Argonne NATIONAL LABORATORY

Emergence of DSEs in Real-World QCD

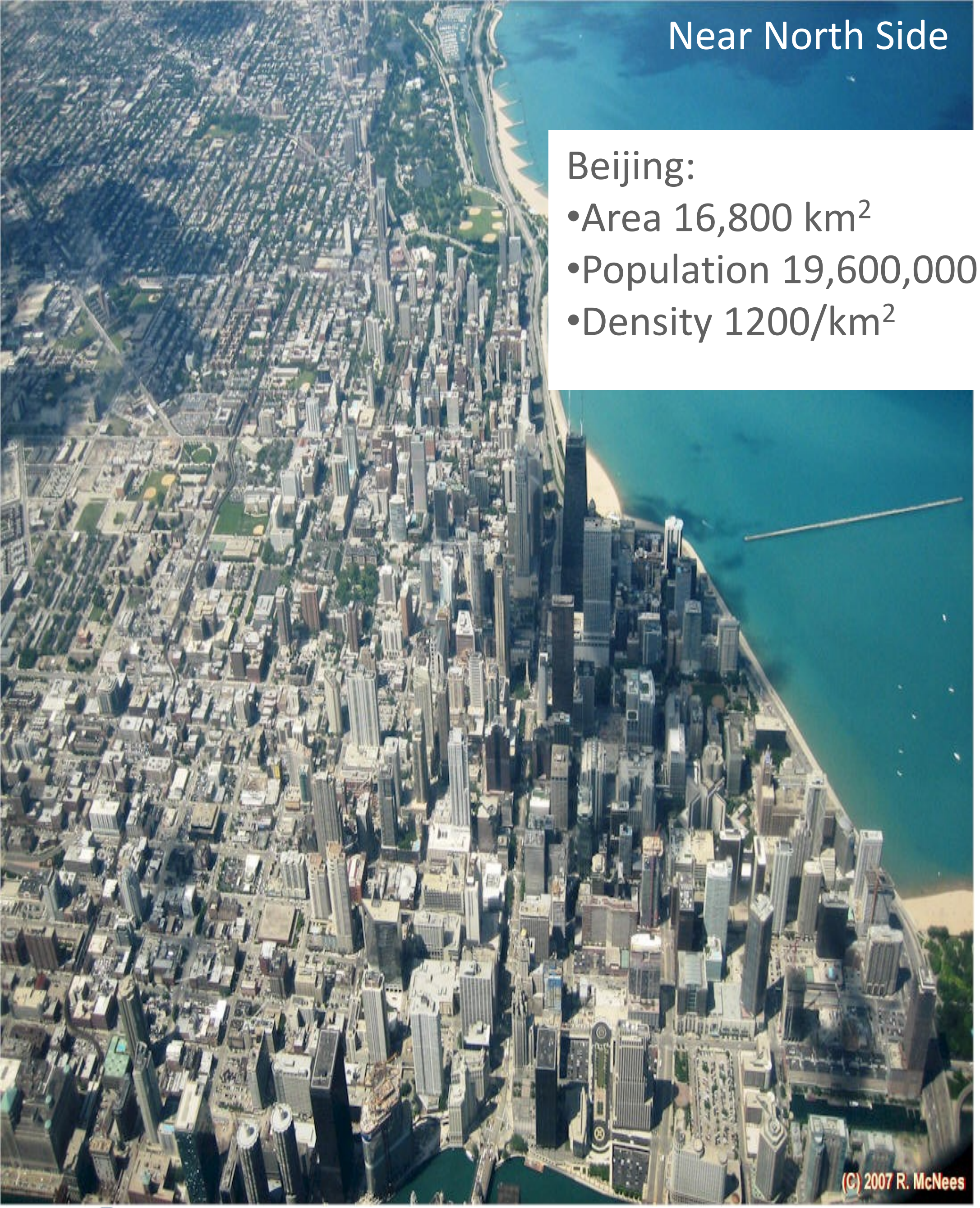
Craig Roberts

Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

National Science Foundation
WHERE DISCOVERIES BEGIN

U.S. DEPARTMENT OF ENERGY



Beijing:
- Area 16,800 km^2
- Population 19,600,000
- Density 1200/km^2

Chicago

Area
- City 234.0 sq mi (606.1 km^2)
- Land 227.2 sq mi (588.4 km^2)
- Water 6.9 sq mi (17.9 km^2) 3.0%
- Urban 2,122.8 sq mi (5,498 km^2)
- Metro 10,874 sq mi (28,163.5 km^2) (*30-times the area of Berlin, 92% of Belgium*)

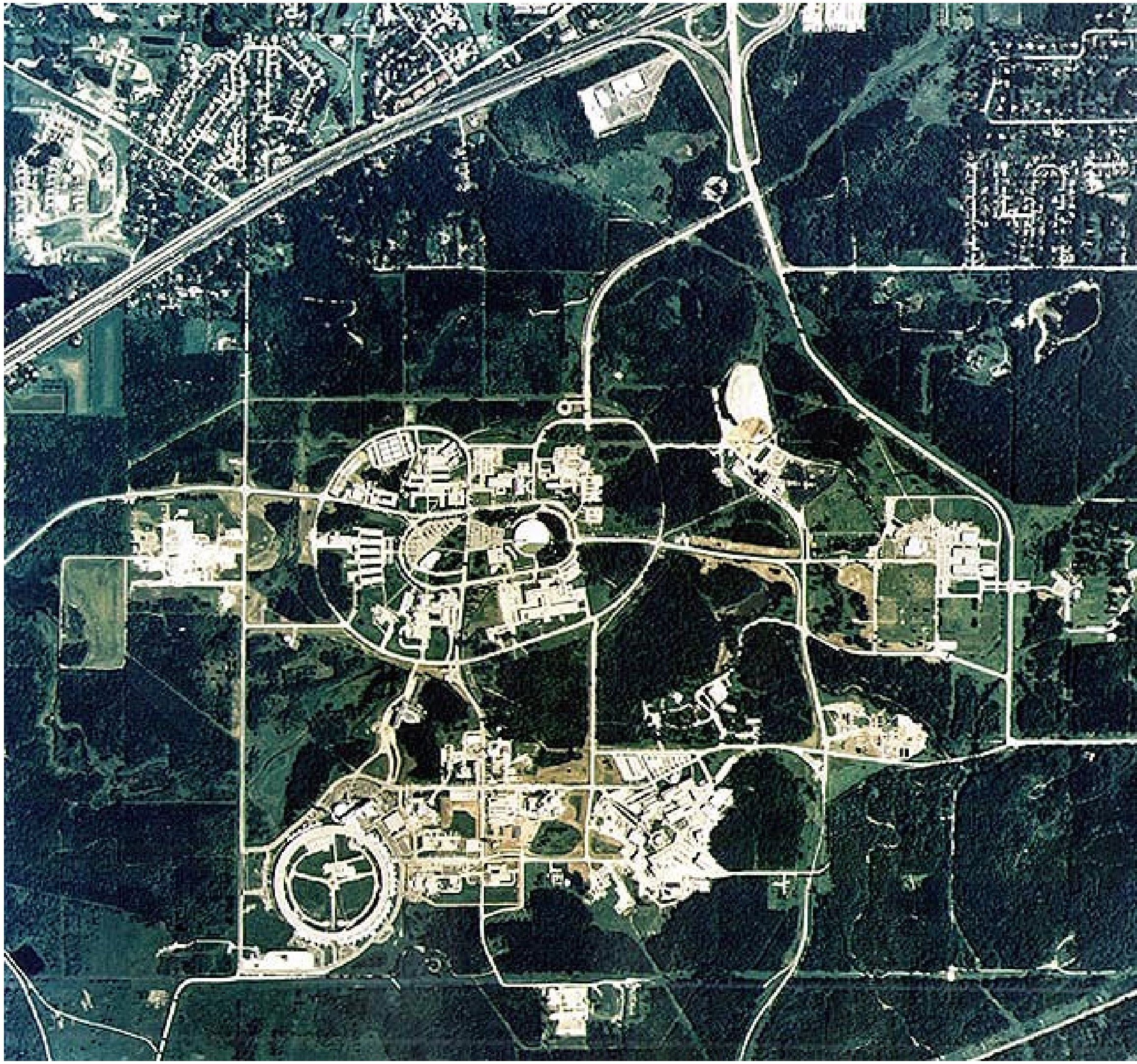
Elevation 597 ft (182 m)

Population (2010 Census)
- City 2,695,598
- Rank 3rd US
- Density 11,864.4/sq mi (4,447.4/km^2) (*⅕ that of Paris*)
- Urban 8,711,000
- Metro 9,461,105 (density 340/km^2) (*Berlin 5,963,998*)

Argonne National Laboratory

- Argonne grew from Enrico Fermi's secret charge — the Manhattan Project — to create the world's first self-sustaining nuclear reaction. Code-named the “Metallurgical Lab”, the team constructed Chicago Pile-1, which achieved criticality on December 2, 1942, underneath the University of Chicago's Stagg football field stands.

  Because the experiments were deemed too dangerous to conduct in a major city, the operations were moved to a spot in nearby Palos Hills and renamed "Argonne" after the surrounding forest.



