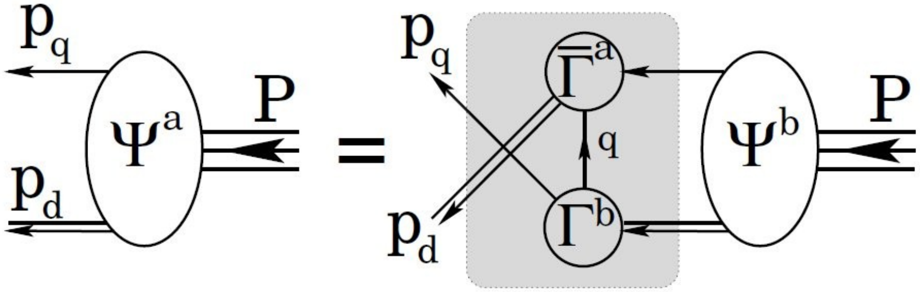


Pion Form Factor $F_\pi(Q^2)$

[Maris & Tandy, Phys.Rev. C62 (2000) 055204]

- DSE computation appeared *before* data; viz., a *prediction*
- pQCD-scale $Q^2F_\pi(Q^2)\to 16\pi\alpha(Q^2)f_\pi^{\,2}$
- VMD-scale: $m_\rho^{\,2}$
- Q^2=10GeV^2 pQCD-scale/VMD-scale = 0.08



