Argonne NATIONAL LABORATORY

Emergence of DSEs in Real-World QCD

Craig Roberts

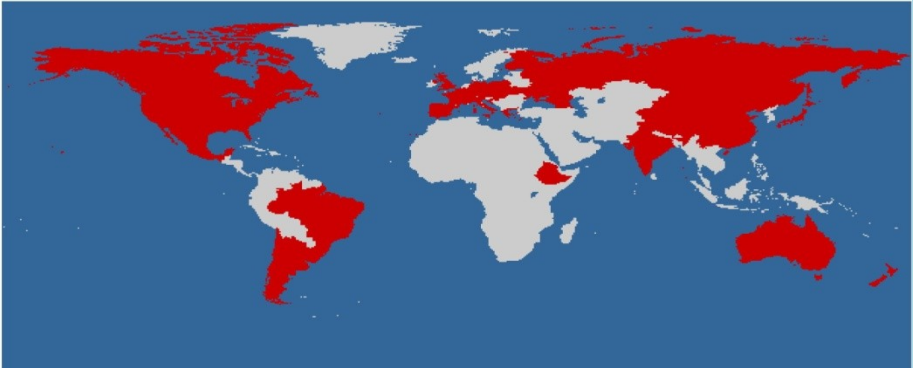
Nuclear Theory

Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

National Science Foundation
WHERE DISCOVERIES BEGIN

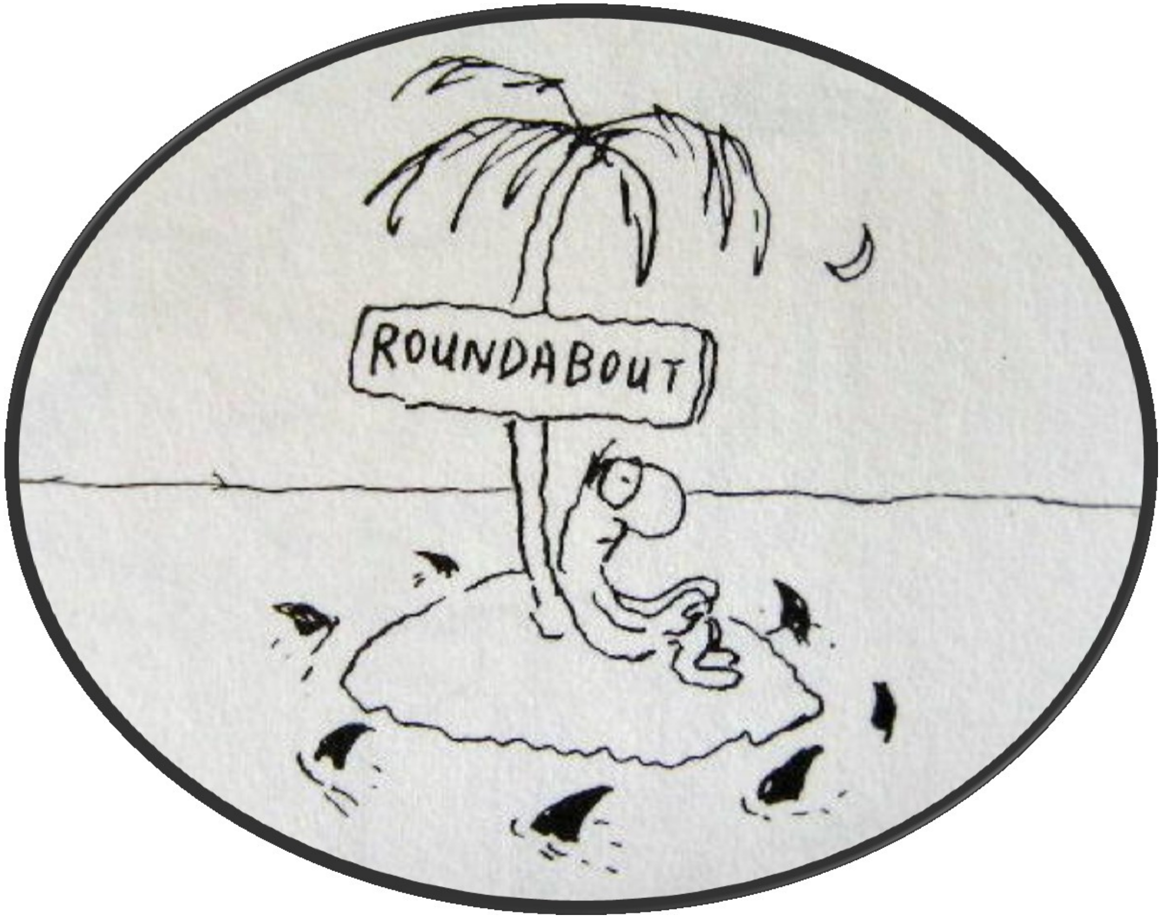
U.S. DEPARTMENT OF ENERGY



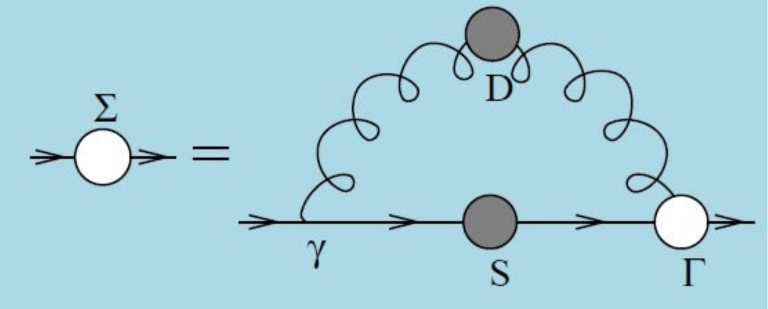
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling



Persistent Challenge
Truncation



Persistent challenge in application of DSEs

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- Infinitely many coupled equations:
  Kernel of the equation for the quark self-energy involves:
  – ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
  – ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***

  each of which satisfies its own DSE, etc…
- Coupling between equations *necessitates* a truncation
  – Weak coupling expansion
  ⇒ produces every diagram in perturbation theory
  – Otherwise ***useless***
  for the nonperturbative problems in which we're interested

Invaluable check on practical truncation schemes

Relationship must be preserved by any truncation

Highly nontrivial constraint

FAILURE has an extremely high cost

– loss of any connection with QCD

Persistent challenge - truncation scheme

- Symmetries associated with conservation of vector and axial-vector currents are critical in arriving at a veracious understanding of hadron structure and interactions
- Example: axial-v
  - Statement of … it's broken in quantum fielc

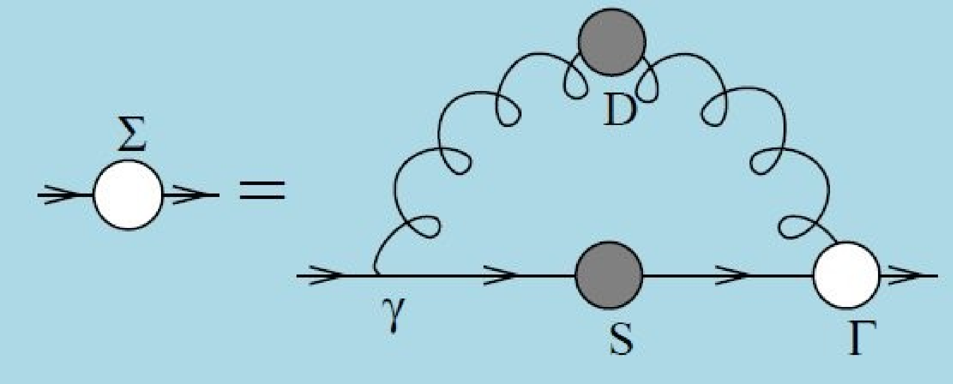
$P_\mu \, \Gamma^l_{5\mu}(k;P$ … $S^{-1}(k_-)$

M_ζ

Axial-Vector vertex
Satisfies an inhomogeneous
Bethe-Salpeter equation

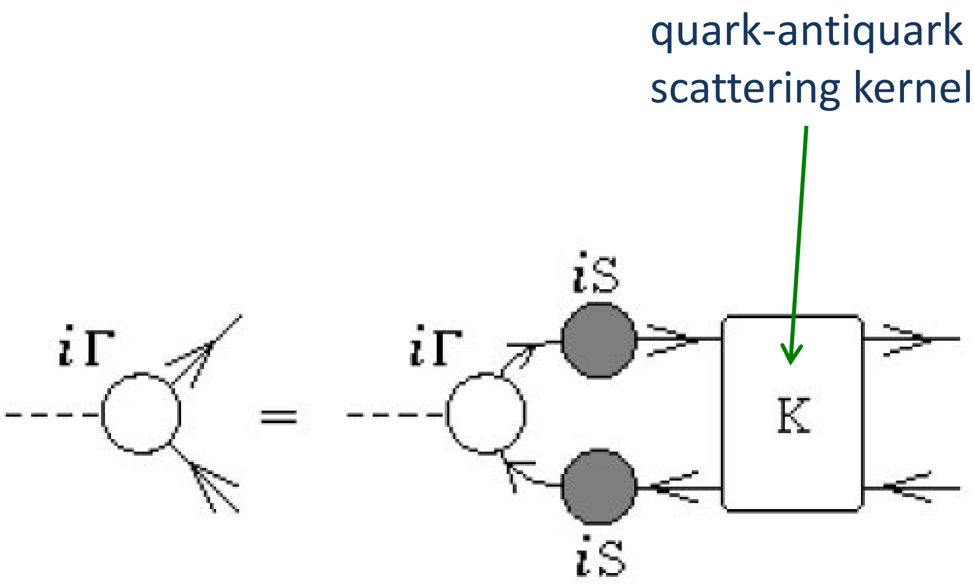
Quark propagator satisfies a gap equation

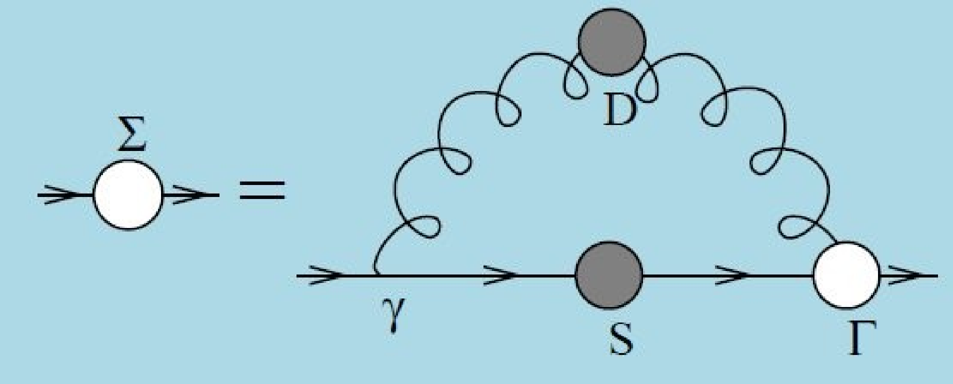
Kernels of these equations are completely different
But they must be intimately related