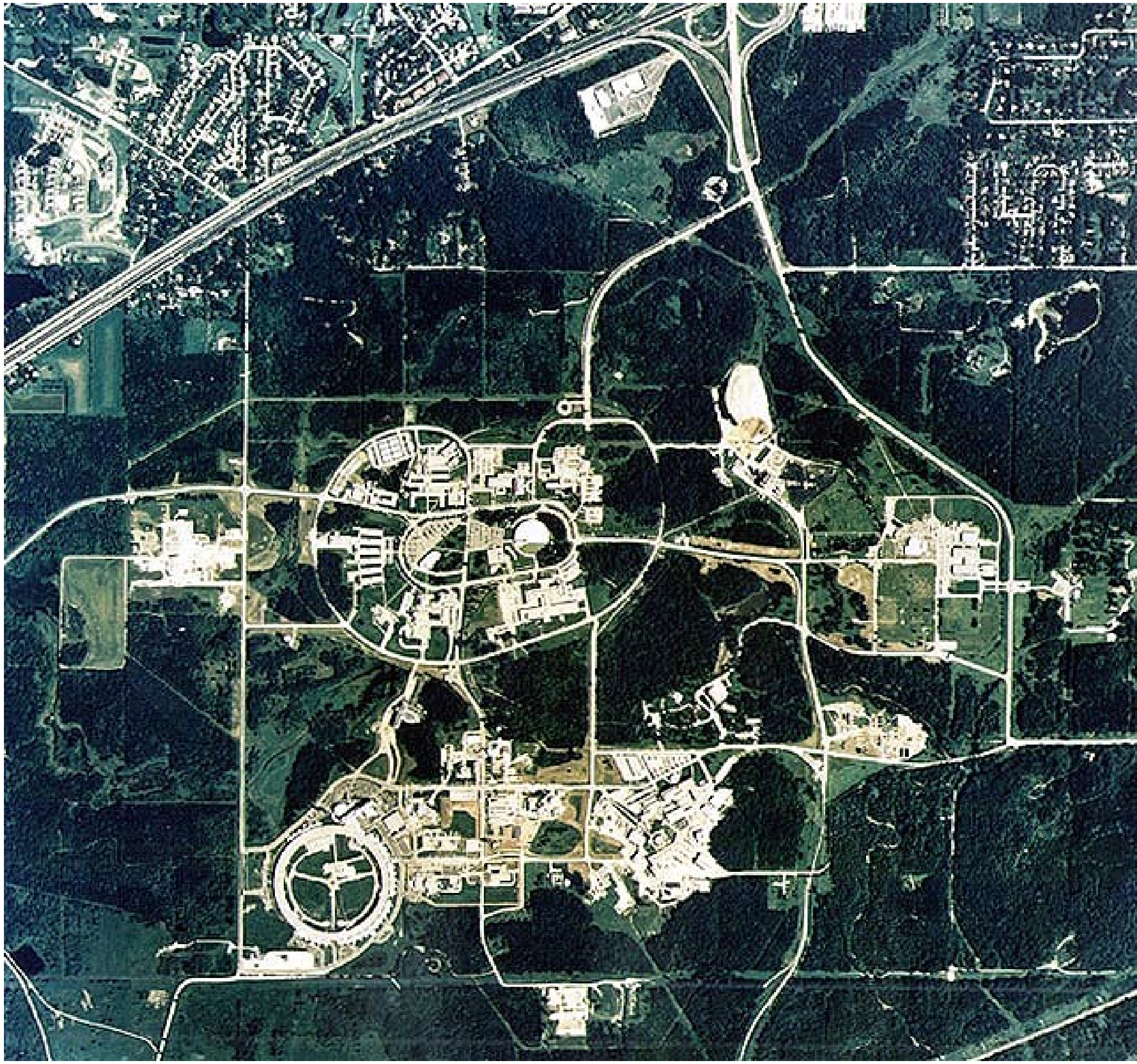
Argonne National Laboratory

- On July 1, 1946, the laboratory was formally chartered as Argonne National Laboratory to conduct “cooperative research in nucleonics.”

  At the request of the U.S. Atomic Energy Commission, it began developing nuclear reactors for the nation's peaceful nuclear energy program.

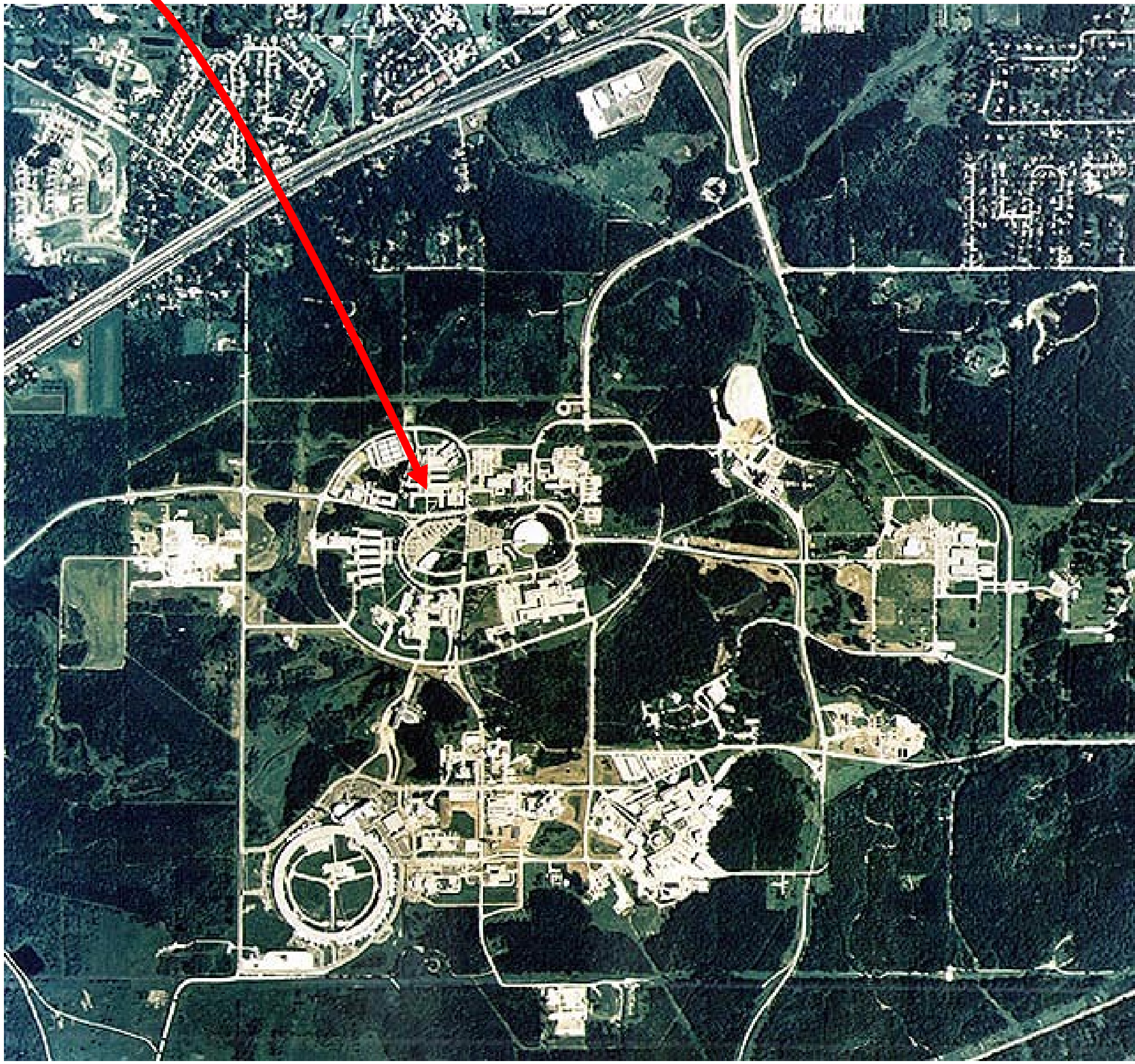
  In the late 1940s and early 1950s, the laboratory moved to a larger location in Lemont, Illinois.

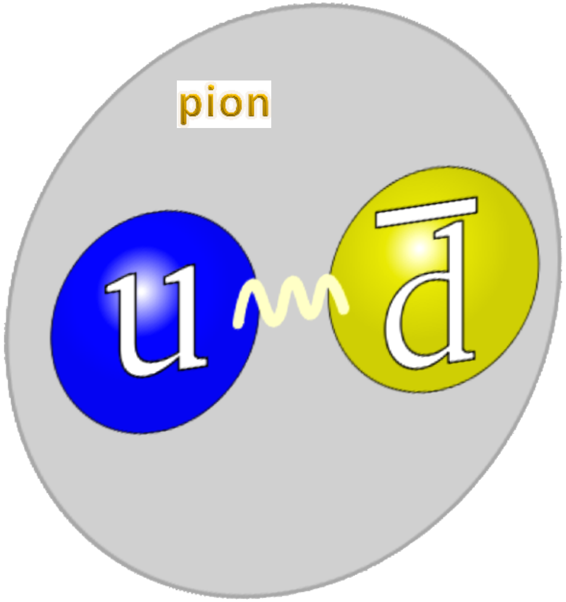
- Annual budget today is $630-million/year, spent on over 200 distinct research programmes

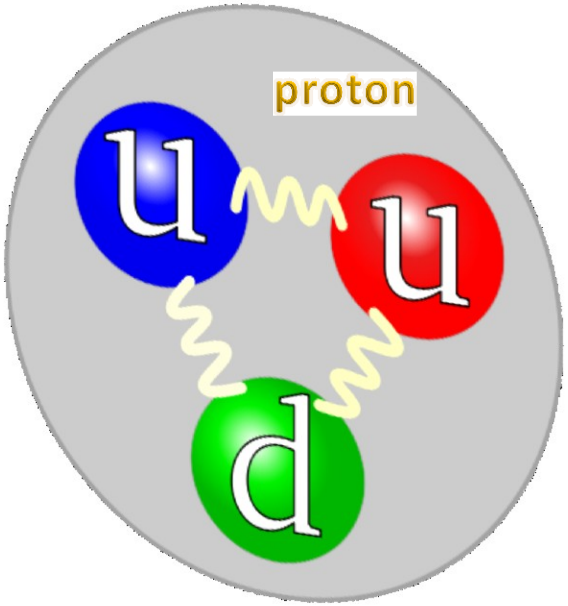


Argonne National Laboratory

- **Physics Division**
  - ATLAS Tandem Linac: International User Facility for Low Energy Nuclear Physics
  - 37 PhD Scientific Staff
  - Annual Budget: $27million







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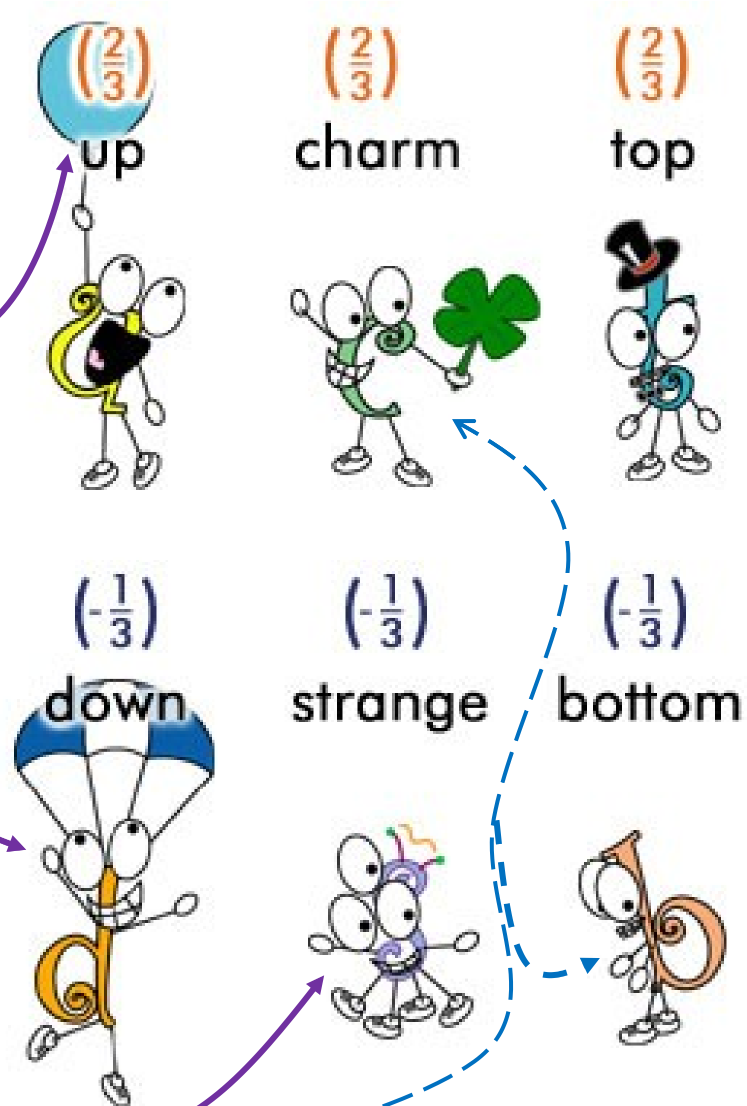
Hadron Theory

Quarks and Nuclear Physics

Standard Model of Particle Physics:
- Six quark flavours

Real World
- Normal matter – only two light-quark flavours are active
- Or, perhaps, three

For numerous good reasons, much research also focuses on accessible heavy-quarks
- Nevertheless, I will mainly focus on the light-quarks; i.e., *u* & *d*.