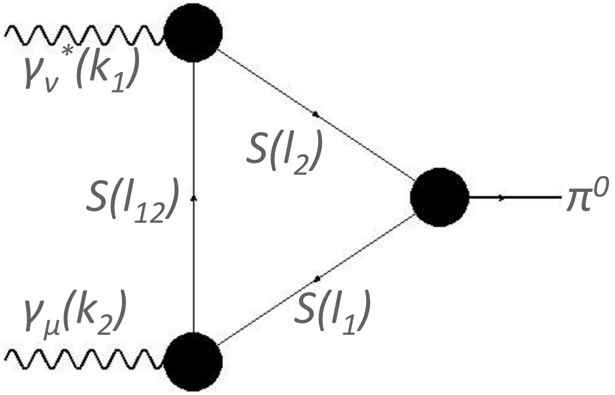


Single-parameter, Internally-consistent Framework

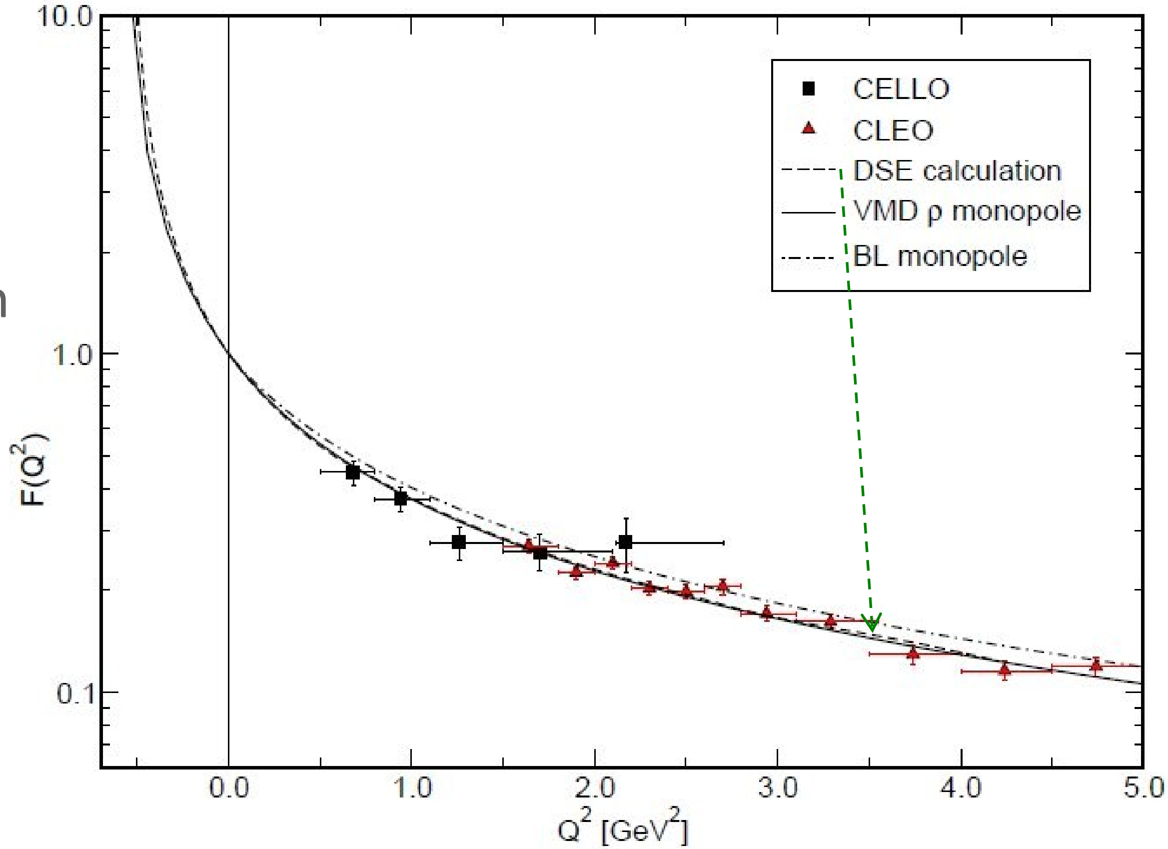
- Dyson-Schwinger Equations – applied extensively to spectrum & interactions of mesons with masses less than 1 GeV; & nucleon & *Δ.*
- On this domain the *rainbow-ladder* approximation – leading-order in systematic, symmetry-preserving truncation scheme, nucl-th/9602012 – is accurate, well-understood tool: e.g.,
  - Prediction of elastic pion and kaon form factors: nucl-th/0005015
  - Pion and kaon valence-quark distribution functions: 1102.2448 [nucl-th]
  - Unification of these and other observables – ππ scattering: hep-ph/0112015
  - Nucleon form factors: arXiv:0810.1222 [nucl-th]
- Readily extended to explain properties of the light neutral pseudoscalar mesons (*η* cf. *ή*): 0708.1118 [nucl-th]
- One parameter: gluon mass-scale = $m_G = 0.8$ GeV

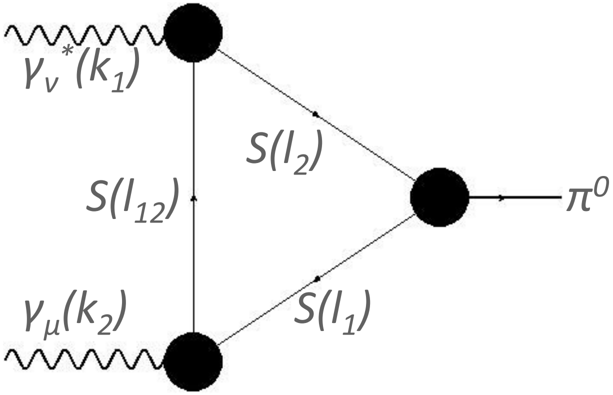
Transition Form Factor $\gamma^*(k_1)\gamma(k_2) \to \pi^0$

Maris & Tandy, Phys.Rev. C65 (2002) 045211



- DSE result
  - no parameters varied;
  - exhibits ρ-pole;
  - perfect agreement with CELLO & CLEO