Persistent challenge - truncation scheme

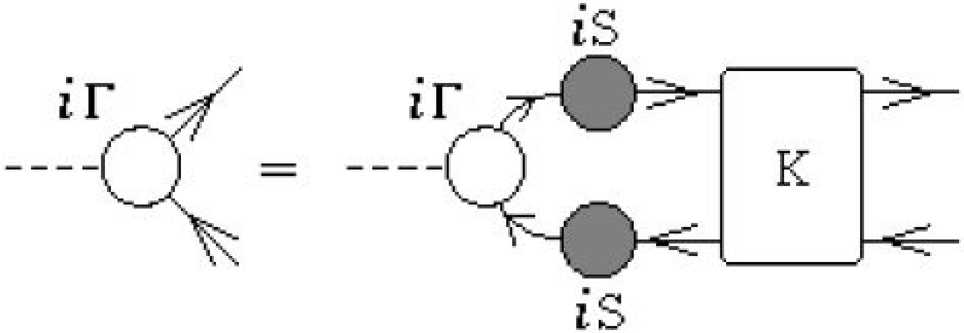
- These observations show that symmetries relate the kernel of the gap equation – nominally a one-body problem, with that of the Bethe-Salpeter equation – considered to be a two-body problem
- Until 1995/1996 people had no idea what to do
- Equations were truncated, sometimes with good phenomenological results, sometimes with poor results
- Neither good nor bad could be explained





Persistent challenge - truncation scheme

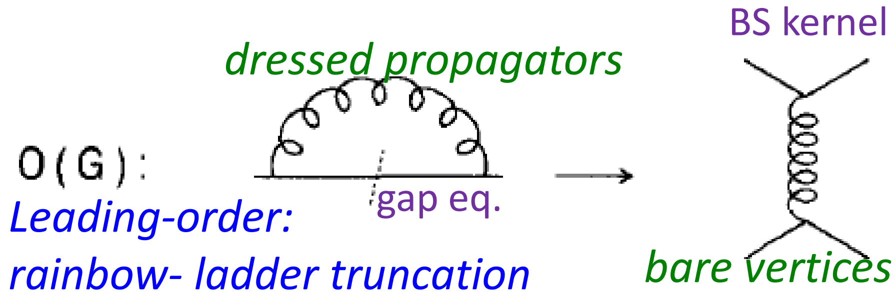
- Happily, that changed, and there is now at least one systematic, ***nonperturbative*** and symmetry preserving truncation scheme
  – H.J. Munczek, Phys. Rev. D **52** (1995) 4736, *Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  – A. Bender, C.D. Roberts and L. von Smekal, Phys.Lett. B **380** (1996) 7, *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*



Modified skeleton expansion in which the propagators are fully-dressed but the vertices are constructed term-by-term

Cutting scheme

- The procedure generates a Bethe-Salpeter kernel from the kernel of any gap equation whose diagrammatic content is known
  - That this is possible and achievable systematically is necessary and sufficient to prove some exact results in QCD



- The procedure also enables the formulation of practical phenomenological models that can be used to illustrate the exact results and provide predictions for experiment with readily quantifiable errors.

Now able to explain the dichotomy of the pion

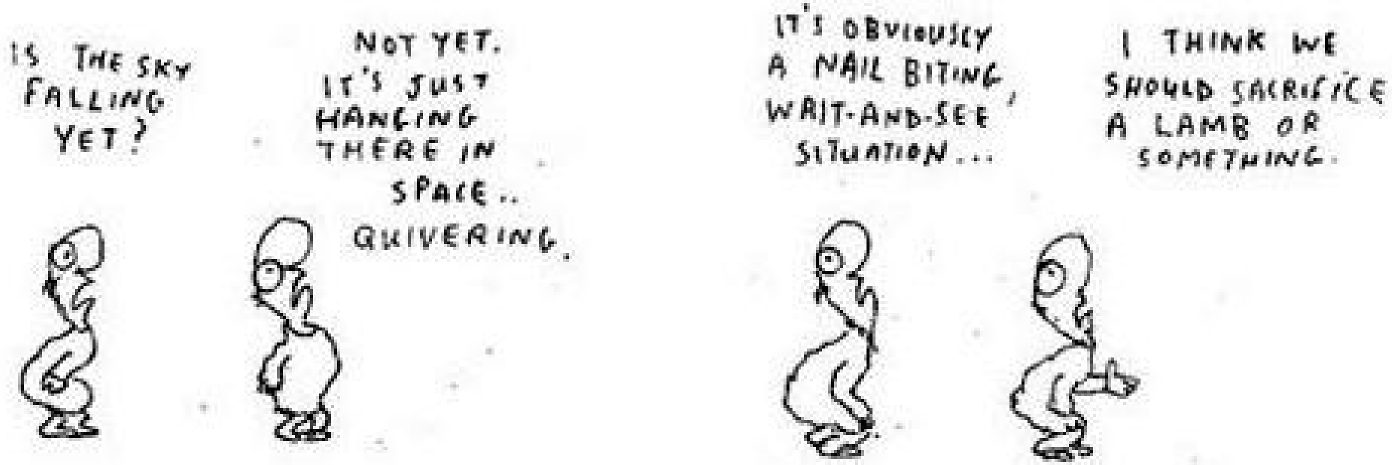
- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless

  – but some are still making this mistake

- However: $m_\pi^2 \propto \imath$

  current-algebra (1968)

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-\ tate} \propto \imath_{constituent}$



Some Exact Results

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

Pseudovector components necessarily nonzero. Cannot be ignored!

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

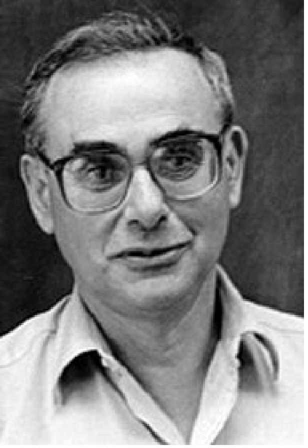
- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(p^2)$$

Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**



Dichotomy of the pion Goldstone mode and bound-state

$$f_\pi \, E_\pi(p^2) = B(p^2)$$

- Goldstone's theorem
  has a pointwise expression in QCD;

  Namely, in the chiral limit the wave-function for the two-body bound-state Goldstone mode is intimately connected with, and almost completely specified by, the fully-dressed one-body propagator of its characteristic constituent

  - The one-body momentum is equated with the relative momentum of the two-body system

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

- Mass-squared of the pseudscalar hadron
- Sum of the current-quark masses of the constituents;

  e.g., pion = $m_u^{\varsigma} + m_d^{\varsigma}$, where "ς" is the renormalisation point

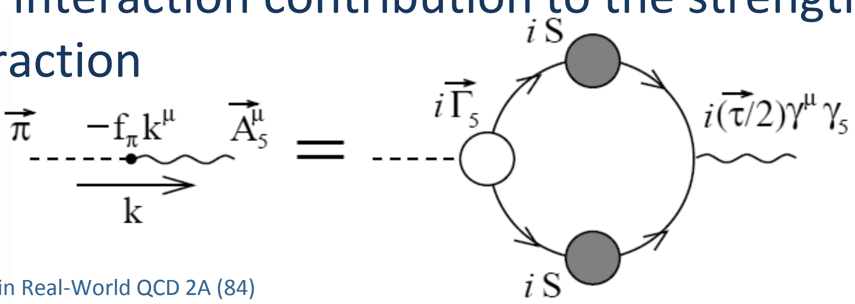
Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

$$f_{H_5} P_\mu = Z_2 \mathrm{tr} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^{\mathrm{t}} \gamma_5 \gamma_\mu S(q + \frac{1}{2}P) \Gamma_{H_5}(q;P) S(q - \frac{1}{2}P)$$

- Pseudovector projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, the pseudoscalar meson's leptonic decay constant, which is the strong interaction contribution to the strength of the meson's weak interaction



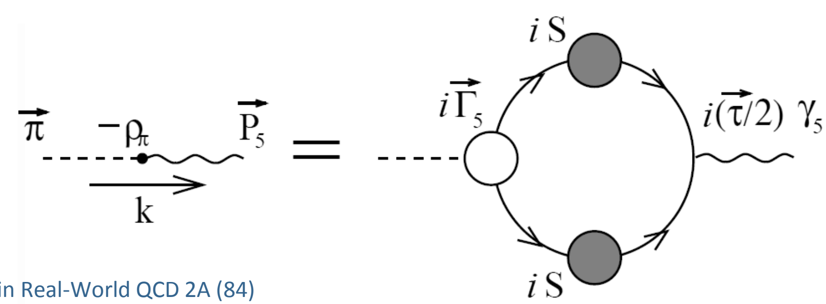
Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0^- Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta}\, \mathcal{M}_{H_5}^{\zeta}$$

$$i\rho_{H_5} = Z_4 \mathrm{tr} \int \frac{d^4 q}{(2\pi)^4}\, \frac{1}{2} (T^{H_5})^{\mathrm{t}} \gamma_5 S(q+\tfrac{1}{2}P)\Gamma_{H_5}(q;P)S(q-\tfrac{1}{2}P)$$

- Pseudoscalar projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, a pseudoscalar analogue of the meson's leptonic decay constant



Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0^- Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

- Consider the case of light quarks; namely, $m_q \approx 0$
  - If chiral symmetry is dynamically broken, then
    - $f_{H5} \rightarrow f_{H5}^{0} \neq 0$
    - $\rho_{H5} \rightarrow -< q\text{-bar } q> / f_{H5}^{0} \neq 0$

  both of which are independent of m_q

The so-called "vacuum quark condensate." More later about this.

- Hence, one arrives at the corollary

Gell-Mann, Oakes, Renner relation 1968

$$m_{H_5}^2 = 2 m_q \frac{-\langle \bar{q} q \rangle}{f_{H_5}^0}$$

$$m_\pi^2 \propto \imath$$

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0^- Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^\zeta_{H_5} \mathcal{M}^\zeta_{H_5}$$

- Consider a different case; namely, one quark mass fixed and the other becoming very large, so that $m_q / m_Q << 1$
- Then
  - $f_{H5} \propto 1/\sqrt{m_{H5}}$
  - $\rho_{H5} \propto \sqrt{m_{H5}}$

  and one arrives at

$$m_{H5} \propto m_Q$$

Provides
QCD proof of
potential model result

Ivanov, Kalinovsky, Roberts
Phys. Rev. D **60**, 034018 (1999) [17 pages]

Radial excitations & Hybrids & Exotics
⇒ wave-functions with support at long-range
⇒ sensitive to confinement interaction
Understanding confinement "remains one of
The greatest intellectual challenges in physics"

Radial excitations of Pseudoscalar meson

- Hadron spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0-)S$] masses below 2GeV: π(140); π(1300); and π(1800)

the pion

- Constituent-Quark Model suggests that these states are the 1st three members of an n^1S_0 trajectory; i.e., ground state plus radial excitations
- But π(1800) is narrow ($\Gamma = 207 \pm 13$); i.e., surprisingly long-lived & decay pattern conflicts with usual quark-model expectations.
  - $S_{Q\text{-}barQ} = 1 \oplus L_{Glue} = 1 \Rightarrow J = 0$
    & $L_{Glue} = 1 \Rightarrow {}^3S_1 \oplus {}^3S_1$ (Q-bar Q) decays are suppressed
  - Perhaps therefore it's a hybrid?

exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
hybrid mesons: normal quantum numbers but non-quark-model decay pattern
BOTH suspected of having "constituent gluon" content

Höll, Krassnigg and Roberts
Phys.Rev. C**70** (2004) 042203(R)

Radial excitations of Pseudoscalar meson

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

Flip side: if no DCSB, then all pseudoscalar mesons decouple from the weak interaction!