Problem: Nature chooses to build things, us included, from matter fields instead of gauge fields.

Quarks & QCD

- Quarks are the problem with QCD
- Pure-glue QCD is far simpler
  - Bosons are the only degrees of freedom
    - Bosons have a classical analogue – see Maxwell's formulation of electrodynamics
  - Generating functional can be formulated as a discrete probability measure that is amenable to direct numerical simulation using Monte-Carlo methods
    - No perniciously nonlocal fermion determinant
- Provides the Area Law & Linearly Rising Potential between static sources, so long identified with confinement

In perturbation theory, quarks don't seem to do much, just a little bit of very-normal charge screening.

K.G. Wilson, formulated lattice-QCD in 1974 paper: "Confinement of quarks".

Wilson Loop

Nobel Prize (1982): *"for his theory for critical phenomena in connection with phase transitions"*.

Contrast with Minkowksi metric: infinitely many four-vectors satisfy $p^2 = p^0p^0 - p^ip^i = 0$;
e.g., $p= \mu\ (1,0,0,1)$, μ any number

Formulating QCD Euclidean Metric

- In order to translate QCD into a computational problem, Wilson had to employ a *Euclidean Metric*

  $x^2 = 0$ possible if and only if $x=(0,0,0,0)$

  because Euclidean-QCD action defines a probability measure, for which many numerical simulation algorithms are available.
- However, working in Euclidean space is more than simply pragmatic:
  - Euclidean lattice field theory is currently a primary candidate for the rigorous definition of an interacting quantum field theory.
  - This relies on it being possible to define the generating functional via a proper limiting procedure.

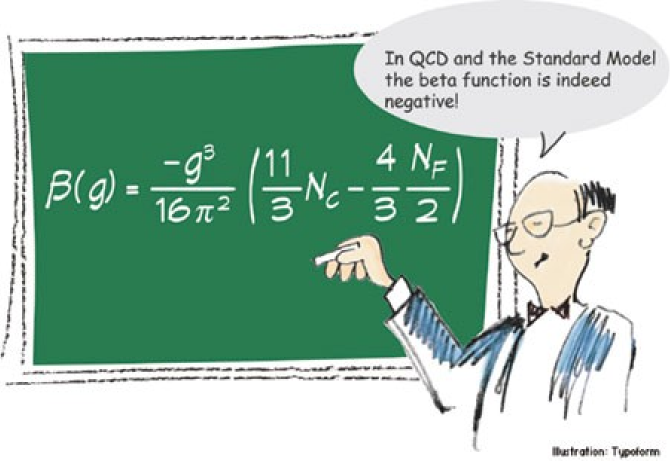
Formulating QCD Euclidean Metric

- The moments of the measure; i.e., "vacuum expectation values" of the fields, are the n-point Schwinger functions; and the quantum field theory is completely determined once all its Schwinger functions are known.
- The time-ordered Green functions of the associated Minkowski space theory can be obtained in a formally well-defined fashion from the Schwinger functions.

This is all *formally* true.

Formulating Quantum Field Theory
Euclidean Metric

- Constructive Field Theory Perspectives:
  - Symanzik, K. (1963) in *Local Quantum Theory (Academic, New York) edited* by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That (Addison-Wesley, Reading, Mass, 3rd edition).*
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics (Springer-Verlag, New York).*
- For some theorists, interested in essentially nonperturbative QCD, this is always in the back of our minds

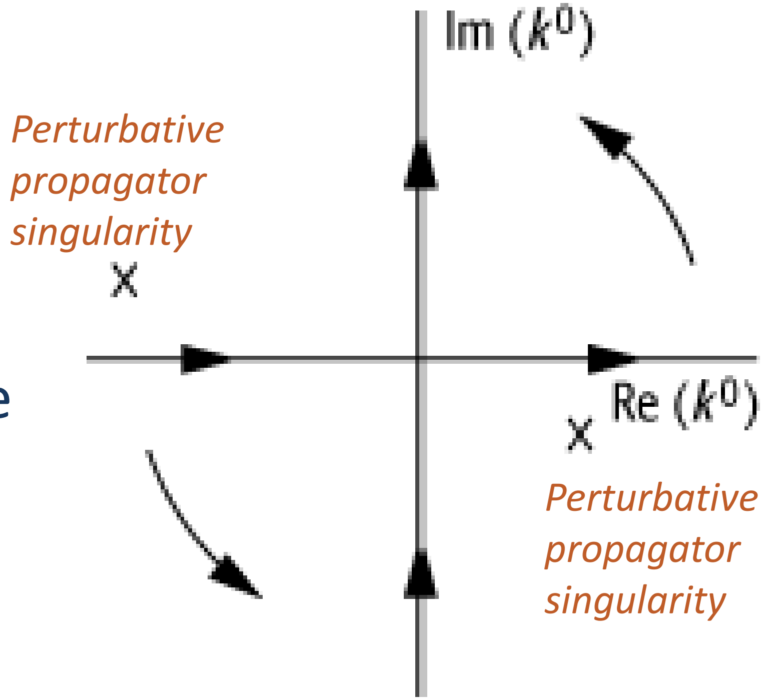


Formulating QCD Euclidean Metric

- However, there is another very important reason to work in Euclidean space; viz.,
  Owing to asymptotic freedom, all results of perturbation theory are strictly valid only at spacelike-momenta.
  - The set of spacelike momenta correspond to a Euclidean vector space
- The continuation to Minkowski space rests on many assumptions about Schwinger functions that are demonstrably valid only in perturbation theory.

Euclidean Metric & Wick Rotation

- It is assumed that a Wick rotation is valid; namely, that QCD dynamics don't nonperturbatively generate anything *unnatural*
- This is a brave assumption, which turns out to be *very, very false* in the case of coloured states.
- Hence, QCD *MUST* be defined in Euclidean space.
- The properties of the real-world are then determined only from a continuation of colour-singlet quantities.



Aside: QED is only defined perturbatively. It possesses an infrared stable fixed point; and masses and couplings are regularised and renormalised in the vicinity of $k^2=0$*. Wick rotation is always valid in this context.*

The Problem with QCD

- This is a RED FLAG in QCD because
  nothing elementary is a colour singlet
- Must somehow solve real-world problems
  - the spectrum and interactions of complex two- and three-body bound-states

  before returning to the real world
- This is going to require a little bit of imagination and a very good toolbox:

Dyson-Schwinger equations

Euclidean Metric Conventions

- To make clear our conventions: for 4-vectors a, b: $a \cdot b := a_\mu \, b_\nu \, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i \, b_i$,

  Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu , \gamma_\nu\} = 2\,\delta_{\mu\nu}$;
  - we use $\gamma_5 := -\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{pmatrix}, \; \gamma_4 = \begin{pmatrix} \tau^0 & 0 \\ 0 & -\tau^0 \end{pmatrix}, \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Euclidean Transcription Formulae

It is possible to derive every equation of Euclidean QCD by assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space

1. $\int^M d^4x^M \to -i \int^E d^4x^E$
2. $\not\partial \to i\gamma^E \cdot \partial^E$
3. $\not A \to -i\gamma^E \cdot A^E$
4. $A_\mu B^\mu \to -A^E \cdot B^E$
5. $x^\mu \partial_\mu \to x^E \cdot \partial^E$

Momentum Space

1. $\int^M d^4k^M \to i \int^E d^4k^E$
2. $\not k \to -i\gamma^E \cdot k^E$
3. $\not A \to -i\gamma^E \cdot A^E$
4. $k_\mu q^\mu \to -k^E \cdot q^E$
5. $k_\mu x^\mu \to -k^E \cdot x^E$

These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

Never before seen by the human eye

Nature's strong messenger - Pion

- 1947 – Pion discovered by Cecil Frank Powell
- Studied tracks made by cosmic rays using photographic emulsion plates
- Despite the fact that Cavendish Lab said method is incapable of "*reliable and reproducible precision measurements.*"
- Mass measured in scattering ≈ $250\text{-}350\ m_e$

Nuclear capture of pion

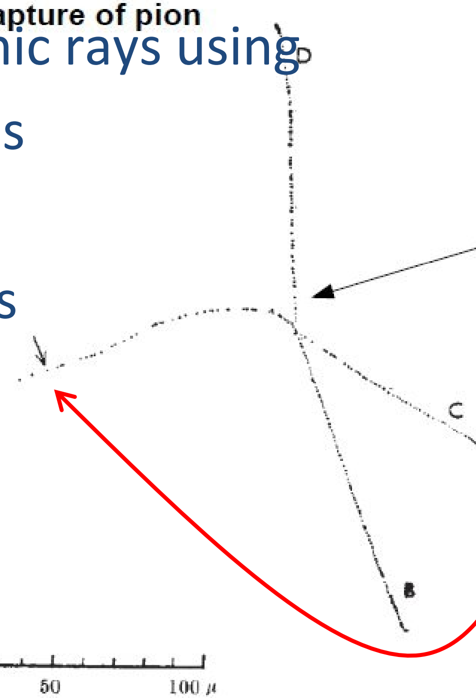
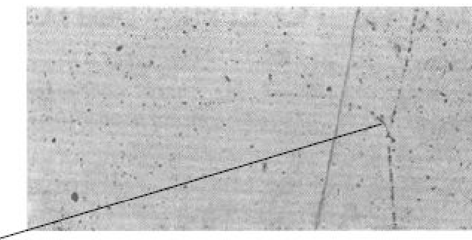


Fig. 1 *b*. TRACE OF COMPLETE STAR ON SCREEN OF PROJECTION MICROSCOPE, SHOWING PROJECTION OF THE TRACKS IN THE PLANE OF THE EMULSION. TRACK *A* CANNOT BE TRACED WITH CERTAINTY BEYOND THE ARROW



g. 1 *a*. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK OF ISON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSION OBJECTIVE. × 500)

- A is the new meson
- B,D,C are likely protons
- Track C goes into the page

Why A is a new meson:
electron: range too large
proton: scattering too large
muon: frequent nuclear interaction

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Nature's strong messenger - Pion

- The beginning of Particle Physics
- Then came
  - Disentanglement of confusion between $_{(1937)}$ muon and pion – similar masses
  - Discovery of particles with "strangeness" (e.g., kaon$_{1947\text{-}1953}$)
- Subsequently, a complete spectrum of mesons and baryons with mass below ≈1 GeV
  - 28 states
- Became clear that pion is "too light"