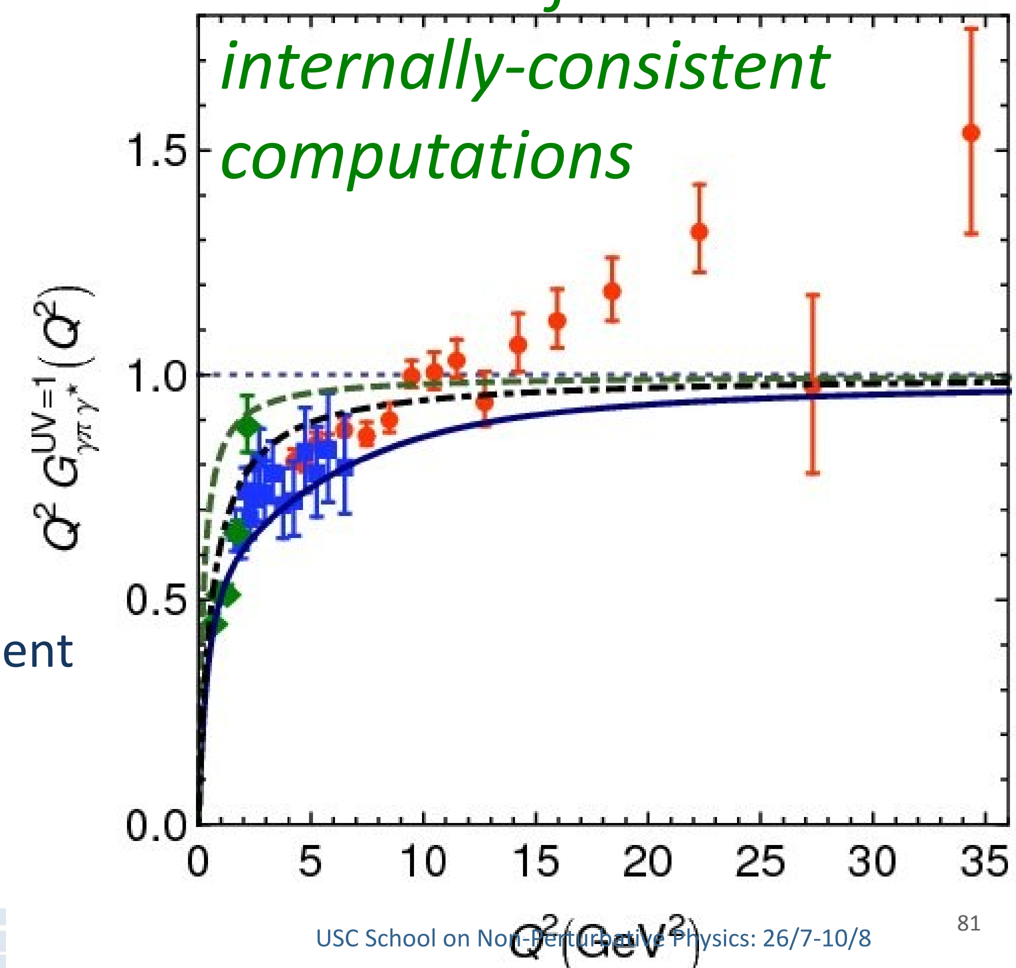
Transition Form Factor

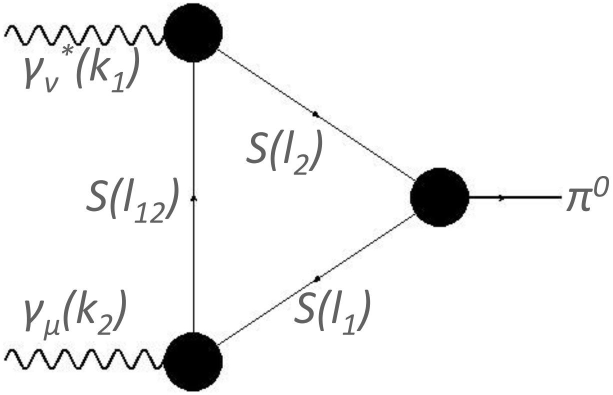
$\gamma^*(k_1)\gamma(k_2) \to \pi^0$

H.L.L. Roberts et al., Phys.Rev. C82 (2010) 065202

- Three , internally-consistent calculations
  - Maris & Tandy
    - Dash-dot: $\gamma^*(k_1)\gamma(k_2) \to \pi^0$
    - Dashed: $\gamma^*(k_1)\gamma^*(k_2) \to \pi^0$
  - H.L.L Roberts *et al.*
    - Solid: $\gamma^*(k_1)\gamma(k_2) \to \pi^0$ contact-interaction, omitting pion's pseudovector component
- All approach UV limit *from below*