- Valid for ALL Pseudoscalar mesons
  - When chiral symmetry is dynamically broken, then
    - ρ_{H5} is finite and nonzero in the chiral limit, $M_{H5} \to 0$
  - A "radial" excitation of the π-meson, is not the ground state, so

    $m^2_{\pi \text{ excited state}} \neq 0 > m^2_{\pi \text{ ground state}} = 0$ (in chiral limit, $M_{H5} \to 0$)
- Putting this things together, it follows that

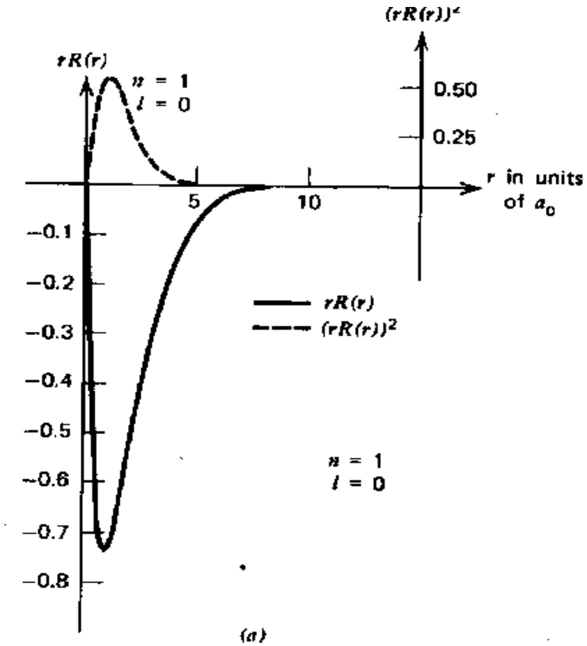
$$f_{H5} = 0$$

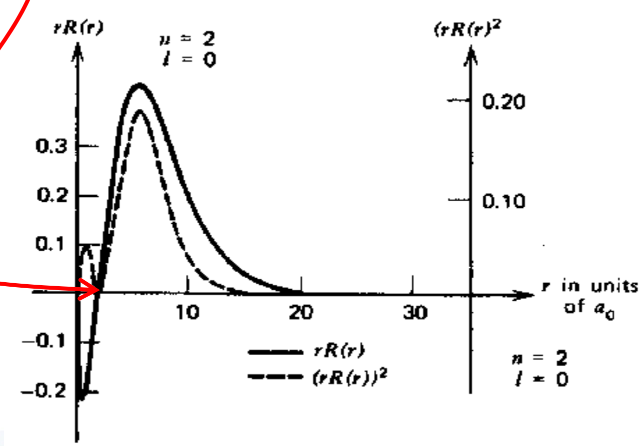
for ALL pseudoscalar mesons, except π(140), in the chiral limit

Dynamical Chiral Symmetry Breaking – Goldstone's Theorem – impacts upon ***every*** *pseudoscalar meson*

Radial excitations of Pseudoscalar meson

- This is fascinating because in quantum mechanics, decay constants of a radial excitation are suppressed by factor of roughly ⅓
  - Radial wave functions possess a zero
  - Hence, integral of "r $R_{n=2}(r)^2$" is quantitatively reduced compared to that of "r $R_{n=1}(r)^2$"
- **HOWEVER, ONLY A SYMMETRY CAN ENSURE THAT SOMETHING VANISHES COMPLETELY**





McNeile and Michael
*Phys.Lett. B**642** (2006) 244-247*

Lattice-QCD & radial excitations of pseudoscalar mesons

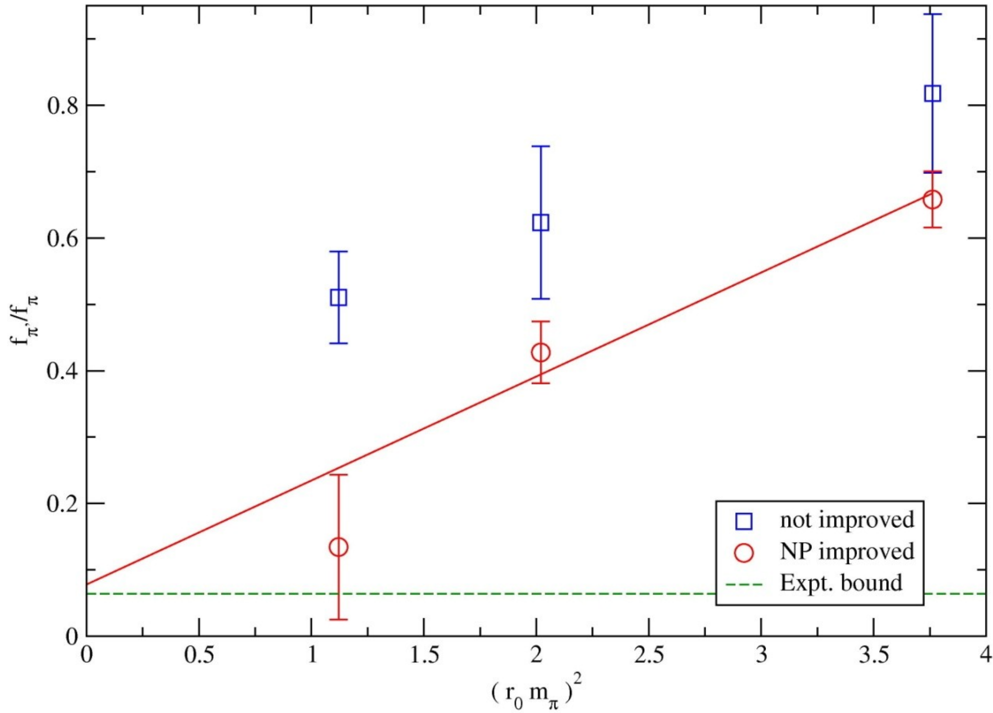
The suppression of $f_{\pi 1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited state mesons.

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$

  $\Rightarrow f_{\pi 1} < 8.4\text{MeV}$

  Diehl & Hiller
  hep-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$-lattice, a ~ 0.1 fm,
  two-flavour, unquenched

  $\Rightarrow f_{\pi 1}/f_\pi = 0.078\ (93)$
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C**76** (2007) 045203

Charge-neutral pseudoscalar mesons

non-Abelian Anomaly and *η*-*η*′ mixing

- Neutral mesons containing s-bar & s are special, in particular

  η & *η*′

- Problem:

  η′ is a pseudoscalar meson but it's much more massive than the other eight pseudoscalars constituted from light-quarks.

  $m_\eta = 548$ MeV

  $m_{\eta'} = 958$ MeV

  Splitting is 75% of η mass!

- Origin:

  While the classical action associated with QCD is invariant under $U_A(N_f)$ (non-Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!

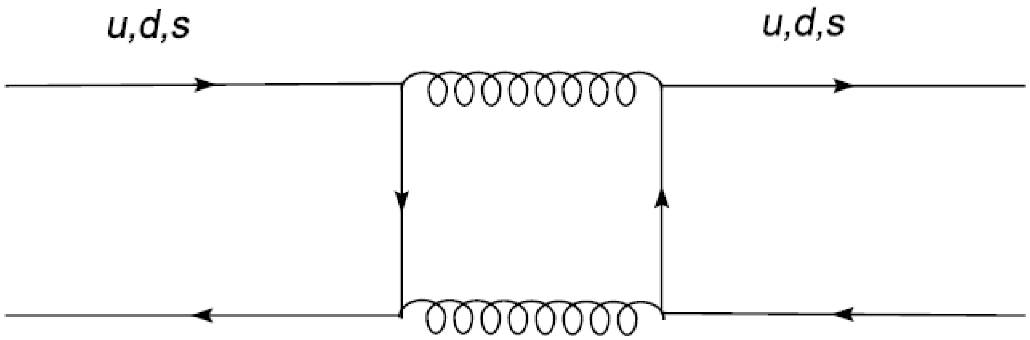
Charge-neutral pseudoscalar mesons

non-Abelian Anomaly and η-η' mixing

- Neutral mesons containing s-bar & s are special, in particular

η & η'

- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$



- Textbooks notwithstanding, this is a perturbative diagram, which has absolutely nothing to do with the essence of the $\eta - \eta'$ problem

Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C**76** (2007) 045203

Charge-neutral pseudoscalar mesons

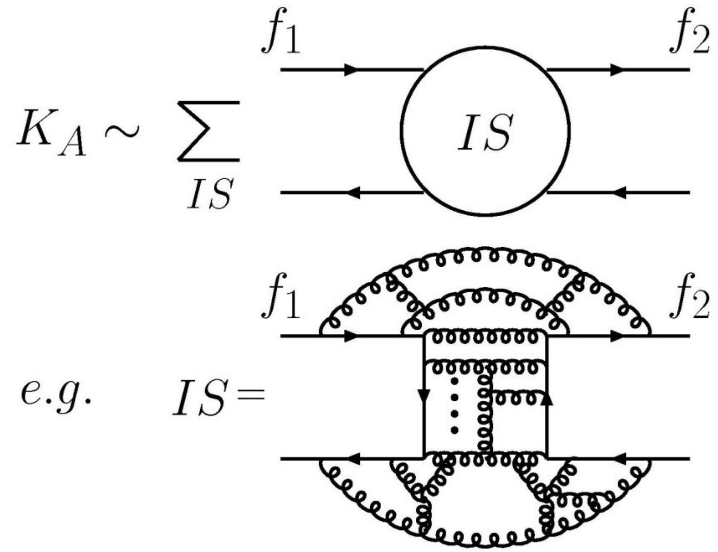
non-Abelian Anomaly and *η*-*η*′ mixing

- Neutral mesons containing s-bar & s are special, in particular

  η & *η*′

- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly.

  All terms have the “hairpin” structure
- *No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.*



Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C76 (2007) 045203

Charge-neutral pseudoscalar mesons

- Axial-Vector Ward-Takahashi identity

$$P_\mu \Gamma^a_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P)$$

- $\{\mathcal{F}^a \,|\, a = 0,\ldots,N_f^2-1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b\right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

- *Expresses the non-Abelian axial anomaly*

Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C**76** (2007) 045203

Charge-neutral pseudoscalar mesons

- *Anomalous* Axial-Vector Ward-Takahashi identity

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

$$\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$$
$$\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\,\mathcal{Q}(0)\,\bar{q}(y)\rangle$$

Important that only A^0 is nonzero

$$\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$$

Anomaly expressed via a mixed vertex

NB. While Q(x) is gauge invariant, the associated Chern-Simons current, K_μ, is not ⇒ in QCD *no physical* boson can couple to K_μ and hence *no physical states can* contribute to resolution of $U_A(1)$ problem.

Bhagwat, Chang, Liu, Roberts, Tandy
*Phys.Rev. C**76** (2007) 045203*

Charge-neutral pseudoscalar mesons

- Only $A^0 \neq 0$ is interesting … otherwise there is no difference between η & η', and all pseudoscalar mesons are Goldstone mode bound states.
- General structure of the anomaly term:

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_\mathcal{A}(k;P) + \gamma\cdot P\mathcal{F}_\mathcal{A}(k;P) + \gamma\cdot k k\cdot P\mathcal{G}_\mathcal{A}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu \mathcal{H}_\mathcal{A}(k;P)\right]$$

- Hence, one can derive generalised Goldberger-Treiman relations

$$2f^0_{\eta'}E_{BS}(k;0) = 2B_0(k^2) - \mathcal{E}_\mathcal{A}(k;0),$$

Follows that $E_A(k;0)=2\ B_0(k^2)$ is necessary and sufficient condition for the absence of a massless η' bound state in the chiral limit, since this ensures $E_{BS} \equiv 0$.