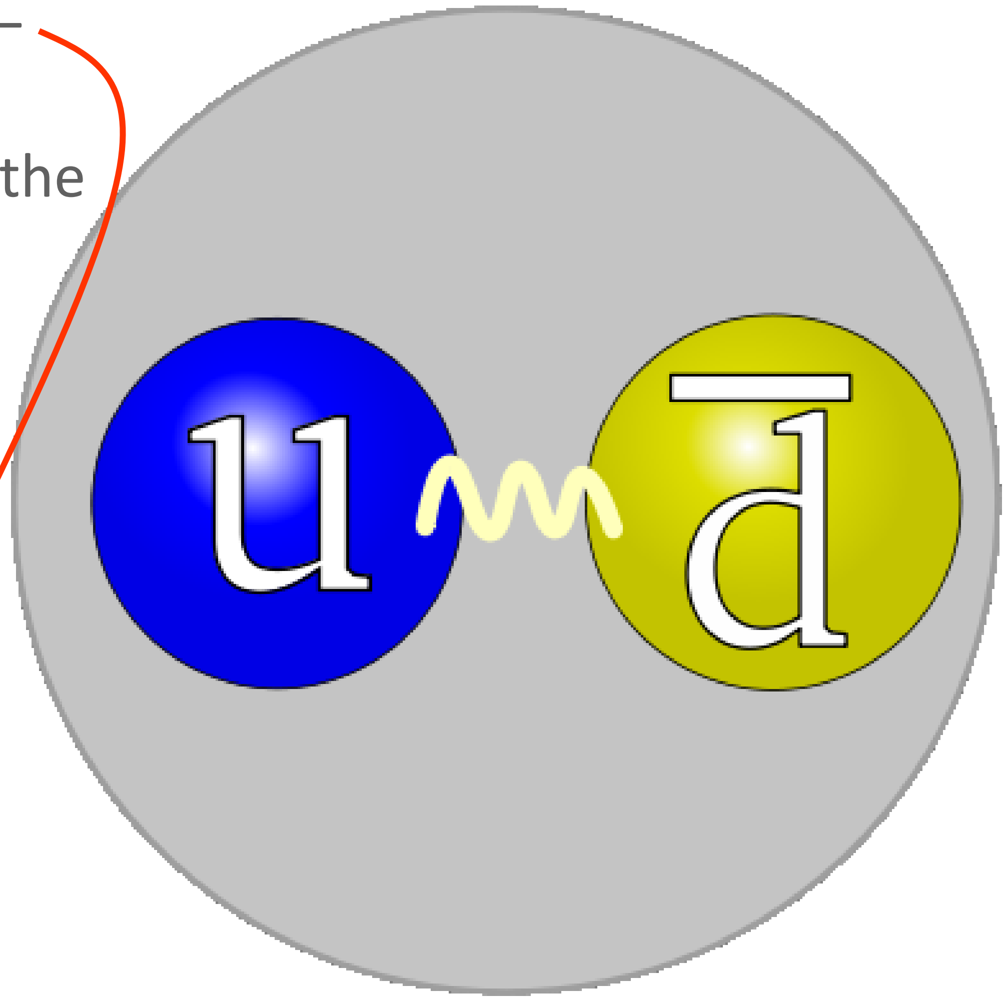
π	140 MeV
ρ	780 MeV
P	940 MeV

\- *hadrons* supposed to be heavy, yet …

Craig Roberts: Emergence of DSEs in Real-World QCD IB (87)

Simple picture - Pion

- Gell-Mann and Ne'eman:
  - Eightfold way$_{(1961)}$ – a picture based on group theory: *SU(3)*
  - Subsequently, quark model – where the *u*-, *d*-, *s*-quarks became the basis vectors in the fundamental representation of *SU(3)*
- Pion = Two *quantum-mechanical* constituent-quarks - particle+antiparticle - interacting via a *potential*



Some of the Light Mesons

LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

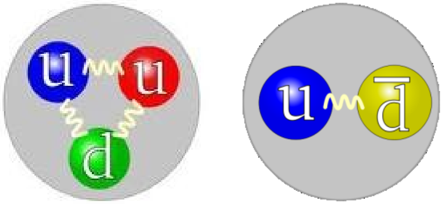
For $I=1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I=0$ (η, η', h, h', ω, φ, f, f'): $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

			$I^G(J^{PC})$
	$\pi^\pm$	140 MeV	$1^-(0^-)$
	π^0		$1^-(0^{-+})$
	η		$0^+(0^{-+})$
	$f_0(600)$ or σ		$0^+(0^{++})$
	$\rho(770)$	780 MeV	$1^+(1^{--})$
	$\omega(782)$		$0^-(1^{--})$
	$\eta'(958)$		$0^+(0^{-+})$
	$f_0(980)$		$0^+(0^{++})$
	$a_0(980)$		$1^-(0^{++})$
	$\varphi(1020)$		$0^-(1^{--})$
	$h_1(1170)$		$0^-(1^{+-})$
	$b_1(1235)$		$1^+(1^{+-})$
	$a_1(1260)$		$1^-(1^{++})$
	$f_2(1270)$		$0^+(2^{++})$
	$f_1(1285)$		$0^+(1^{++})$
	$\eta(1295)$		$0^+(0^{-+})$
	$\pi(1300)$		$1^-(0^{-+})$
	$a_2(1320)$		$1^-(2^{++})$
	$f_0(1370)$		$0^+(0^{++})$
•	$h_1(1380)$		$?^-(1^{+-})$
	$\pi_1(1400)$		$1^-(1^{-+})$
	$\eta(1405)$		$0^+(0^{-+})$
	$f_1(1420)$		$0^+(1^{++})$
	$\omega(1420)$		$0^-(1^{--})$
•	$f_2(1430)$		$0^+(2^{++})$
	$a_0(1450)$		$1^-(0^{++})$
	$\rho(1450)$		$1^+(1^{--})$

	$\eta(1475)$	$0^+(0^{-+})$
	$f_0(1500)$	$0^+(0^{++})$
•	$f_1(1510)$	$0^+(1^{++})$
	$f_2'(1525)$	$0^+(2^{++})$
•	$f_2(1565)$	$0^+(2^{++})$
•	$\rho(1570)$	$1^+(1^{--})$
•	$h_1(1595)$	$0^-(1^{+-})$
	$\pi_1(1600)$	$1^-(1^{-+})$
•	$a_1(1640)$	$1^-(1^{++})$
•	$f_2(1640)$	$0^+(2^{++})$
	$\eta_2(1645)$	$0^+(2^{-+})$
	$\omega(1650)$	$0^-(1^{--})$
	$\omega_3(1670)$	$0^-(3^{--})$
	$\pi_2(1670)$	$1^-(2^{-+})$
	$\varphi(1680)$	$0^-(1^{--})$
	$\rho_3(1690)$	$1^+(3^{--})$
	$\rho(1700)$	$1^+(1^{--})$
•	$a_2(1700)$	$1^-(2^{++})$
	$f_0(1710)$	$0^+(0^{++})$
•	$\eta(1760)$	$0^+(0^{-+})$
	$\pi(1800)$	$1^-(0^{-+})$
•	$f_2(1810)$	$0^+(2^{++})$
•	$X(1835)$	$?^?(?^{-+})$
	$\varphi_3(1850)$	$0^-(3^{--})$
•	$\eta_2(1870)$	$0^+(2^{-+})$
	$\pi_2(1880)$	$1^-(2^{-+})$
•	$\rho(1900)$	$1^+(1^{--})$

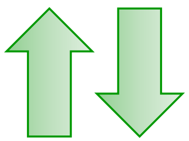
•	$f_2(1910)$	$0^+(2^{++})$
	$f_2(1950)$	$0^+(2^{++})$
•	$\rho_3(1990)$	$1^+(3^{--})$
	$f_2(2010)$	$0^+(2^{++})$
•	$f_0(2020)$	$0^+(0^{++})$
	$a_4(2040)$	$1^-(4^{++})$
	$f_4(2050)$	$0^+(4^{++})$
•	$\pi_2(2100)$	$1^-(2^{-+})$
•	$f_0(2100)$	$0^+(0^{++})$
•	$f_2(2150)$	$0^+(2^{++})$
•	$\rho(2150)$	$1^+(1^{--})$
	$\varphi(2170)$	$0^-(1^{--})$
•	$f_0(2200)$	$0^+(0^{++})$
•	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
•	$\eta(2225)$	$0^+(0^{-+})$
•	$\rho_3(2250)$	$1^+(3^{--})$
	$f_2(2300)$	$0^+(2^{++})$
•	$f_4(2300)$	$0^+(4^{++})$
•	$f_0(2330)$	$0^+(0^{++})$
	$f_2(2340)$	$0^+(2^{++})$
•	$\rho_5(2350)$	$1^+(5^{--})$
•	$a_6(2450)$	$1^-(6^{++})$
•	$f_6(2510)$	$0^+(6^{++})$

• — OMITTED FROM SUMMARY TABLE

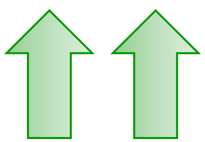


Modern Miracles in Hadron Physics

- proton = three constituent quarks
  - $M_{proton} \approx 1\text{GeV}$
  - Therefore guess $M_{constituent-quark} \approx ⅓ \times \text{GeV} \approx 350\text{MeV}$
- pion = constituent quark + constituent antiquark



  - Guess $M_{pion} \approx ⅔ \times M_{proton} \approx 700\text{MeV}$
- WRONG . . . . $M_{pion} = 140\text{MeV}$
- Rho-meson



  - Also *constituent quark + constituent antiquark*
    – just pion with spin of one constituent flipped
  - $M_{rho} \approx 770\text{MeV} \approx 2 \times M_{constituent-quark}$

What is "wrong" with the pion?

Dichotomy of the pion

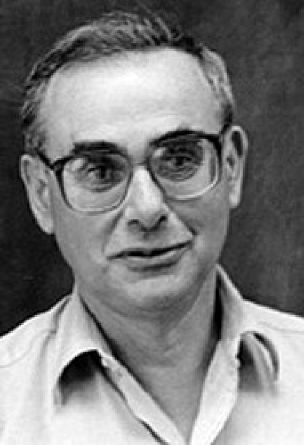
- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless

  – but some are still making this mistake

- However: $m_{\pi}^{2} \propto \imath$

  current-algebra (1968)

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-\ tate} \propto \imath_{constituent}$



Dichotomy of the pion
Goldstone mode and bound-state

- The *correct understanding* of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
  - well-defined and valid chiral limit;
  - and an accurate realisation of dynamical chiral symmetry breaking.

HIGHLY NONTRIVIAL

Impossible in quantum mechanics

Only possible in asymptotically-free gauge theories

Chiral QCD

- Action, in terms of local Lagrangian density:

$$S[G^a_\mu,\bar q,q] = \int d^4x\,\{\frac{1}{4}G^a_{\mu\nu}(x)G^a_{\mu\nu}(x)+\frac{1}{2\xi}\partial_\mu G^a_\mu(x)\,\partial_\nu G^a_\nu(x)+\bar q(x)[\gamma_\mu D_\mu+M]q(x)\}$$

  - Chromomagnetic Field Strength Tensor: $\partial_\mu G^a_\nu(x) - \partial_\nu G^a_\mu(x) + g f^{abc} G^b_\mu(x) G^c_\nu(x)$
  - Covariant Derivative: $D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}G^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \dots \\ 0 & m_d & 0 & \dots \\ 0 & 0 & m_s & \dots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

$m_t = 40{,}000\ m_u$
Why?