Hallmark of internally-consistent computations





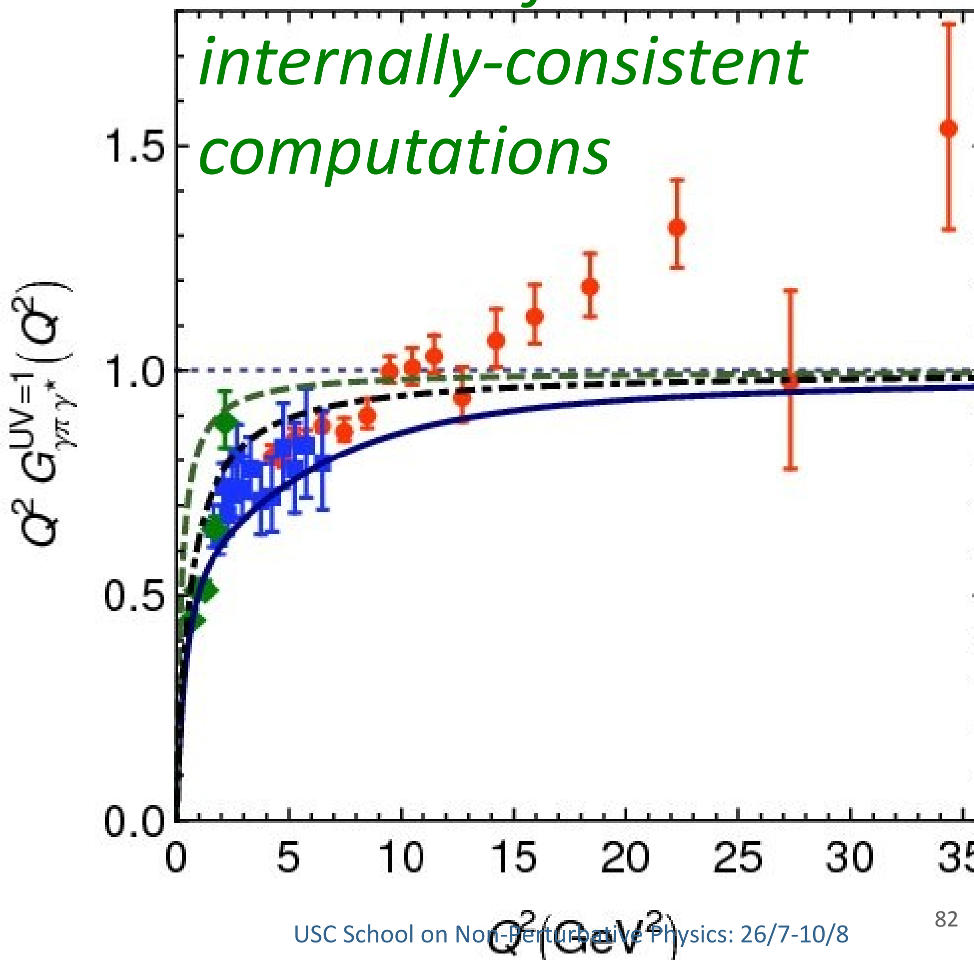
Transition Form Factor

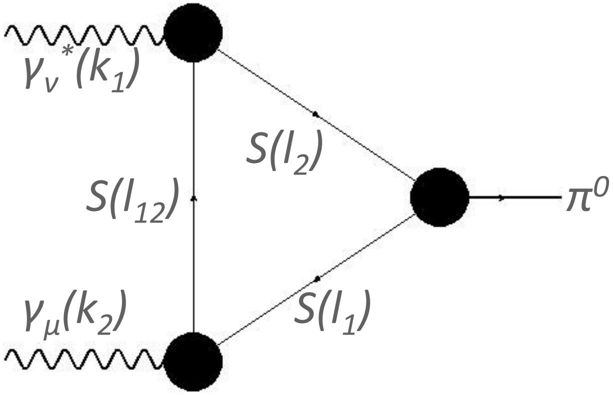
$\gamma^*(k_1)\gamma(k_2) \rightarrow \pi^0$

H.L.L. Roberts et al., Phys.Rev. C82 (2010) 065202

- All approach UV limit *from below*
- UV scale in this case is 10-times larger than for $F_\pi(Q^2)$:
  - $8\pi^2 f_\pi^2$ = (0.82 GeV)^2
  - cf. m_ρ^2 = (0.78 GeV)^2
- Hence, internally-consistent computations *can and do* approach the UV-limit from below.

Hallmark of internally-consistent computations





Transition Form Factor

$\gamma^*(k_1)\gamma(k_2) \rightarrow \pi^0$

H.L.L. Roberts et al., Phys.Rev. C82 (2010) 065202

- UV-behaviour: light-cone OPE

$$Q^2 G(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 4\pi^2 f_\pi^2 J(w=1)$$

$$w = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2} \overset{k_1^2=Q^2,k_2^2=0}{=} 1\,,$$

$$J(w) = \frac{1}{3}\int_0^1 dx \frac{1}{1-x}\phi_\pi(x)$$

- Integrand sensitive to endpoint: x=1
  - Perhaps $\varphi_\pi(x) \neq 6x(1-x)$?
  - Instead, $\varphi_\pi(x) \approx$ constant?