A_0 and B_0 characterise gap equation's chiral-limit solution

Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C76 (2007) 045203

Charge-neutral pseudoscalar mesons

- $E_A(k; 0) = 2\, B_0(k^2)$

  We're discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of an intimate connection between DCSB and an expectation value of the topological charge density
- Further highlighted . . . proved

$$\langle \bar{q} q \rangle_\zeta^0 = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q, \zeta)$$

$$= \frac{N_f}{2} \int d^4x \, \langle \bar{q}(x) i \gamma_5 q(x) \mathcal{Q}(0) \rangle^0,$$

So-called quark condensate linked inextricably with a mixed vacuum polarisation, which measures the topological structure within hadrons

Charge-neutral pseudoscalar mesons

- AVWTI ⇒ QCD mass formulae for all pseudoscalar mesons, including those which are charge-neutral

$$m_{\pi_i}^2 f_{\pi_i}^a = 2\,\mathcal{M}^{ab}\rho_{\pi_i}^b + \delta^{a0} n_{\pi_i}$$

- Consider the limit of a *U(N$_f$)*-symmetric mass matrix, then this formula yields:

$$m_\eta^2 f_\eta = 2m(\zeta)\rho_\eta^0(\zeta) \qquad n_{\eta'} = \sqrt{\frac{N_f}{2}}\nu_{\eta'}\,,\; \nu_{\eta'} = \langle 0|\mathcal{Q}|\eta'\rangle$$

$$m_{\eta'}^2 f_{\eta'}^0 = 2m(\zeta)\rho_{\eta'}^0(\zeta) + n_{\eta'}$$

- Plainly, the *η – η'* mass splitting is nonzero in the chiral limit so long as $\nu_{\eta'} \neq 0$ … viz., so long as the topological content of the *η'* is nonzero!
- We know that, for large N_c,
  - $f_{\eta'} \propto N_c^{1/2} \propto \rho_{\eta'}^0$
  - $\nu_{\eta'} \propto 1/N_c^{1/2}$

Consequently, the η – η' mass splitting vanishes in the large-N$_c$ limit!

Charge-neutral pseudoscalar mesons

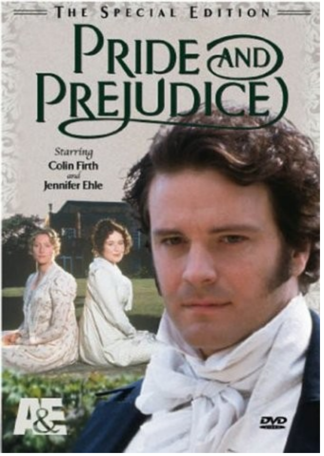
- AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons
- In "Bhagwat *et al.*," implications of mass formulae were illustrated using an elementary dynamical model, which includes a one-parameter *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
  - Employed in an analysis of pseudoscalar- and vector-meson bound-states
- Despite its simplicity, the model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)

$$|\eta\rangle \sim 0.55(\bar{u}u + \bar{d}d) - 0.63\bar{s}s,$$
$$|\eta'\rangle \sim 0.45(\bar{u}u + \bar{d}d) + 0.78\bar{s}s.$$

  - π^0–η angles of $\sim 1.2^\circ$ (Expt. from reaction $p\, d \to {}^3He\, \pi^0$: $0.6^\circ \pm 0.3^\circ$)

Dynamical Chiral Symmetry Breaking

Vacuum Condensates?



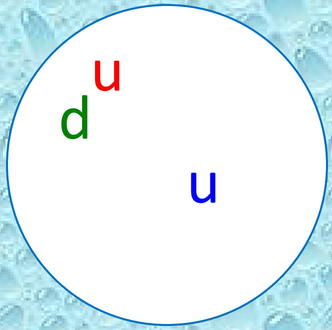
Universal *Conventions*

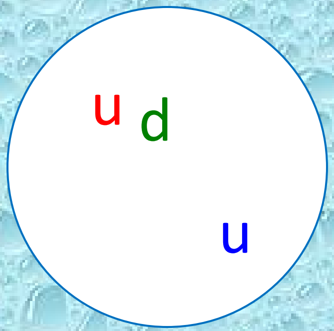
- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

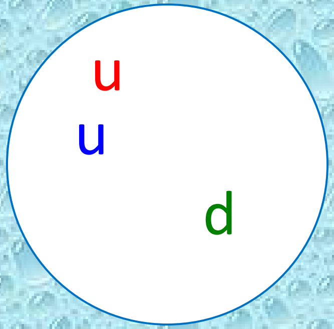
  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."

"Orthodox Vacuum"

- Vacuum = "frothing sea"
- Hadrons = bubbles in that "sea", containing nothing but quarks & gluons interacting perturbatively, unless they're near the bubble's boundary, whereat they feel they're trapped!







Background

- Worth noting that nonzero vacuum expectation values of local operators in QCD—the so-called vacuum condensates—are phenomenological parameters, which were introduced at a time of limited computational resources in order to assist with the theoretical estimation of essentially nonperturbative strong-interaction matrix elements.
- A universality of these condensates was assumed, namely, that the properties of all hadrons could be expanded in terms of the same condensates. While this helps to retard proliferation, there are nevertheless infinitely many of them.
- As qualities associated with an unmeasurable state (the vacuum), such condensates do not admit direct measurement. Practitioners have attempted to assign values to them via an internally consistent treatment of many separate empirical observables.
- However, only one, the so-called quark condensate, is attributed a value with any confidence.

PHYSICAL REVIEW C **85**, 065202 (2012)

Confinement contains condensates

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

[2] *Centre for Particle Physics Phenomenology: CP³-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

[3] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[4] *Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616, USA*

[5] *C. N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

[6] *Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 27 February 2012; published 21 June 2012)

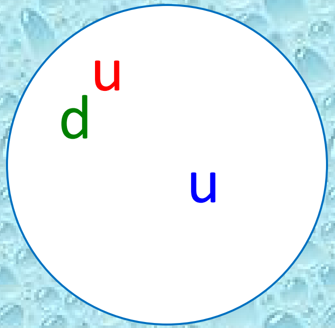
Dynamical chiral symmetry breaking and its connection to the generation of hadron masses has historically been viewed as a vacuum phenomenon. We argue that confinement makes such a position untenable. If quark-hadron duality is a reality in QCD, then condensates, those quantities that have commonly been viewed as constant empirical mass scales that fill all space-time, are instead wholly contained within hadrons; i.e., they are a property of hadrons themselves and expressed, e.g., in their Bethe-Salpeter or light-front wave functions. We explain that this paradigm is consistent with empirical evidence and incidentally expose misconceptions in a recent Comment.

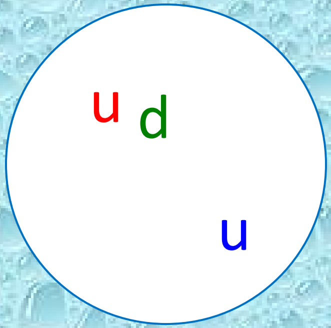
DOI: 10.1103/PhysRevC.85.065202 PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

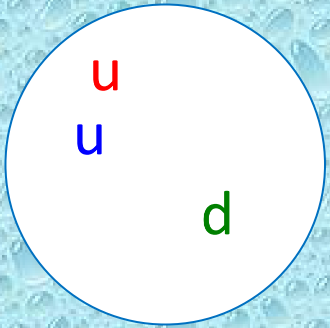
Confinement contains condensates

“Orthodox Vacuum”

- Vacuum = “frothing sea”
- Hadrons = bubbles in that “sea”, containing nothing but quarks & gluons interacting perturbatively, unless they’re near the bubble’s boundary, whereat they feel they’re trapped!

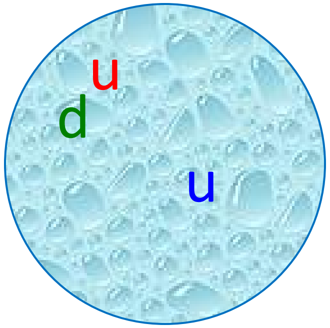


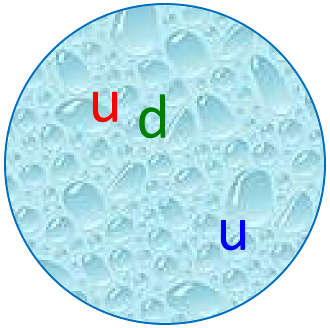


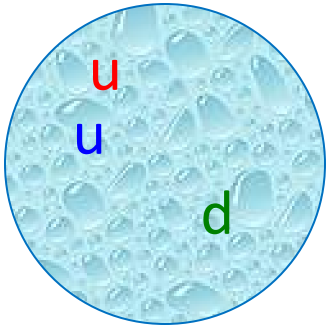


New Paradigm

- Vacuum = hadronic fluctuations
  but no condensates
- Hadrons = complex, interacting systems within which perturbative behaviour is restricted to just 2% of the interior







Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

- C
  –

We now have sufficient information to address the question of just what is this so-called "vacuum quark condensate."

- H

elation
1968

$$m_{H_5}^2 = 2 m_q \frac{-\langle \bar{q} q \rangle}{f_{H_5}^0}$$

$$m_\pi^2 \propto \imath$$

Spontaneous(Dynamical) Chiral Symmetry Breaking

The 2008 Nobel Prize in Physics was divided, one half awarded to Yoichiro Nambu

> *"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"*

Nambu - Jona-Lasinio Model

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345–358
Dynamical Model Of Elementary Particles Based On An Analogy With Superconductivity. II
Y. Nambu, G. Jona-Lasinio, Phys.Rev. 124 (1961) 246-254

- Treats a chirally-invariant four-fermion Lagrangian & solves the gap equation in Hartree-Fock approximation (analogous to rainbow truncation)
- Poss …d
- *Esse…* …nd Nam… & de… ther… dege… Nambu vacua, related by a chiral rotation
- ***Nontrivial Vacuum is "Born"***

The following Lagrangian density will be assumed ($\hbar = c = 1$):

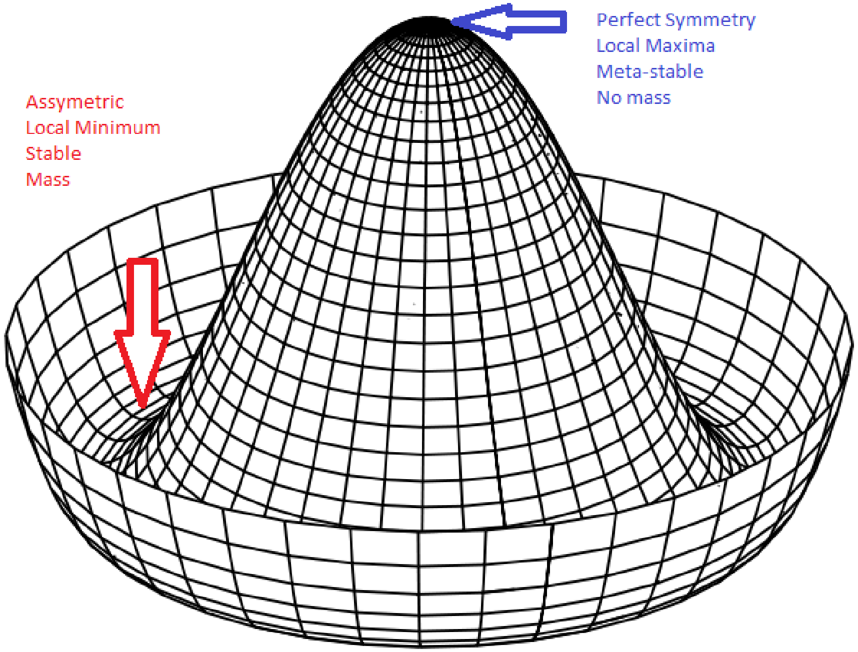
$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi + g_0[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \qquad (2.6)$$