- Understanding Hadron Physics means knowing all that this Action predicts.

- Current-quark masses
  - External paramaters in QCD
  - Generated by the Higgs boson, within the Standard Model
  - Raises more questions than it answers

Chiral Symmetry

- Interacting gauge theories, in which it makes sense to speak of massless fermions, have a nonperturbative chiral symmetry
- A related concept is *Helicity,* which is the projection of a particle's spin, *J*, onto it's direction of motion:

λ c • ɔ

- For a massless particle, helicity is a Lorentz-invariant *spin-observable* $\lambda = \pm$; i.e., it's parallel or antiparallel to the direction of motion
  - Obvious:
    - massless particles travel at speed of light
    - hence no observer can overtake the particle and thereby view its momentum as having changed sign

Chiral Symmetry

- Chirality operator is γ_5
  - Chiral transformation: $\Psi(x) \to \exp(i\,\gamma_5\,\vartheta)\,\Psi(x)$
  - Chiral rotation through $\vartheta = \frac{1}{4}\pi$
    - Composite particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to the operation of parity conjugation
- *Therefore, a prediction of chiral symmetry is the existence of degenerate parity partners in the theory's spectrum*

Chiral Symmetry

- Perturbative QCD: *u*- & *d*- quarks are very light

  $m_u/m_d \approx 0.5$ & $m_d \approx 4$ MeV

  (a generation of high-energy experiments)

  H. Leutwyler, [*0911.1416 [hep-ph]*]

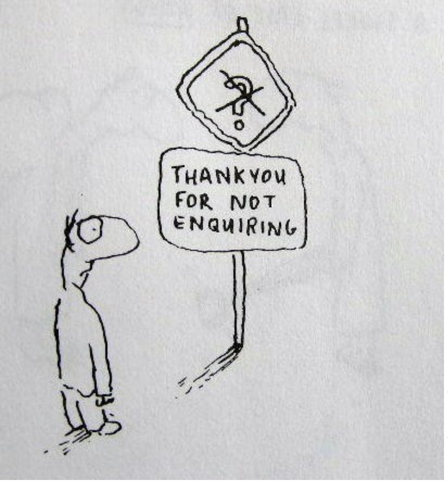
- However, splitting between parity partners is greater-than 100-times this mass-scale; e.g.,

J^P	$\frac{1}{2}^+$ (p)	$\frac{1}{2}^-$
Mass	940 MeV	1535 MeV

Dynamical
Chiral Symmetry Breaking

- Something is happening in QCD
  - some inherent dynamical effect is dramatically changing the pattern by which the Lagrangian's chiral symmetry is expressed
- Qualitatively different from spontaneous symmetry breaking *aka* the Higgs mechanism
  - *Nothing is added to the theory*
  - *Have only fermions & gauge-bosons*

  Yet, the mass-operator generated by the theory produces a spectrum with no sign of chiral symmetry



QCD's Challenges

Understand emergent phenomena

- *Quark and Gluon Confinement*
  No matter how hard one strikes the proton,
  one cannot liberate an individual quark or gluon
- *Dynamical Chiral Symmetry Breaking*
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  . . . no degeneracy between $J^P=+$ and $J^P=-$ *(parity partners)*
- *Neither of these phenomena is apparent in QCD's Lagrangian*
  Yet they are the dominant determining characteristics of real-world QCD.
- QCD
  – Complex behaviour arises from apparently simple rules.

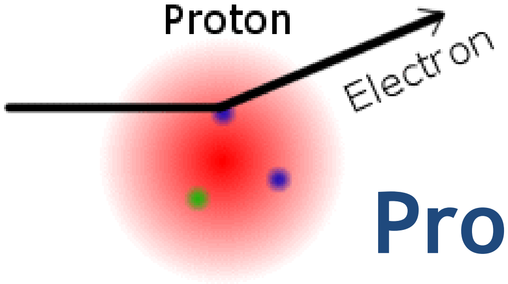
The study of nonperturbative QCD is the puriew of …

Hadron Physics

Nucleon … Two Key Hadrons Proton and Neutron

- Fermions – two static properties:
  proton electric charge = +1; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933;
  Stern awarded Nobel Prize (1943): *"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"*.
- Dirac (1928) – pointlike fermion: $$\mu_p = \frac{e\hbar}{2M}$$
- Stern (1933) – $$\mu_p = (1 + 1.79)\frac{e\hbar}{2M}$$
- Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
- Quark discovery via e-p-scattering at SLAC in 1968
  - the elementary quanta of QCD

Friedman, Kendall, Taylor, Nobel Prize (1990): "for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"



Nucleon Structure Probed in scattering experiments

- Electron is a good probe because it is structureless

  Electron's relativistic current is

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\quad Q = P' - P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$

Structureless fermion, or simply-structured fermion, $F_1=1$ & $F_2=0$, so that $G_E=G_M$ and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$\begin{aligned} J_\mu(P',P) &= ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,, \\ &= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P) \end{aligned}$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

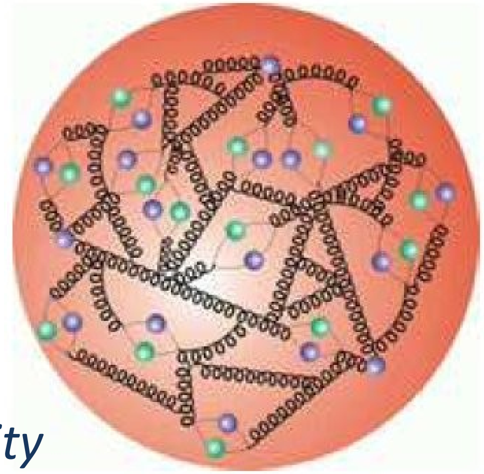
G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magnetic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

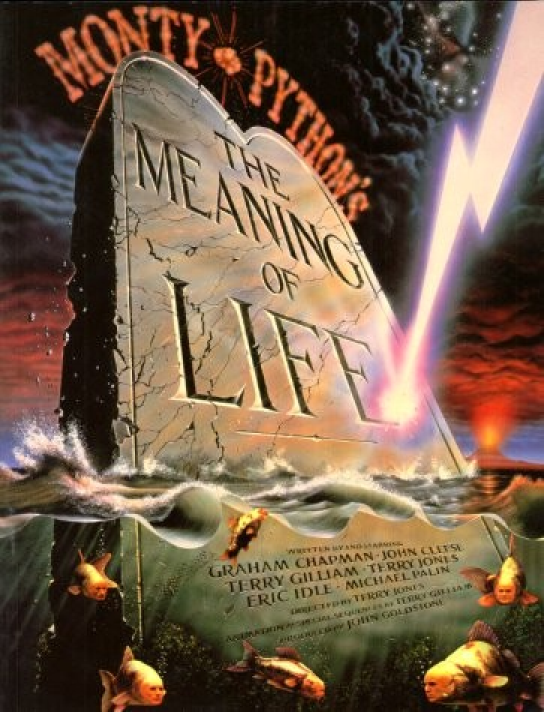
Nuclear Science Advisory Council Long Range Plan

- A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD
- So, what's the problem?

  They are legion …
  - *Confinement*
  - *Dynamical chiral symmetry breaking*
  - *A fundamental theory of unprecedented complexity*



- QCD defines the difference between nuclear and particle physicists:
  - Nuclear physicists try to solve this theory
  - Particle physicists run away to a place where tree-level computations are all that's necessary; *perturbation theory is the last refuge of a scoundrel*



Understanding NSAC's Long Range Plan

- *What are the quarks and gluons of QCD?*
- *Is there such a thing as a constituent quark, a constituent-gluon?*

  After all, these are the concepts for which Gell-Mann won the Nobel Prize.

- *Do they – can they – correspond to well-defined quasi-particle degrees-of-freedom?*
- *If not, with what should they be replaced?*

What is the meaning of the NSAC Challenge?

Recall the dichotomy of the pion

- How does one make an almost massless particle from two massive constituent-quarks?
- One can always tune a potential in quantum mechanics so that the ground-state is massless

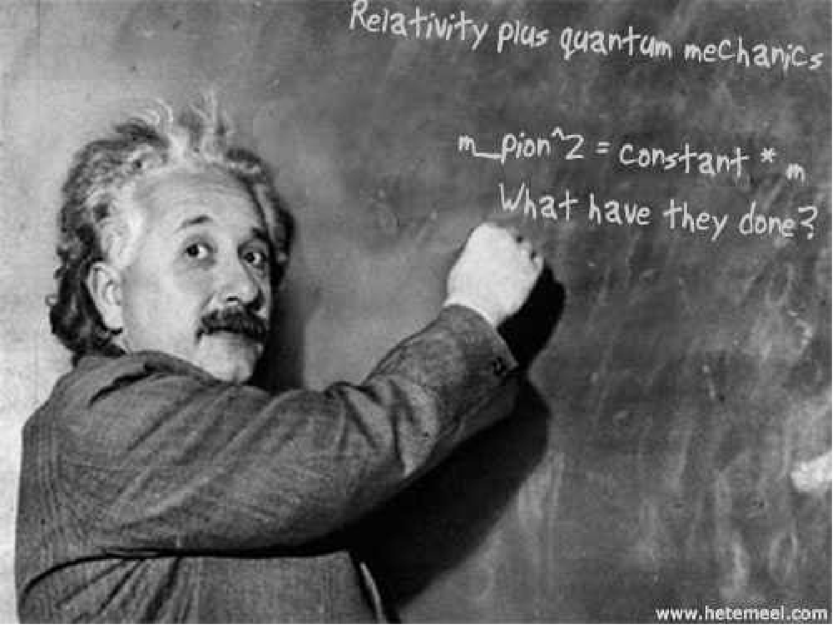
  – and some are still making this mistake

- However: $m_{\pi}^{2} \propto \iota$

  current-algebra (1968)

Models based on constituent-quarks cannot produce this outcome. They must be fine tuned in order to produce the empirical splitting between the π & ρ mesons

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-\ tate} \propto \iota_{constituent}$



What is the meaning of all this?

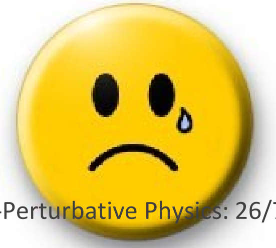
If m_π=m_ρ, then repulsive and attractive forces in the Nucleon-Nucleon potential have the SAME range and there is NO intermediate range attraction.