- There is one-to-one correspondence between behaviour of $\varphi_\pi(x)$ and short-range interaction between quarks
- $\varphi_\pi(x)$ = constant is achieved if, and only if, the interaction between quarks is momentum-independent; namely, of the Nambu – Jona-Lasinio form

Guttiérez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P)\right.$$
$$\left. + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)\right]$$

Remains!

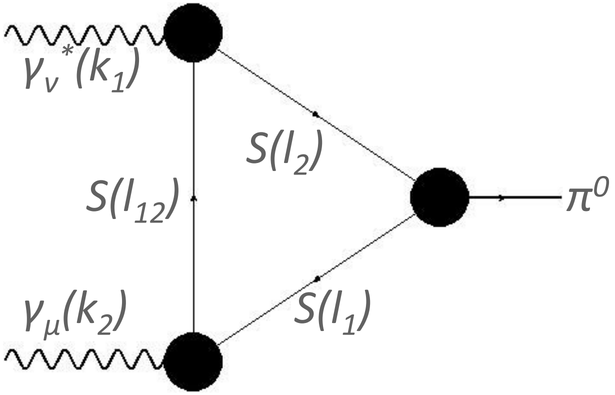
- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

$A \to 1$, $B \to M_Q$

- Bethe-Salpeter amplitude can't depend on relative momentum; propagator can't be momentum-dependent
- Solved gap and Bethe-Salpeter equations