The coupling parameter g_0 is positive, and has dimensions $[\text{mass}]^{-2}$. The γ_5 invariance property of the interaction is evident from Eq. (2.5). According to the …

…rlds described …3.22) are then … (3.31)

which form another orthogonal set. Since the original total H commutes with X, it will have no matrix elements connecting different "worlds." Moreover, as

Craig Roberts: Emergence of DSEs in Real-World QCD 2A (84)



Higgs Mechanism

Broken Symmetries and the Masses of Gauge Bosons
P.W. Higgs, Phys. Rev. Lett. 13, 508–509 (1964)

Quotes are in the original

Higgs:

- *Consider the equations [...] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:* (246 GeV!)
- *In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory.*

PHYSICAL REVIEW VOLUME 175, NUMBER 5 25 NOVEMBER 1968

Behavior of Current Divergences under $SU_3 \times SU_3$*†

MURRAY GELL-MANN
California Institute of Technology, Pasadena, California 91109

AND

R. J. OAKES
Northwestern University, Evanston, Illinois 60201

AND

B. RENNER‡
California Institute of Technology, Pasadena, California 91109

(Received 22 July 1968)

GMOR Relation

Gell-Mann - Oakes - Renner Relation

Behavior of current divergences under SU(3) x SU(3).
Murray Gell-Mann, R.J. Oakes , B. Renner
Phys.Rev. 175 (1968) 2195-2199

- This paper derives a relation between

  m_π^2 and the expectation-value $<\pi|u_0|\pi>$,

  where u_o is an operator that is linear in the putative Hamiltonian's explicit chiral-symmetry breaking term
  - NB. QCD's current-quarks were not yet invented, so u_0 was not expressed in terms of current-quark fields
- PCAC-hypothesis (partial conservation of axial current) is used in the derivation
- Subsequently, the concepts of soft-pion theory
  - Operator expectation values do not change as $t=m_\pi^2 \to t=0$

  to take $<\pi|u_0|\pi> \to <0|u_0|0>$ … *in-pion → in-vacuum*

Gell-Mann - Oakes - Renner Relation

Behavior of current divergences under SU(3) x SU(3).
Murray Gell-Mann, R.J. Oakes , B. Renner
Phys.Rev. 175 (1968) 2195-2199

- PCAC hypothesis; viz., pion field dominates the divergence of the axial-vector current

Zhou Guangzhao 周光召
Born 1929 Changsha, Hunan province

$$\partial_\mu A_\mu \propto \phi_\pi$$

- Soft-pion theorem

$$\begin{aligned} & \langle\alpha|\mathcal{O}|\beta\pi(q)\rangle \approx \langle\alpha|[Q_5,\mathcal{O}]|\beta\rangle \\ \Rightarrow\ & \langle\pi|\mathcal{O}|\pi\rangle \approx \langle 0|[Q_5,[Q_5,\mathcal{O}]]|0\rangle \\ \Rightarrow\ & \langle\pi(q)|\mathcal{H}|\pi(q)\rangle \approx \langle 0|[Q_5,[Q_5,\mathcal{H}]]|0\rangle \\ \propto\ & \langle 0|\mathcal{H}_{\rm chiral-symmetry-breaking}|0\rangle \end{aligned}$$

Commutator is chiral rotation
Therefore, isolates explicit chiral-symmetry breaking term in the putative Hamiltonian

- In QCD,

this is $m\bar{q}q$ and one therefore has $m_\pi^2 \propto m\langle 0|\bar{q}q|0\rangle$

Gell-Mann - Oakes - Renner Relation

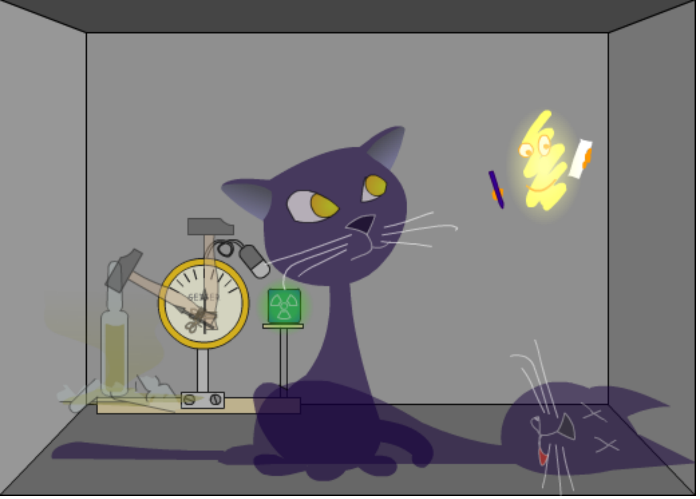
$$m_\pi^2 \propto m\langle 0|\bar{q}q|0\rangle$$

$-\ (0.25\text{GeV})^3$

- Theoretical physics at its best.
- But no one is thinking about how properly to consider or define what will come to be called the

  vacuum quark condensate

- So long as the condensate is just a mass-dimensioned constant, which approximates another well-defined matrix element, there is no problem.
- Problem arises if one over-interprets this number, which textbooks have been doing for a *VERY LONG TIME*.



Note of Warning

Chiral Magnetism (or Magnetohadrochironics)

A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

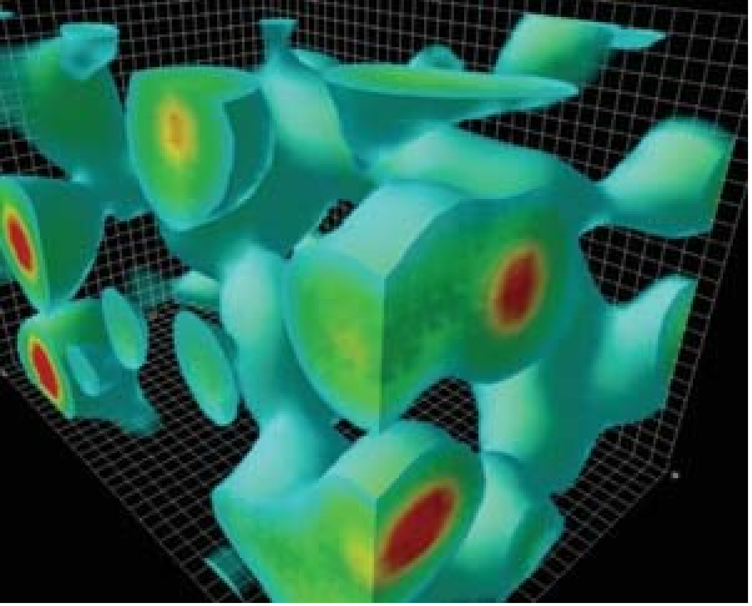
The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations

- *These authors argue that dynamical chiral-symmetry breaking can be realised as a property of hadrons, instead of via a nontrivial vacuum exterior to the measurable degrees of freedom*

The essential ingredient required for a spontaneous symmetry breakdown in a composite system is the existence of a divergent number of constituents

– DIS provided evidence for divergent sea of low-momentum partons – parton model.



QCD Sum Rules

QCD and Resonance Physics. Sum Rules.
M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov
Nucl.Phys. B147 (1979) 385-447; citations: 3713

- Introduction of the gluon vacuum condensate

$$\frac{\alpha}{\pi}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle = (0.33\,\mathrm{GeV})^4$$

and development of "sum rules" relating properties of low-lying hadronic states to vacuum condensates

QCD Sum Rules

QCD and Resonance Physics. Sum Rules.
M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov
Nucl.Phys. B147 (1979) 385-447; citations: 3781

- Introduction of the gluon vacuum condensate

$$\frac{\alpha}{\pi}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle = (0.33\,\mathrm{GeV})^4$$

and development of "sum rules" relating properties of low-lying hadronic states to vacuum condensates

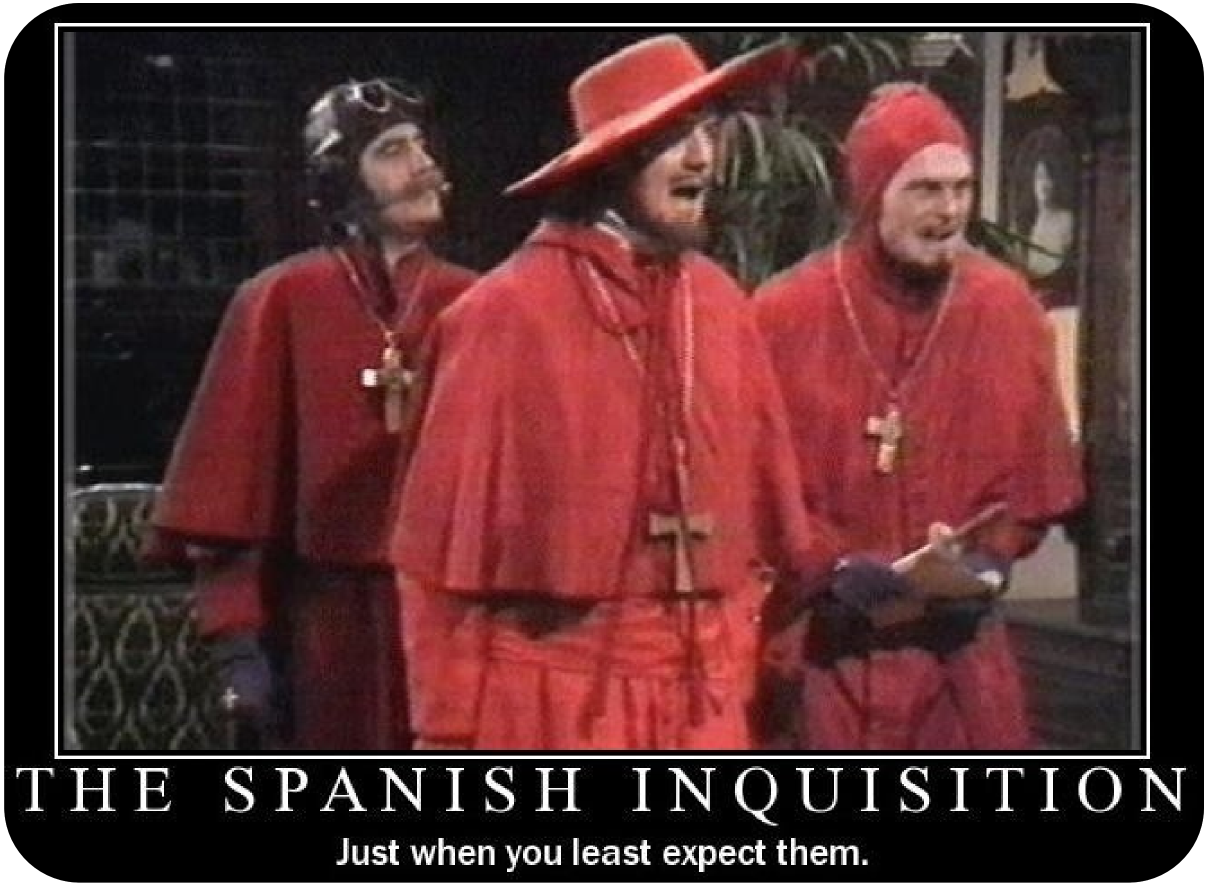
- *At this point (1979), the cat was out of the bag: a physical reality was seriously attributed to a plethora of vacuum condensates*