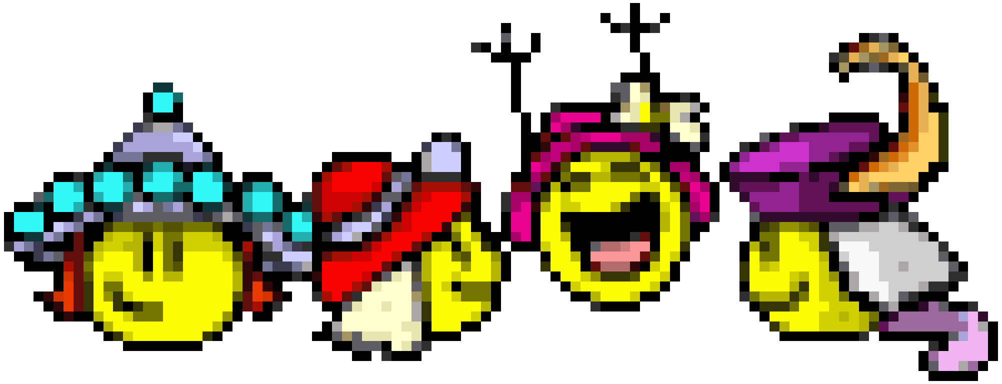
Under these circumstances:

- *Can ^{12}C be produced, can it be stable?*
- *Is the deuteron stable; can Big-Bang Nucleosynthesis occur?*

(Many more existential questions …)

Probably not … but it wouldn't matter because we wouldn't be around to worry about it.

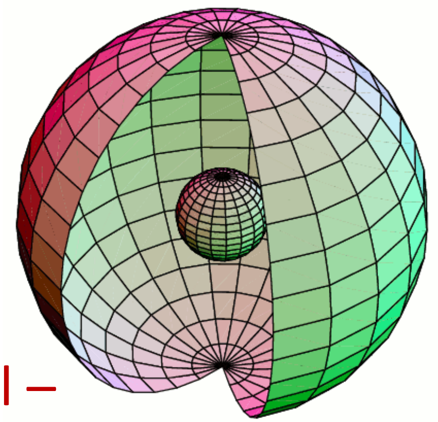




Why don’t we just stop talking and solve the problem?

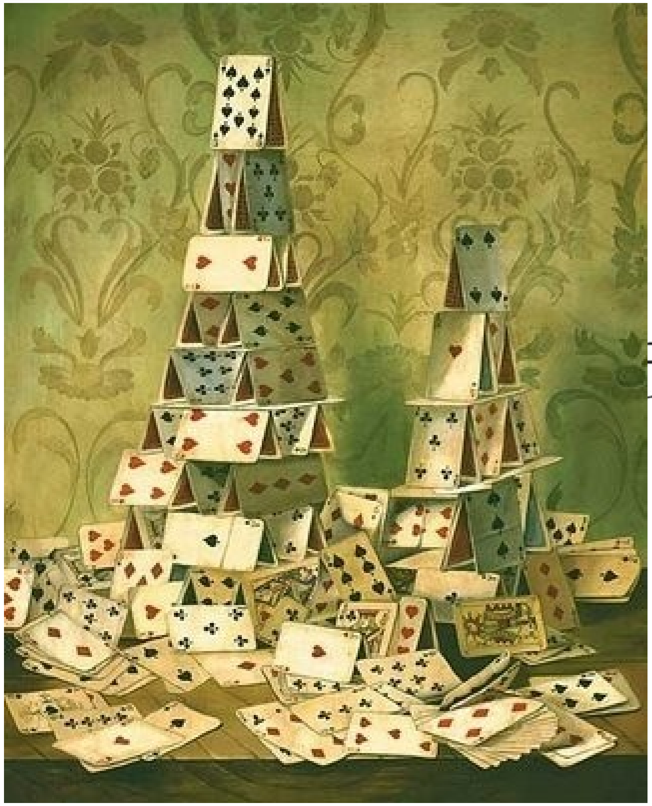
Just get on with it!

- But … QCD's emergent phenomena can't be studied using perturbation theory
  - *So what? Same is true of bound-state problems in quantum mechanics!*
- Differences:
  - Here relativistic effects are crucial – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential –
    Unknown throughout > 98% of the pion's/proton's volume!
- Understanding requires *ab initio* nonperturbative solution of fully-fledged interacting relativistic quantum field theory, something which Mathematics and Theoretical Physics are a long way from achieving.

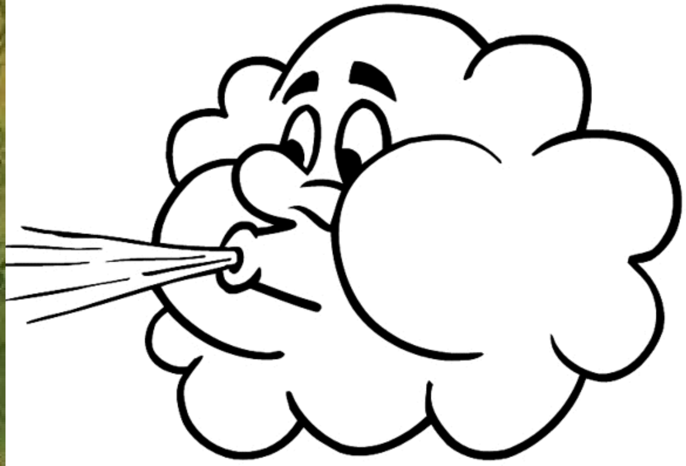


How can we tackle the SM's Strongly-interacting piece?

The Traditional Approach



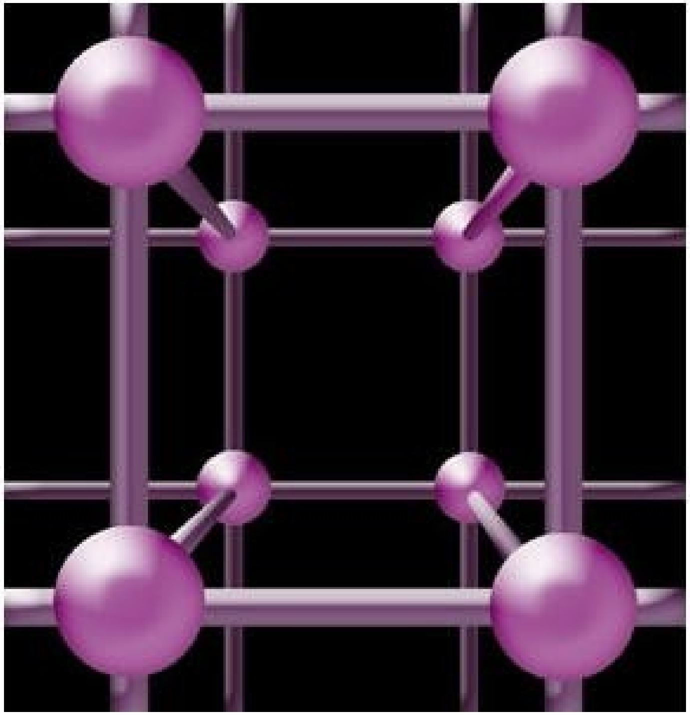
– Modelling



– has its problems.

How can we tackle the SM's Strongly-interacting piece?

Lattice-QCD

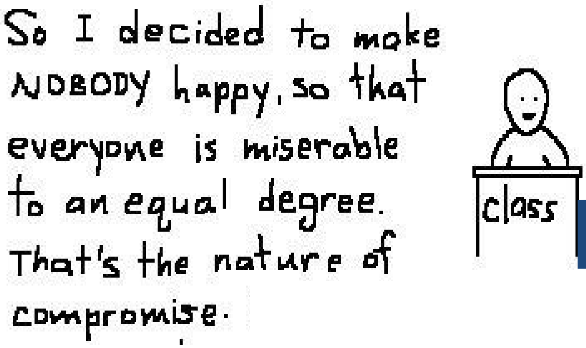


– Spacetime becomes an hypercubic lattice
– Computational challenge, many millions of degrees of freedom

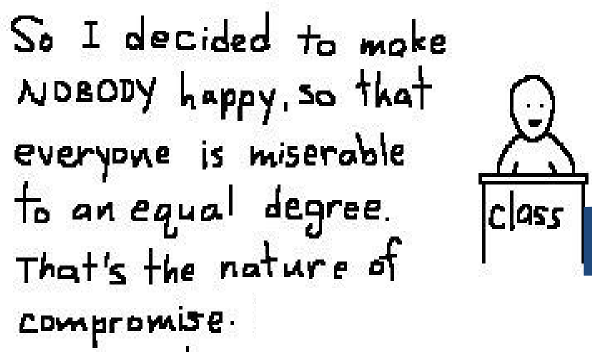
How can we tackle the SM's Strongly-interacting piece?

Lattice-QCD

- Spacetime becomes an hypercubic lattice
- Computational challenge, many millions of degrees of freedom
- *Approximately 500 people worldwide & 20-30 people per collaboration.*

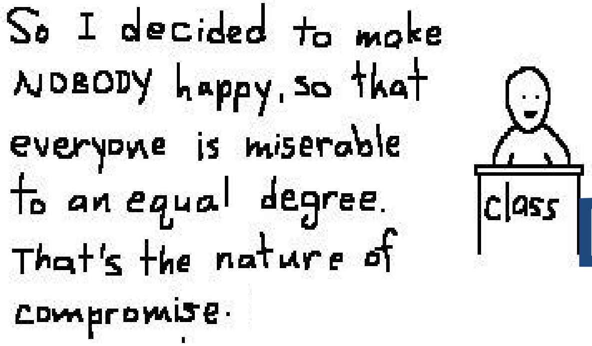


A Compromise?
Dyson-Schwinger Equations



A Compromise? Dyson-Schwinger Equations

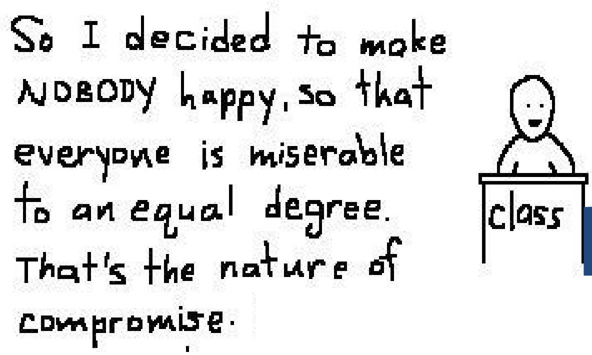
- 1994 . . . *"As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . . ."*



A Compromise? Dyson-Schwinger Equations

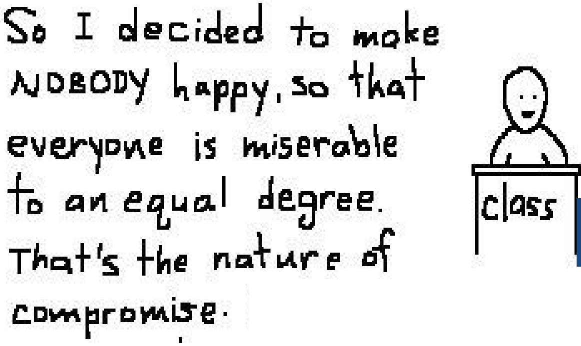
- 1994 . . . *"However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF (VA) and RHIC (NY), there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . . "*

Craig Roberts: Emergence of DSEs in Real-World QCD IB (87)