$P^2=0$: $M_Q=0.4$GeV, $E_\pi=0.098$, $F_\pi=0.5M_Q$

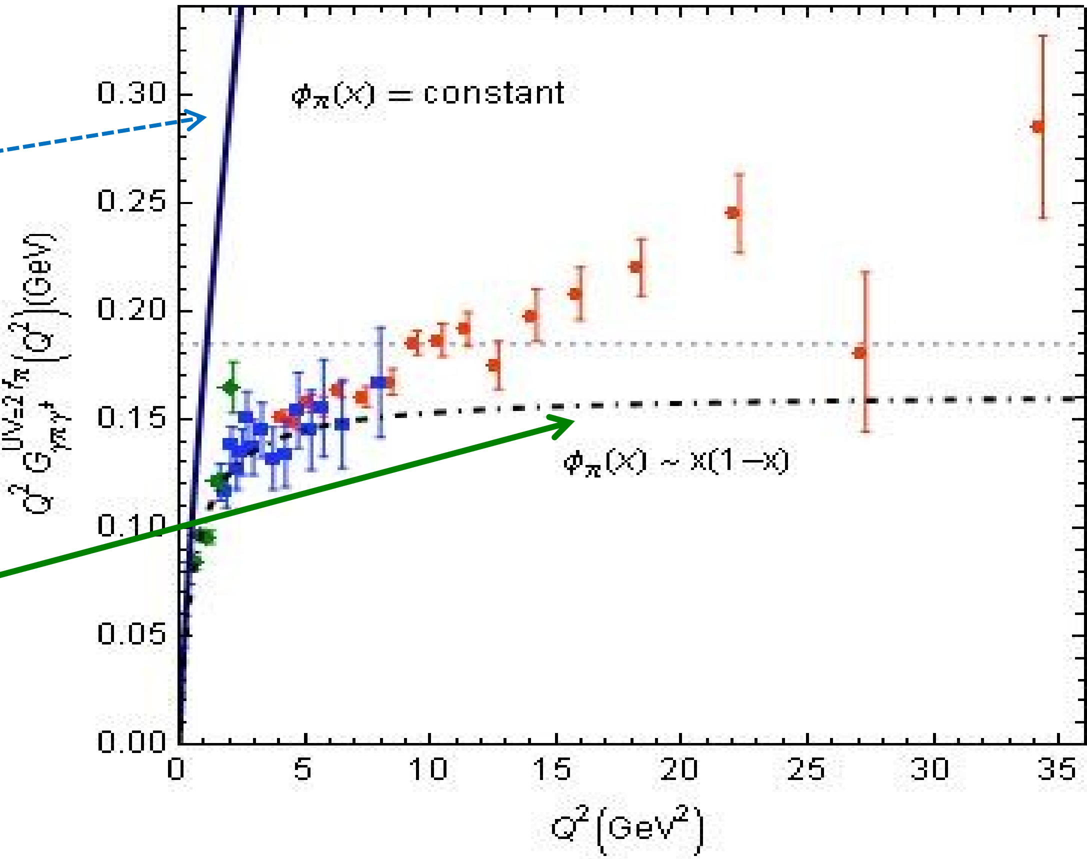
Nonzero and significant



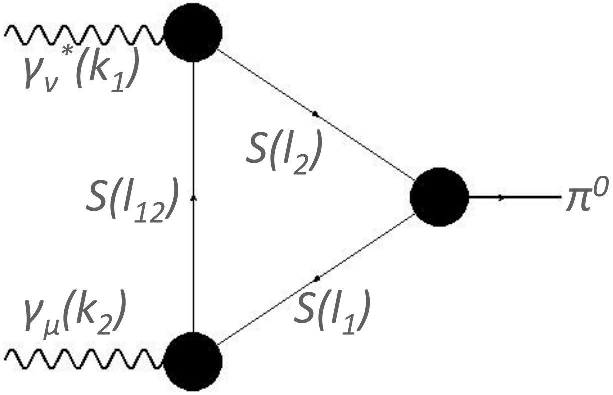
Transition Form Factor
$\gamma^*(k_1)\gamma(k_2) \to \pi^0$

H.L.L. Roberts et al., Phys.Rev. C82 (2010) 065202

- Comparison between Internally-consistent calculations:
- $\varphi_\pi(x) \approx$ constant, in conflict with large-Q^2 data here, as it is in all cases
  - Contact interaction cannot describe scattering of quarks at large-Q^2
- $\varphi_\pi(x) = 6x(1-x)$ yields pQCD limit, approaches *from below*