"quark condensate" 1960-1980

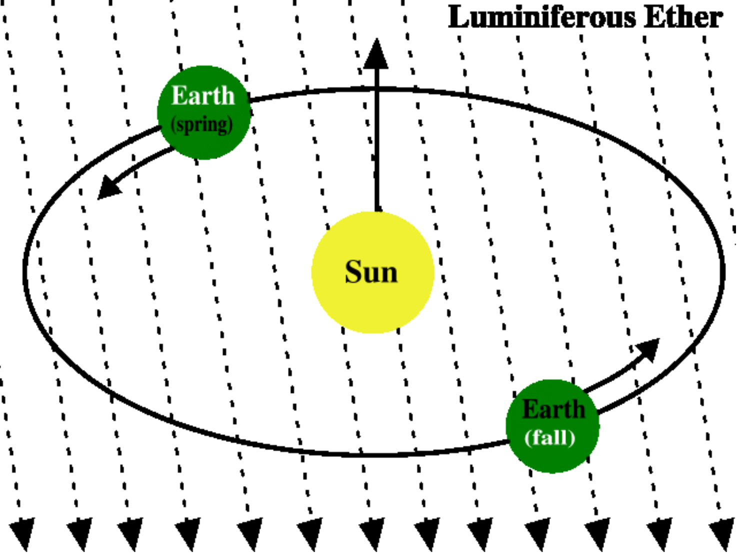
- Instantons in non-perturbative QCD vacuum,
  MA Shifman, AI Vainshtein… - Nuclear Physics B, 1980
- Instanton density in a theory with massless quarks,
  MA Shifman, AI Vainshtein… - Nuclear Physics B, 1980
- Exotic new quarks and dynamical symmetry breaking,
  WJ Marciano - Physical Review D, 1980
- The pion in QCD
  J Finger, JE Mandula… - Physics Letters B, 1980

No references to this phrase before 1980

7330+ REFERENCES TO THIS PHRASE SINCE 1980



Precedent?



Precedent- *Luminiferous Aether*

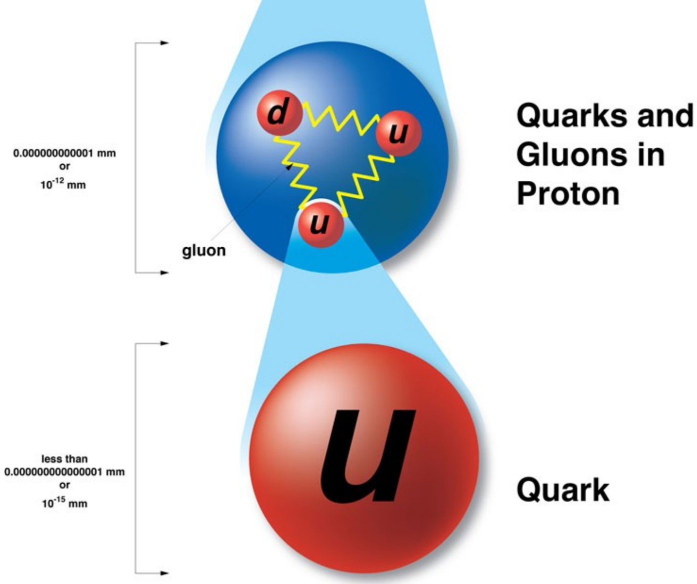
Pre-1887

~~Since the Earth is in motion, the flow of aether across the Earth should produce a detectable "aether wind"~~

- Physics theories of the late 19th century postulated that, just as water waves must have a medium to move across (water), and audible sound waves require a medium to move through (such as air or water), so also light waves require a medium, the "*luminiferous aether*".

Apparently unassailable logic

- Until, of course, "… *the most famous failed experiment to date.*" *On the Relative Motion of the Earth and the Luminiferous Ether*
  Michelson, Albert Abraham & Morley, Edward Williams
  American Journal of Science 34 (1887) 333–345.



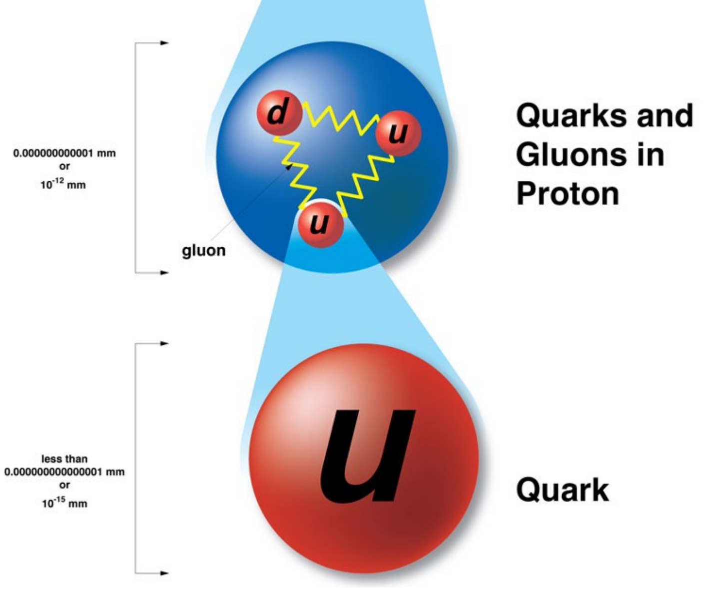
QCD

1973-1974

$$\langle 0|\bar{q}q|0\rangle$$

- How should one approach this problem, understand it, within Quantum ChromoDynamics?

1) Are the quark and gluon "condensates" theoretically well-defined?
2) Is there a physical meaning to this quantity or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?

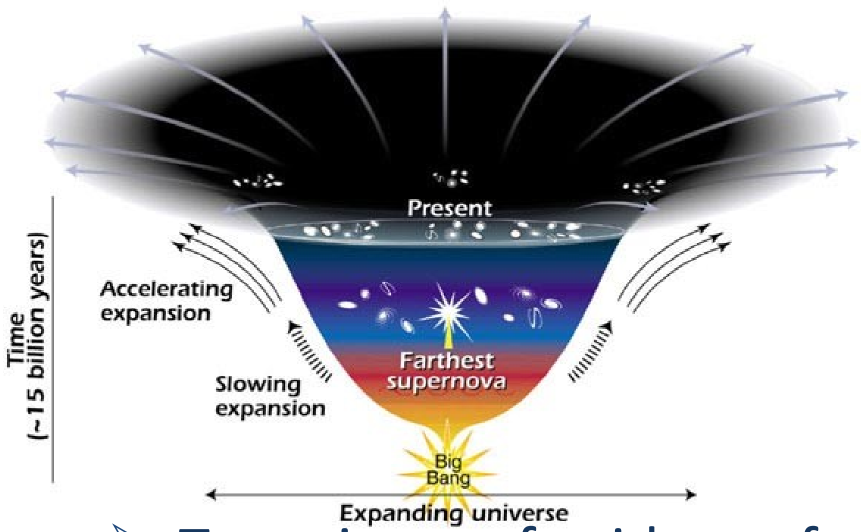


$$\langle 0 | \bar{q}q | 0 \rangle$$

QCD

1973-1974

Why does it matter?



"Dark Energy"

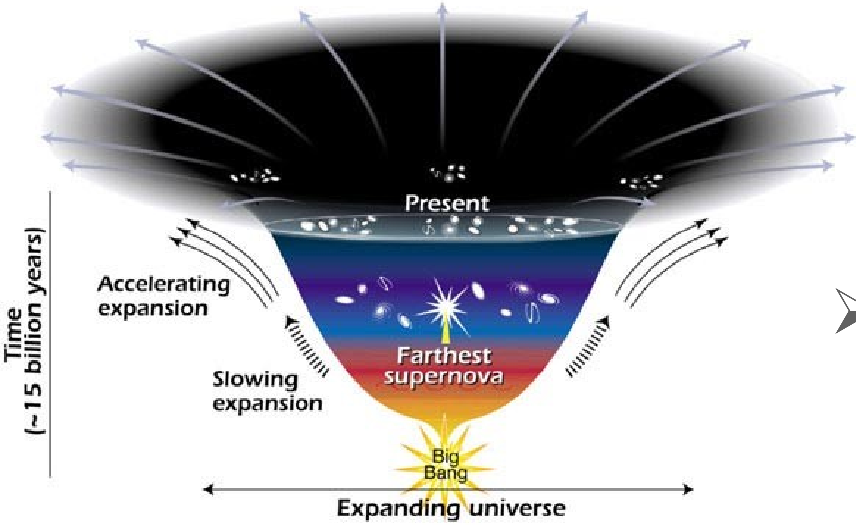
- Two pieces of evidence for an accelerating universe
  1) Observations of type Ia supernovae
     → *the rate of expansion of the Universe is growing*
  2) Measurements of the composition of the Universe point to a missing energy component with negative pressure:
     CMB anisotropy measurements indicate that the Universe is at

     $$\Omega_0 = 1 \,{}^{+}\!/\!_{-}\, 0.04.$$

     In a flat Universe, the matter density and energy density must sum to the critical density. However, matter only contributes about ⅓ of the critical density,

     $$\Omega_M = 0.33 \,{}^{+}\!/\!_{-}\, 0.04.$$

     Thus, *⅔ of the critical density is missing*.



"Dark Energy"

- In order to have escaped detection, the missing energy must be smoothly distributed.

- In order not to interfere with the formation of structure (by inhibiting the growth of density perturbations) the energy density in this component must change more slowly than matter (so that it was subdominant in the past).

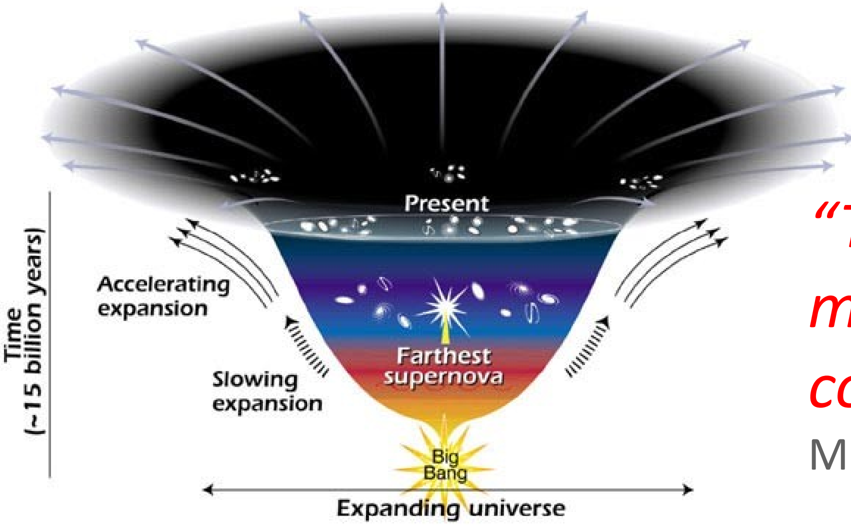
- Accelerated expansion can be accommodated in General Relativity through the Cosmological Constant, *Λ*.