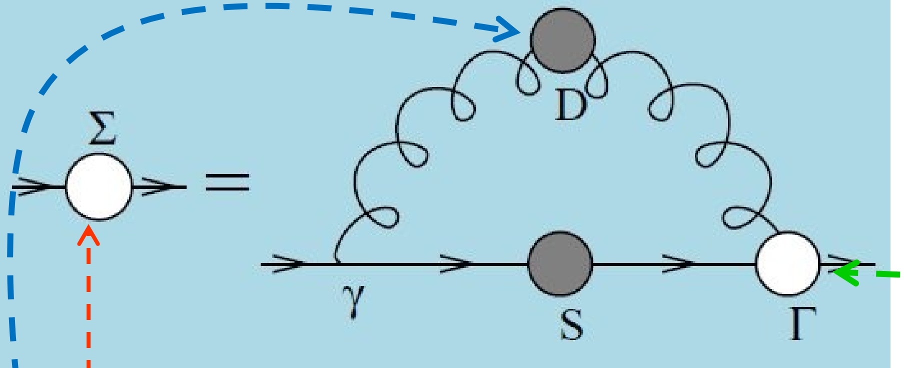
A Compromise? Dyson-Schwinger Equations

- 1994 . . . *"Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD."*



A Compromise? Dyson-Schwinger Equations

- 1994 . . . *"Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD."*
- C. D. Roberts and ~~A. G. Williams~~, "Dyson-Schwinger equations and their application to hadronic physics,"
  Prog. Part. Nucl. Phys. **33**, 477 (1994) [arXiv:hep-ph/9403224].
  (555 citations)



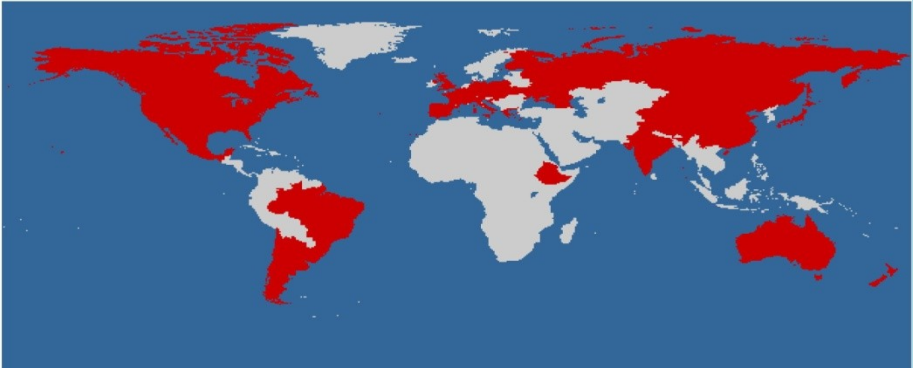
A Compromise? DSEs

- Dyson (1949) & Schwinger (1951) . . . One can derive a system of coupled integral equations relating all the Green functions for a theory, one to another.

  Gap equation:
  - fermion self energy
  - gauge-boson propagator
  - fermion-gauge-boson vertex

$$S(p) = \frac{1}{i\gamma \cdot p + \Sigma(p)}$$

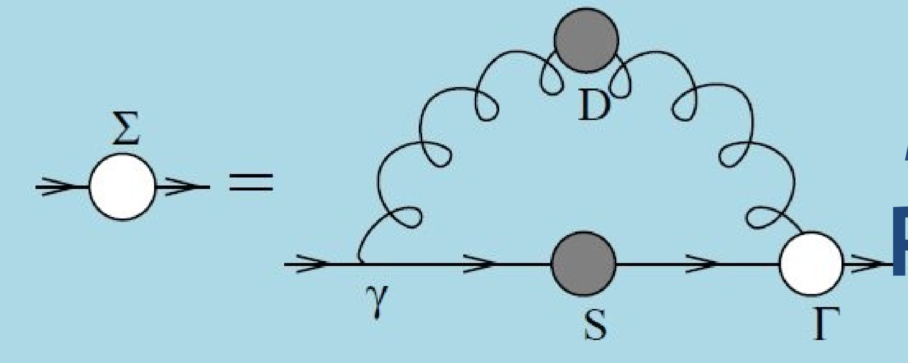
- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.



Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling



Mass from Nothing?! Perturbation Theory

- QCD is asymptotically-free (2004 Nobel Prize)
  - Chiral-limit is well-defined;
    i.e., one can truly speak of a massless quark.
  - NB. This is nonperturbatively *impossible* in QED.
- Dressed-quark propagator: $$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$
- Weak coupling expansion of
  gap equation yields every diagram in perturbation theory
- In perturbation theory:
  If *m=0*, then *M(p²)=0*
  Start with no mass,
  Always have no mass.

$$M(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{\Lambda^2}\right] + \ldots\right)$$

0

Dynamical Chiral Symmetry Breaking

Nambu—Jona-Lasinio Model

- Recall the gap equation

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+ \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\, \gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p)$$

NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\, \frac{1}{m_G^2}\, \theta(\Lambda^2 - \ell^2)$$

- Model is not renormalisable
  $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- NJL gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2)$$
$$= \quad i\gamma\cdot p + m + \frac{4}{3}\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\, \theta(\Lambda^2 - \ell^2)\, \gamma_\mu \frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\, \gamma_\mu$$

Nambu—Jona-Lasinio Model

- Multiply the NJL gap equation by $(-i\gamma\cdot p)$; trace over Dirac indices:

$$p^2\, A(p^2) = p^2 + \frac{8}{3}\,\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2 - \ell^2)\, p\cdot\ell\, \frac{A(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}$$

  - Angular integral vanishes, therefore $A(p^2) = 1$.
  - This owes to the fact that the NJL model is defined by a four-fermion contact-interaction in configuration space, which entails a momentum-independent interaction in momentum space.
- Simply take Dirac trace of NJL gap equation:

$$B(p^2) = m + \frac{16}{3}\,\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2 - \ell^2)\, \frac{B(\ell^2)}{\ell^2 + B^2(\ell^2)}$$

  - Integrand is p^2-independent, therefore the only solution is $B(p^2)$ = constant = M.
- General form of the propagator for a fermion dressed by the NJL interaction: $S(p) = 1/[\, i\,\gamma\cdot p + M\,]$

Critical coupling for dynamical mass generation?

NJL model & a mass gap?

- Evaluate the integrals

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2,\Lambda^2),$$
$$\mathcal{C}(M^2,\Lambda^2) = \Lambda^2 - M^2 \ln\left[1+\Lambda^2/M^2\right].$$

- *Λ* defines the model's mass-scale. Henceforth set *Λ = 1*, then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2,1)$$

- Chiral limit, *m=0*
  - Solutions?
    - One is obvious; viz., *M=0*
      This is the *perturbative result*

    … start with no mass, end up with no mass

- Chiral limit, *m=0*
  - Suppose, on the other hand that *M≠0*, and thus may be cancelled
    - This nontrivial solution can exist if-and-only-if one may satisfy

$$3\pi^2\, m_G^2 = \mathcal{C}(M^2,1)$$

Critical coupling for dynamical mass generation!

NJL model & a mass gap?

- Can one satisfy $3\pi^2 m_G^2 = C(M^2,1)$?
  - $C(M^2, 1) = 1 - M^2 \ln [1 + 1/M^2]$
    - Monotonically decreasing function of M
    - Maximum value at $M = 0$; viz., $C(M^2=0, 1) = 1$
- Consequently, there is a solution *iff* $3\pi^2 m_G^2 < 1$
  - Typical scale for hadron physics: $\Lambda = 1$ GeV
    - There is a M≠0 solution iff $m_G^2 < (\Lambda/(3\pi^2)) = (0.2\ \text{GeV})^2$
- Interaction strength is proportional to $1/m_G^2$
  - Hence, if interaction is strong enough, then one can start with no mass but end up with a massive, perhaps very massive fermion

Dynamical Chiral Symmetry Breaking

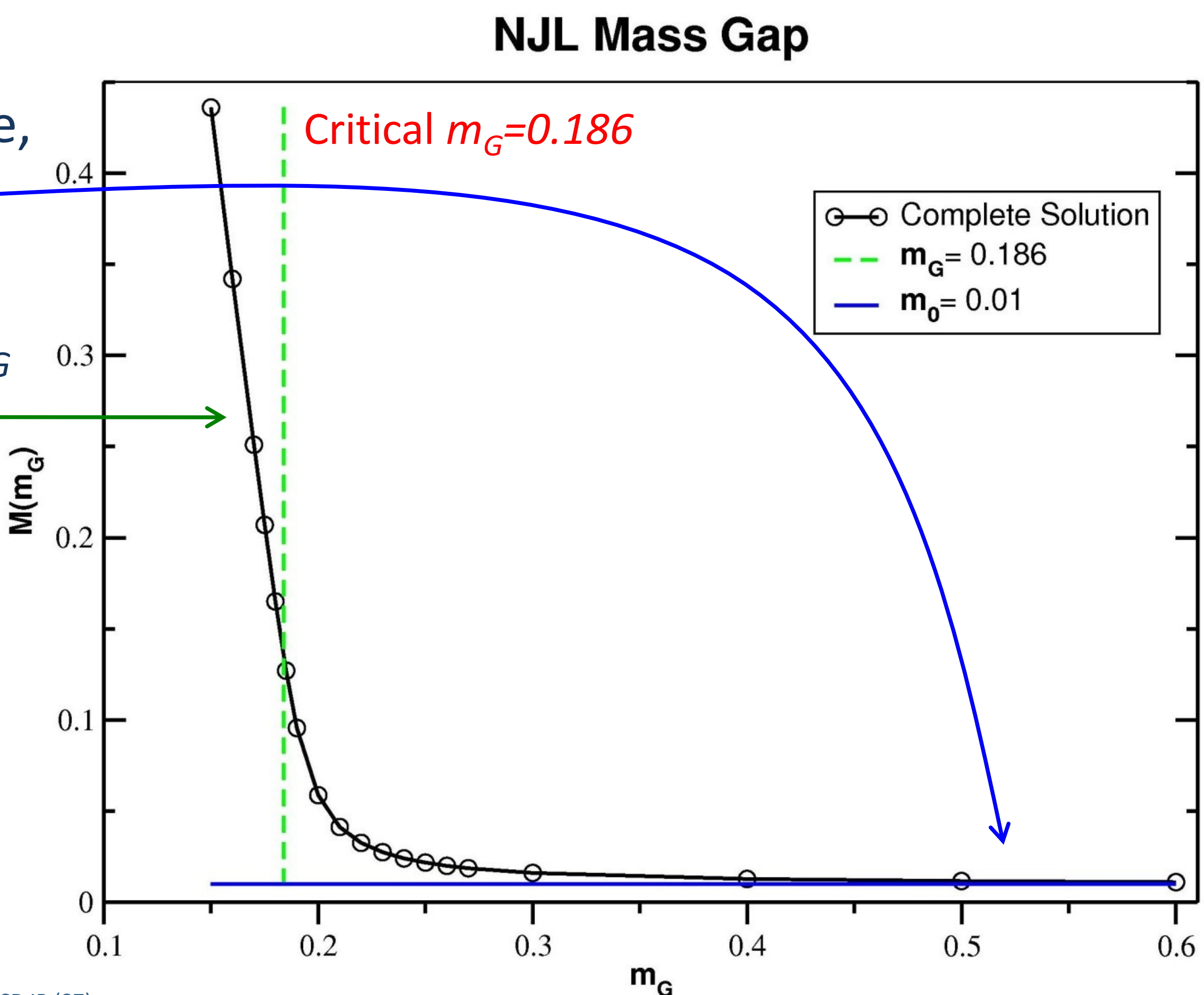
Solution of gap equation

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

NJL Model Dynamical Mass

- Weak coupling corresponds to m_G large, in which case $M \approx m$
- On the other hand, strong coupling; i.e., m_G small, $M >> m$

This is the defining characteristic of dynamical chiral symmetry breaking



NJL Model and Confinement?