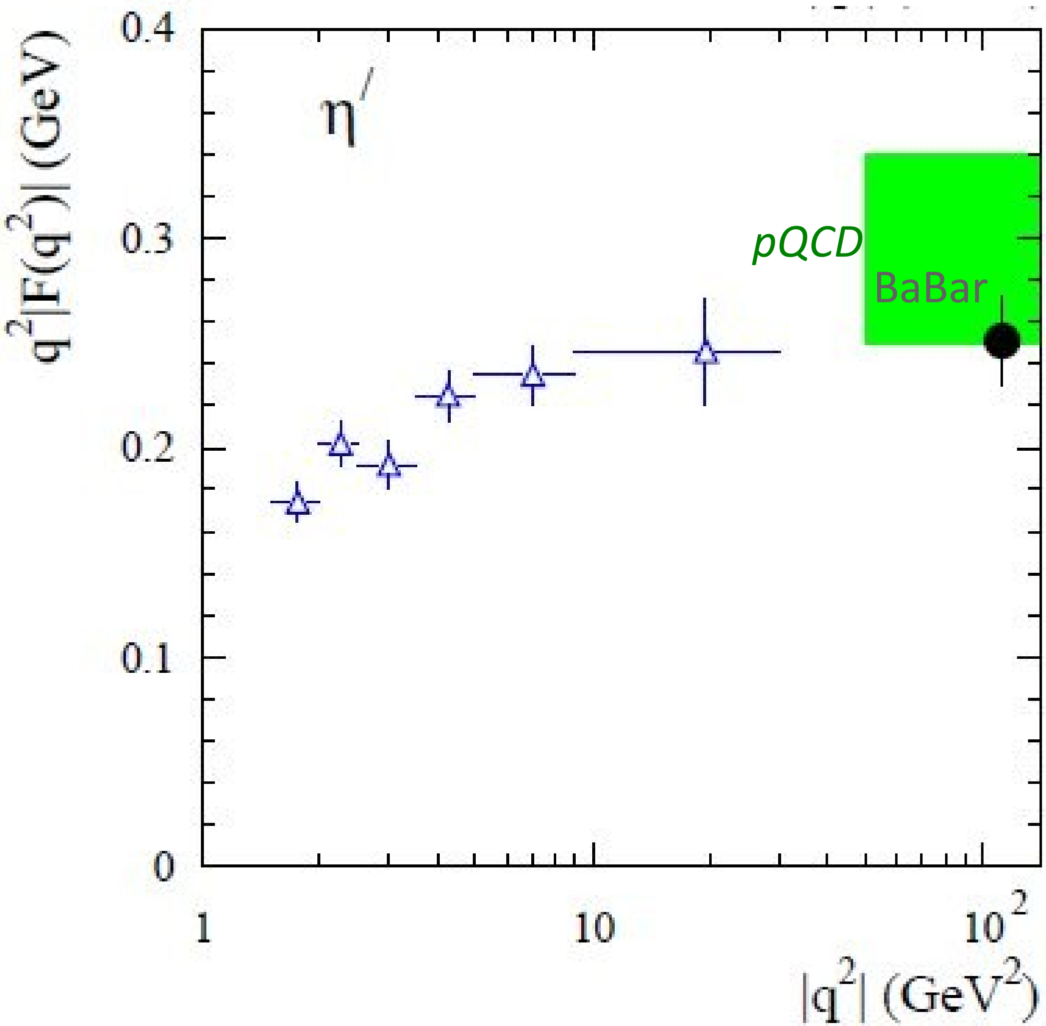


Transition Form Factor
$\gamma^*(k_1)\gamma(k_2) \rightarrow \pi^0$



H.L.L. Roberts et al., Phys.Rev. C82 (2010) 065202

- 2σ shift of any one of the last three high-points
  - one has quite a different picture
- η production **CLEO △**
- η' production
- *Both η & η' production in perfect agreement with pQCD*



Measurement of γγ∗→π0 transition form factor at Belle. arXiv:1205.3249 [hep-ex]

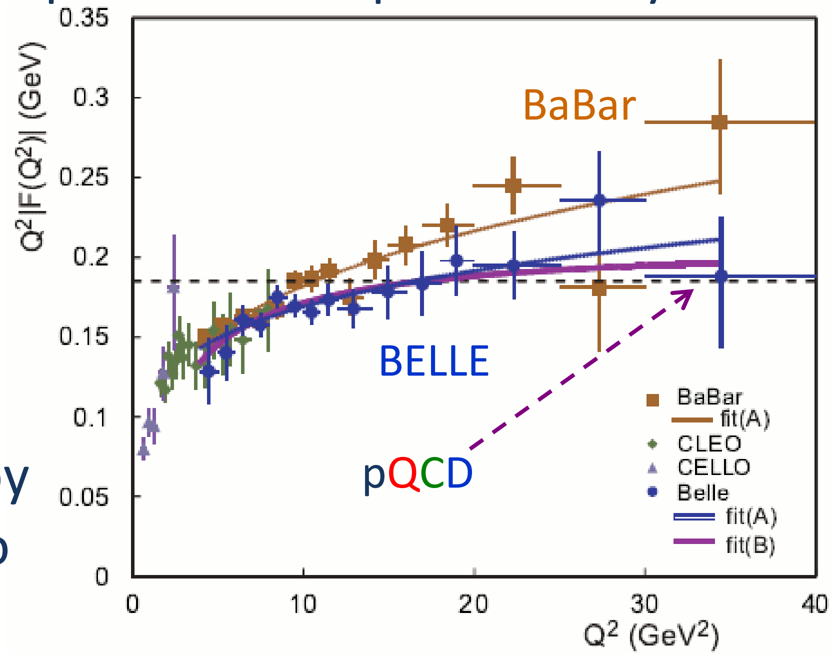
BELLE: Transition Form Factor

Finale … $\gamma^*(k_1)\gamma(k_2) \to \pi^0$

- Abstract:

  The measured values of $Q^2|F(Q^2)|$ agree well with the previous measurements below $Q^2 = 9$ GeV^2 but do not exhibit the rapid growth in the higher Q^2 region seen in another recent measurement, which exceeds the asymptotic QCD expectation by as much as 50%.

- BELLE data is fully consistent with pQCD
  - BaBar data are not a measure of the transition form factor
- Bad theory done in order to “explain” BaBar data. Worse being done now by some of these same authors, trying to salvage their misguided models.



Charting the interaction

Charting the interaction

- How may one use experiment to reveal the nature of the interaction that underlies the strong interaction?
- Remember:
  Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- We've already seen a bit of this:
  Using DSEs, one may predict observables using different types of interactions. The correct theory is then selected by experiment, so long as a unified description – free from parameter fiddling – is used in the theoretical analysis.