  - Contemporary cosmological observations mean:

$$\rho_{obs} = \frac{\Lambda}{8\pi G} \leq (10^{-2}\, GeV)^4$$

  - Einstein introduced the repulsive effect of the cosmological constant in order to balance the attractive gravity of matter so that a static universe was possible. He promptly discarded it after the discovery of the expansion of the Universe.

"Dark Energy"



"The advent of quantum field theory made consideration of the cosmological constant obligatory not optional."
Michael Turner, "Dark Energy and the New Cosmology"

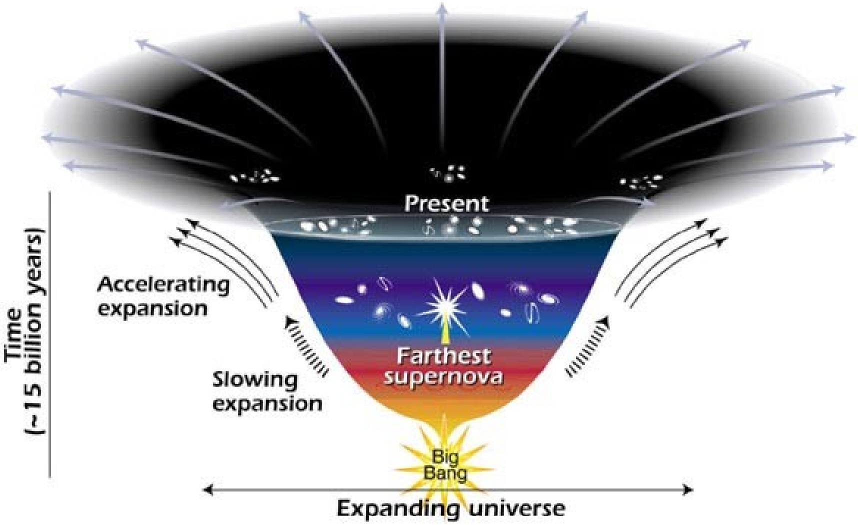
- The only possible covariant form for the energy of the (quantum) vacuum; viz.,

$$T^{\mu\nu}_{\rm VAC} = \rho_{\rm VAC}\, g^{\mu\nu}$$

  is mathematically equivalent to the cosmological constant.

"It is a perfect fluid and precisely spatially uniform"

"Vacuum energy is almost <u>the perfect candidate</u> for dark energy."



"Dark Energy"

$$T^{\mu\nu}_{\rm VAC} = \rho_{\rm VAC} g^{\mu\nu}$$

Enormous and even greater contribution from Higgs VEV!

- *QCD vacuum contribution*
  - If chiral symmetry breaking is expressed in a nonzero expectation value of the quark bilinear, then the energy difference between the symmetric and broken phases is of order

$$M_{QCD} \approx 0.3 \text{ GeV}$$

Mass-scale generated by spacetime-independent condensate

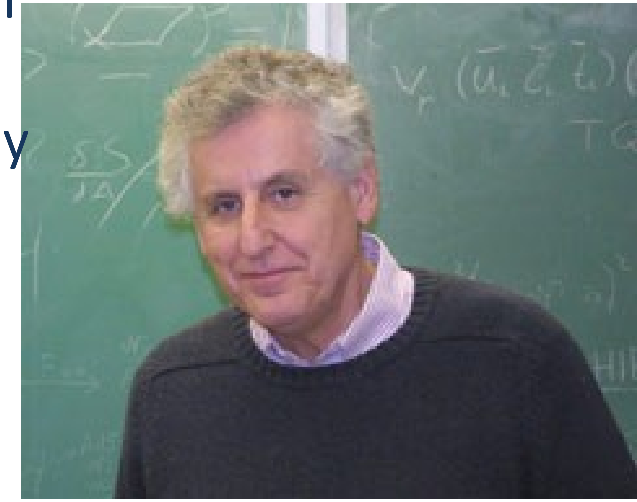
  - One obtains therefrom:

$$\rho^{QCD} = 10^{46} \rho^{obs}$$

"The biggest embarrassment in theoretical physics."

Resolution?

- *Quantum Healing Central*:
  - “KSU physics professor [Peter Tandy] publishes groundbreaking research on inconsistency in Einstein theory.”
- *Paranormal Psychic Forums:*
  - “Now Stanley Brodsky of the SLAC National Accelerator Laboratory in Menlo Park, California, and colleagues have found a way to get rid of the discrepancy. “People have just been taking it on faith that this quark condensate is present throughout the vacuum,” says Brodsky.



$$\langle 0 | \bar{q}q | 0 \rangle$$

QCD

1973-1974

Are the condensates real?

- Is there a physical meaning to the vacuum quark condensate (and others)?
- Or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?

S. Weinberg, Physica 96A (1979)
Elements of truth in this perspective

What is measurable?

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^{\zeta}_{H_5} \mathcal{M}^{\zeta}_{H_5}$$

- C
  –

cuum
e." More

We now have sufficient information to address the question of just what is this so-called "vacuum quark condensate."

- H

elation
1968

$$m^2_{H_5} = 2 m_q \frac{-\langle \bar{q} q \rangle}{f^0_{H_5}}$$

$$m_\pi^2 \propto \imath$$

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts
nucl-th/9708029

- Pseudoscalar projection of pion's Bethe-Salpeter wave-function onto the origin in configuration space: ***|Ψ$_\pi$^{PS}(0)|***
  – or the pseudoscalar pion-to-vacuum matrix element

$$i\rho_\pi = -\langle 0|\bar{q} i\gamma_5 q|\pi\rangle$$
$$= Z_4(\zeta,\Lambda)\, \mathrm{tr}_{\rm CD} \int^\Lambda \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-)$$

- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
  - For arbitrary current-quark masses
  - For any pseudoscalar meson

$$f_\pi m_\pi^2 = 2\, m(\zeta)\rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts
nucl-th/9708029

- Pseudovector projection of pion's Bethe-Salpeter wave-function onto the origin in configuration space: ***|Ψ$_\pi^{AV}$(0)|***
  – or the pseudoscalar pion-to-vacuum matrix element
  – or the pion's leptonic decay constant

$$i f_\pi P_\mu = \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle$$
$$= Z_2(\zeta,\Lambda)\, \mathrm{tr}_{\mathrm{CD}} \int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-)$$

- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
  - For arbitrary current-quark masses
  - For any pseudoscalar meson

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts
nucl-th/9708029

- Define $-\langle \bar{q} q \rangle_\zeta^\pi \equiv -f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$
- Then, using the pion Goldberger-Treiman relations (equivalence of 1- and 2-body problems), one derives, in the chiral limit

Chiral limit

$$\kappa_\pi(0;\zeta) = -\langle \bar{q} q \rangle_\zeta$$

- Namely, the so-called *vacuum quark condensate* is the *chiral-limit value* of the *in-pion condensate*

$|\Psi_\pi^{PS}(0)| * |\Psi_\pi^{AV}(0)|$

- The *in-pion condensate* is the *only well-defined function of current-quark mass* in QCD that is *smoothly connected to the vacuum quark condensate*.

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

There is *only one* condensate

Langeld, Roberts *et al*.
nucl-th/0301024,
Phys.Rev. C67 (2003) 065206

I. Casher Banks formula:

Density of eigenvalues of Dirac operator

$$-\langle 0|\bar{q}q|0\rangle = 2m \int_0^\infty d\lambda \, \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

II. Constant in the Operator Product Expansion:

$$M(p^2) \overset{\text{large}-p^2}{=} \frac{2\pi^2\gamma_m}{3} \frac{\left(-\langle\bar{q}q\rangle^0\right)}{p^2\left(\frac{1}{2}\ln\left[\frac{p^2}{\Lambda^2_{\rm QCD}}\right]\right)^{1-\gamma_m}}$$

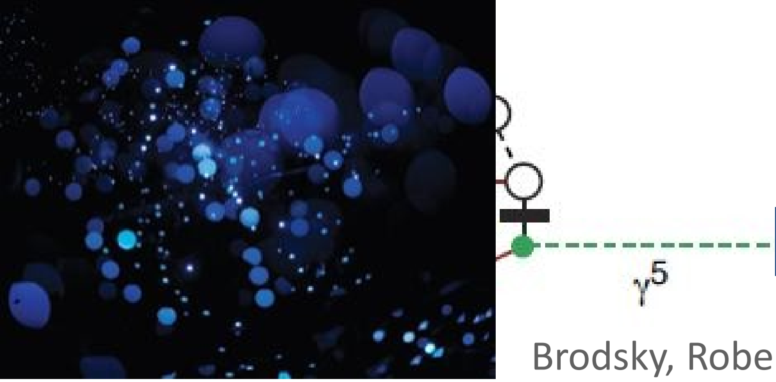
III. Trace of the dressed-quark propagator:

$$\tilde{\sigma}(m) := N_c \, {\rm tr}_D \int_p^\Lambda \tilde{S}_m(p)$$

$m \to 0$

Algebraic proof that these are all the same. So, no matter how one chooses to calculate it, one is always calculating the same thing; viz.,

$|\Psi_\pi^{PS}(0)| * |\Psi_\pi^{AV}(0)|$



Paradigm shift: In-Hadron Condensates

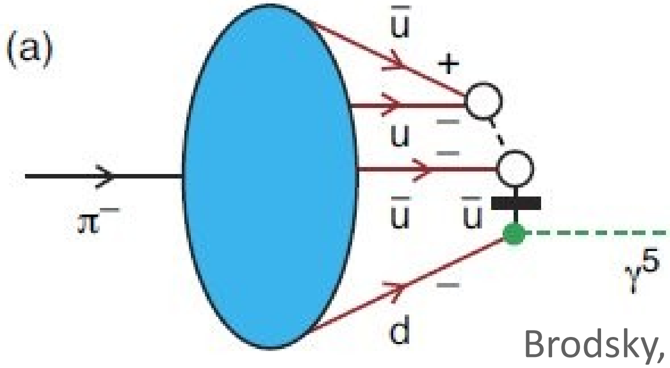
Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, PNAS **108**, 45 (2011)

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **GMOR**
    cf.

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

QCD

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$



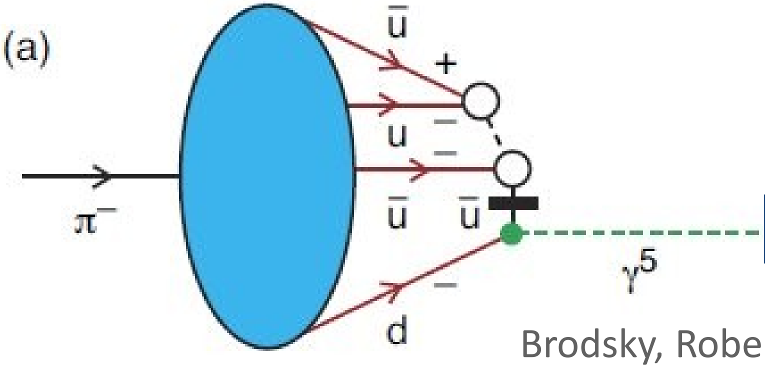
Paradigm shift: In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, PNAS **108**, 45 (2011)

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**

$$i f_\pi P_\mu = \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle = Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \tag{5}$$

$$i\rho_\pi = -\langle 0|\bar{q}i\gamma_5 q|\pi\rangle = Z_4(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4}\gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-). \tag{6}$$



Paradigm shift: In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, PNAS **108**, 45 (2011)

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**
  - **And**

Chiral limit

$$\kappa_\pi(0;\zeta) = -<\bar{q}q>_\zeta$$

$$-\langle \bar{q}q \rangle_\zeta^\pi \equiv -f_\pi \langle 0|\bar{q}\gamma_5 q|\pi\rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$$

Topological charge of “vacuum”

- *Wikipedia:* Instanton effects are important in understanding the formation of condensates in the vacuum of quantum chromodynamics (QCD)
- *Wikipedia:* The difference between the mass of the η and that of the η' is larger than the quark model can naturally explain. This “η-η' puzzle” is resolved by instantons.
- Claimed that some lattice simulations demonstrate nontrivial topological structures in QCD vacuum
- *Now illustrate new paradigm perspective …*

Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C76 (2007) 045203

Charge-neutral pseudoscalar mesons

- AVWTI ⇒ QCD mass formulae for all pseudoscalar mesons, including those which are charge-neutral

$$m_{\pi_i}^2 f_{\pi_i}^a = 2\,\mathcal{M}^{ab}\rho_{\pi_i}^b + \delta^{a0} n_{\pi_i}$$

- Consider the limit of a $U(N_f)$-symmetric mass matrix, then this formula yields:

$$m_\eta^2 f_\eta = 2m(\zeta)\rho_\eta^0(\zeta)$$
$$m_{\eta'}^2 f_{\eta'}^0 = 2m(\zeta)\rho_{\eta'}^0(\zeta) + n_{\eta'}$$

Algebraic result. Very different than requiring QCD's vacuum to possess nontrivial topological structure

Qualitatively the same as f_π, a property of the bound-state

$$n_{\eta'} = \sqrt{\frac{N_f}{2}}\,\nu_{\eta'}\,,\ \nu_{\eta'} = \langle 0|\mathcal{Q}|\eta'\rangle$$

Topological charge density: $Q(x) = i(\alpha_s/4\pi)\, tr_C\, \varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$

- Plainly, the $\eta - \eta'$ mass splitting is nonzero in the chiral limit so long as $\nu_{\eta'} \neq 0$ … viz.,
  so long as the topological content of the η' is nonzero!

Topology and the "condensate"

- Exact result in QCD, algebraic proof:

$$\langle \bar{q} q \rangle_\zeta^0 = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \mathrm{tr}_{CD} \int_q^\Lambda S^0(q, \zeta)$$

$$= \frac{N_f}{2} \int d^4x \langle \bar{q}(x) i \gamma_5 q(x) \mathcal{Q}(0) \rangle^0,$$

- "chiral condensate" = in-pion condensate

  the zeroth moment of a mixed vacuum polarisation

  – connecting topological charge with the pseudoscalar quark operator

- This connection is required if one is to avoid *ή* appearing as a Goldstone boson

PHYSICAL REVIEW VOLUME 175, NUMBER 5 25 NOVEMBER 1968

Behavior of Current Divergences under $SU_3 \times SU_3$*†

MURRAY GELL-MANN
California Institute of Technology, Pasadena, California 91109

AND

R. J. OAKES
Northwestern University, Evanston, Illinois 60201

AND

B. RENNER‡
California Institute of Technology, Pasadena, California 91109
(Received 22 July 1968)

GMOR Relation

GMOR Relation

- Valuable to highlight the precise form of the Gell-Mann–Oakes–Renner (GMOR) relation: Eq. (3.4) in Phys.Rev. 175 (1968) 2195

$$m_\pi^2 = \lim_{P' \to P \to 0} \langle \pi(P') | \mathcal{H}_{\chi\mathrm{sb}} | \pi(P) \rangle$$

  - m_π *is the pion's mass*
  - $H_{\chi sb}$ *is that part of the* hadronic Hamiltonian density which explicitly breaks chiral symmetry.
- Crucial to observe that the operator expectation value in this equation is evaluated between pion states.
- Moreover, the virtual low-energy limit expressed in the equation is purely formal. It does not describe an achievable empirical situation.

GMOR Relation

- In terms of QCD quantities, GMOR relation entails

$$\forall m_{ud} \sim 0\,,\; m_{\pi^\pm}^2 = m_{ud}^\zeta\, \mathcal{S}_\pi^\zeta(0)\,,$$
$$\mathcal{S}_\pi^\zeta(0) \;=\; -\langle \pi(P)|\tfrac{1}{2}(\bar{u}u + \bar{d}d)|\pi(P)\rangle$$

  - $m_{ud}^\zeta = m_u^\zeta + m_d^\zeta$ … the current-quark masses
  - $S_\pi^\zeta(0)$ is the pion's scalar form factor at zero momentum transfer, $Q^2=0$
- RHS is proportional to the pion σ-term
- Consequently, using the connection between the σ-term and the Feynman-Hellmann theorem, GMOR relation is actually the statement

$$\forall m_{ud} \sim 0\,,\; m_\pi^2 = m_{ud}^\zeta \frac{\partial}{\partial m_{ud}^\zeta} m_\pi^2$$

Expanding the concept of in-hadron condensates
Lei Chang, Craig D. Roberts and Peter C. Tandy
arXiv:1109.2903 [nucl-th], Phys. Rev. C**85** (2012) 012201(R)

GMOR Relation

Maris, Roberts and Tandy
nucl-th/9707003,
*Phys.Lett. B**420** (1998) 267-273*

- Using $$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

  it follows that

$$\mathcal{S}_{\pi}^{\zeta}(0) = \frac{\partial}{\partial m_{ud}^{\zeta}} m_{\pi}^2 = \frac{\partial}{\partial m_{ud}^{\zeta}} \left[m_{ud}^{\zeta} \frac{\rho_{\pi}^{\zeta}}{f_{\pi}} \right]$$

- This equation is valid for any values of $m_{u,d}$, including the neighbourhood of the chiral limit, wherein

$$\frac{\partial}{\partial m_{ud}^{\zeta}} \left[m_{ud}^{\zeta} \frac{\rho_{\pi}^{\zeta}}{f_{\pi}} \right]_{m_{ud}=0} = \frac{\rho_{\pi}^{\zeta 0}}{f_{\pi}^{0}}$$

Expanding the concept of in-hadron condensates
Lei Chang, Craig D. Roberts and Peter C. Tandy
arXiv:1109.2903 [nucl-th], Phys. Rev. C**85** (2012) 012201(R)

GMOR Relation

- Consequently, in the neighbourhood of the chiral limit

$$m_{\pi^\pm}^2 = -m_{ud}^\zeta \frac{\langle \bar{q}q \rangle^{\zeta 0}}{(f_\pi^0)^2} + \mathrm{O}(m_{ud}^2)$$

- This is a QCD derivation of the commonly recognised form of the GMOR relation.
- Neither PCAC nor soft-pion theorems were employed in the analysis.
- Nature of each factor in the expression is abundantly clear; viz., chiral limit values of matrix elements that explicitly involve the hadron.

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **85**, 012201(R) (2012)

Expanding the concept of in-hadron condensates

Lei Chang,[1] Craig D. Roberts,[1,2,3] and Peter C. Tandy[4]

[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[2]*Department of Physics, Center for High Energy Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[3]*Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616-3793, USA*

[4]*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 13 September 2011; published 23 January 2012)

The in-pseudoscalar-meson condensate can be represented through the pseudoscalar meson's scalar form factor at zero-momentum transfer. With the aid of a mass formula for scalar mesons, revealed herein, the analog is shown to be true for in-scalar-meson condensates. The concept is readily extended to all hadrons so that, via the zero-momentum-transfer value of any hadron's scalar form factor, one can readily extract the value for a quark condensate in that hadron which is a measure of dynamical chiral symmetry breaking.

DOI: 10.1103/PhysRevC.85.012201 PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

Expanding the Concept

Expanding the concept of in-hadron condensates
Lei Chang, Craig D. Roberts and Peter C. Tandy
arXiv:1109.2903 [nucl-th], Phys. Rev. C**85** (2012) 012201(R)

In-Hadron Condensates

- Plainly, the in-pseudoscalar-meson condensate can be represented through the pseudoscalar meson's scalar form factor at zero momentum transfer $Q^2 = 0$.
- Using an *exact mass formula for scalar mesons*, one proves the in-scalar-meson condensate can be represented in precisely the same way.

$$f_{S_{qQ}} m^2_{S_{qQ}} = -\check{m}_{qQ} \rho^{\zeta}_{S_{qQ}}$$

- By analogy, and with appeal to demonstrable results of heavy-quark symmetry, the $Q^2 = 0$ values of vector- and pseudovector-meson scalar form factors also determine the in-hadron condensates in these cases.
- This expression for the concept of in-hadron quark condensates is readily extended to the case of baryons.
- Via the $Q^2 = 0$ value of any hadron's scalar form factor, one can extract the value for a quark condensate in that hadron which is a reasonable and realistic measure of dynamical chiral symmetry breaking.

$$\langle H(p')|\bar{q}\mathcal{O}q|H(p)\rangle$$

Hadron Charges

- Hadron Form factor matrix elements
- *Scalar charge of a hadron is an intrinsic property of that hadron … no more a property of the vacuum than the hadron's electric charge, axial charge, tensor charge, etc. …*

Confinement

Confinement Contains Condensates
S.J. Brodsky, C.D. Roberts, R. Shrock and P.C. Tandy
arXiv:1202.2376 [nucl-th], Phys. Rev. C **85**, 065202 (2012) [9 pages]

Confinement

- Confinement is essential to the validity of the notion of in-hadron condensates.
- Confinement makes it impossible to construct gluon or quark quasiparticle operators that are nonperturbatively valid.
- So, although one can define a perturbative (bare) vacuum for QCD, it is impossible to rigorously define a ground state for QCD upon a foundation of gluon and quark quasiparticle operators.
- Likewise, it is impossible to construct an interacting vacuum – a BCS-like trial state – and hence DCSB in QCD cannot rigorously be expressed via a spacetime-independent coherent state built upon the ground state of perturbative QCD.
- Whilst this does not prevent one from following this path to build practical models for use in hadron physics phenomenology, it does invalidate any claim that theoretical artifices in such models are empirical.

Paradigm shift: In-Hadron Condensates

"Void that is truly empty
solves dark energy puzzle"
Rachel Courtland, New Scientist 4th Sept. 2010

"EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe."

$$\Omega_{QCD-condensates} = 3\pi G_N \frac{\Lambda_{QCD}}{3H_0^2} \cong 10^{46}$$

Cosmological Constant:

- ✓ ***Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}***
- ✓ ***Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model***

Discovering Technicolor
J. R. Andersen *et al.*,
Eur. Phys. J. Plus 126 (2011) 81

Modern Technicolour

- Even with the Higgs discovered, perhaps, the Standard Model (SM) has both conceptual problems and phenomenological shortcomings.
- The SM is incomplete, at least, since it cannot even account for a number of basic observations.
  - Neutrino's have a small mass. We do not yet know if the neutrinos have a Dirac or a Majorana nature
  - Origin of dark mass in the universe
  - Matter-antimatter asymmetry. We exist. Therefore, excess of matter over antimatter. SM can't describe this
- Technicolour: electroweak symmetry breaks via a fermion bilinear operator in a strongly interacting theory. Higgs sector of the SM becomes an effective description of a more fundamental fermionic theory, similar to the Ginzburg-Landau theory of superconductivity

Dynamical Chiral Symmetry Breaking

Importance of being well-dressed for quarks & mesons