- **Confinement**: no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1] + [B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2}$$

- This is merely a free-particle-like propagator with a shifted mass

  $p^2 + M^2 = 0 \rightarrow$ Minkowski-space mass = M

- Hence, whilst **NJL model** exhibits dynamical chiral symmetry breaking it **does not confine**.

NJL-fermion still propagates as a plane wave

Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983),
  "The Ground State q-q.bar Mass Spectrum In QCD,"
  Phys. Rev. ***D 28, 181.***

Antithesis of NJL model; viz.,
Delta-function in momentum space
NOT in configuration space.

In this case, G sets the mass scale

- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \frac{\lambda^a}{2};$

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4\, G\, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]$$

- MN Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,\gamma_\mu$$

MN Model's Gap Equation

- The gap equation yields the following pair of coupled, algebraic equations (set *G = 1* GeV^2)

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$

$$\boxed{B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,,}$$

- Consider the chiral limit form of the equation for $B(p^2)$
  - Obviously, one has the trivial solution $B(p^2) = 0$
  - However, is there another?

MN model and DCSB

- The existence of a $B(p^2) \neq 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

- Substituting this result into the equation for $A(p^2)$ one finds

  $$A(p^2) - 1 = \tfrac{1}{2} A(p^2) \rightarrow A(p^2) = 2,$$

  which in turn entails

  $$B(p^2) = 2\,(\,1 - p^2\,)^{1/2}$$

- Physical requirement: quark self-energy is real on the domain of spacelike momenta → complete chiral limit solution

$$A(p^2) = \begin{cases} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1 + \sqrt{1 + 8/p^2}\right)\,; & p^2 > 1 \end{cases}$$

$$B(p^2) = \begin{cases} \sqrt{1 - p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{cases}$$

NB. Self energies are momentum-dependent because the interaction is momentum-dependent. Should expect the same in QCD.

MN Model and Confinement?

- Solution we've found is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta
- Examine the propagator's denominator

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

  This is greater-than zero for all p^2 …
  - There are no zeros
  - So, the *propagator has no pole*
- This is nothing like a free-particle propagator.
  It can be interpreted as describing a **confined degree-of-freedom**
- Note that, in addition there is no critical coupling:

  The nontrivial solution exists so long as $G > 0$.
- Conjecture: ***All confining theories exhibit DCSB***
  - NJL model demonstrates that converse is not true.

Massive solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\,B(p^2)}{m + B(p^2)}\,,$$

$$B(p^2) = m + \frac{4\,[m + B(p^2)]^2}{B(p^2)([m + B(p^2)]^2 + 4p^2)}\,.$$

- Second line is a quartic equation for $B(p^2)$.

  Can be solved algebraically with four solutions, available in a closed form.

- Only one solution has the correct $p^2 \to \infty$ limit; viz.,

  $B(p^2) \to m$.

  This is the *unique physical* solution.

- NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

Munczek-Nemirovsky Dynamical Mass

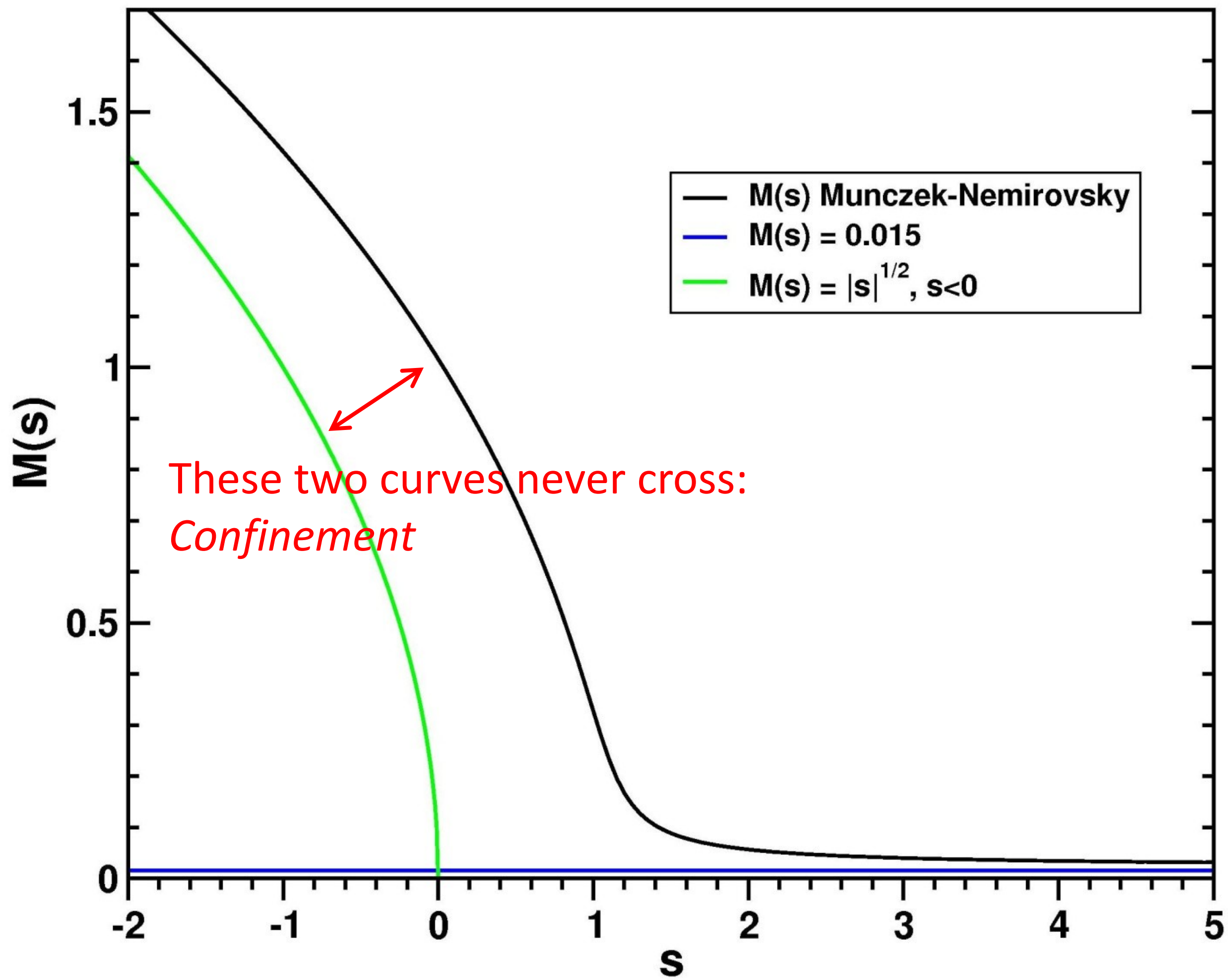
- Large-s: $M(s) \sim m$
- Small-s: $M(s) \gg m$

  This is the essential characteristic of DCSB
- We will see that p^2-dependent mass-functions are a quintessential feature of QCD.
- No solution of

  $$s + M(s)^2 = 0$$

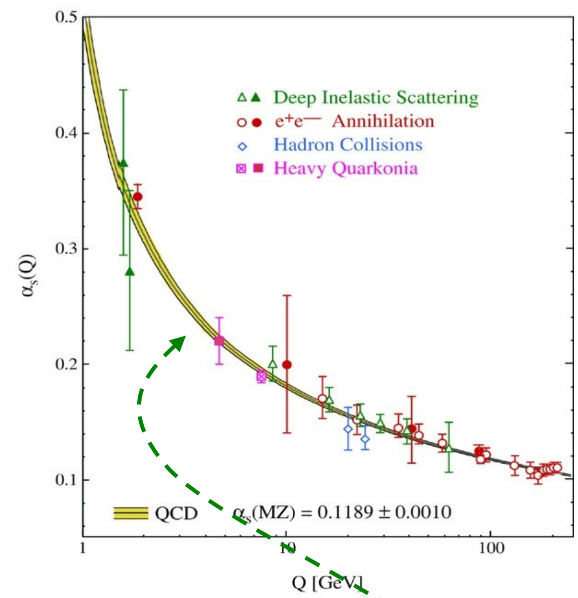
  → *No plane-wave propagation*

  Confinement?!

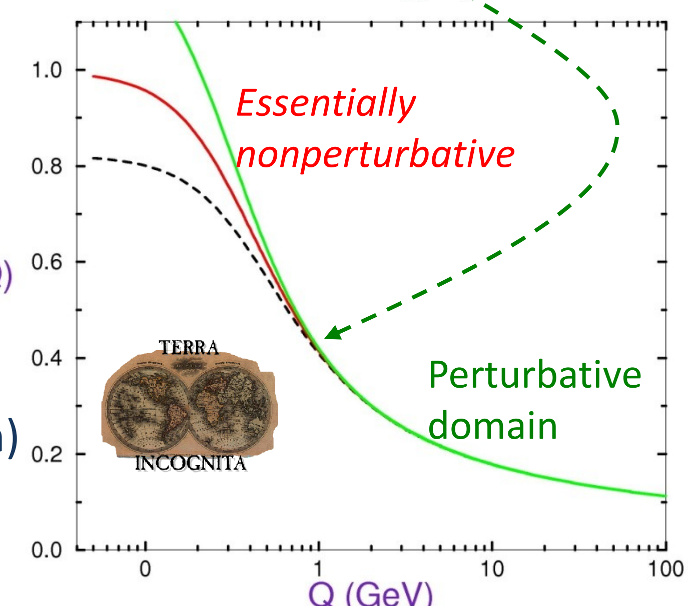


What happens in the real world?

- Strong-interaction: QCD
  - Asymptotically free
    - Perturbation theory is valid and accurate tool at large-Q^2 & hence chiral limit is defined
  - Essentially nonperturbative for $Q^2 < 2$ GeV^2
    - *Nature's only example of truly nonperturbative, fundamental theory*
    - *A-priori, no idea as to what such a theory can produce*



- Possibilities?
  - G(0) < 1: M(s) ≡ 0 is only solution for m = 0.
  - G(0) ≥ 1: M(s) ≠ 0 is possible and energetically favoured: DCSB.
  - M(0) ≠ 0 is a new, dynamically generated mass-scale. If it's large enough, can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.



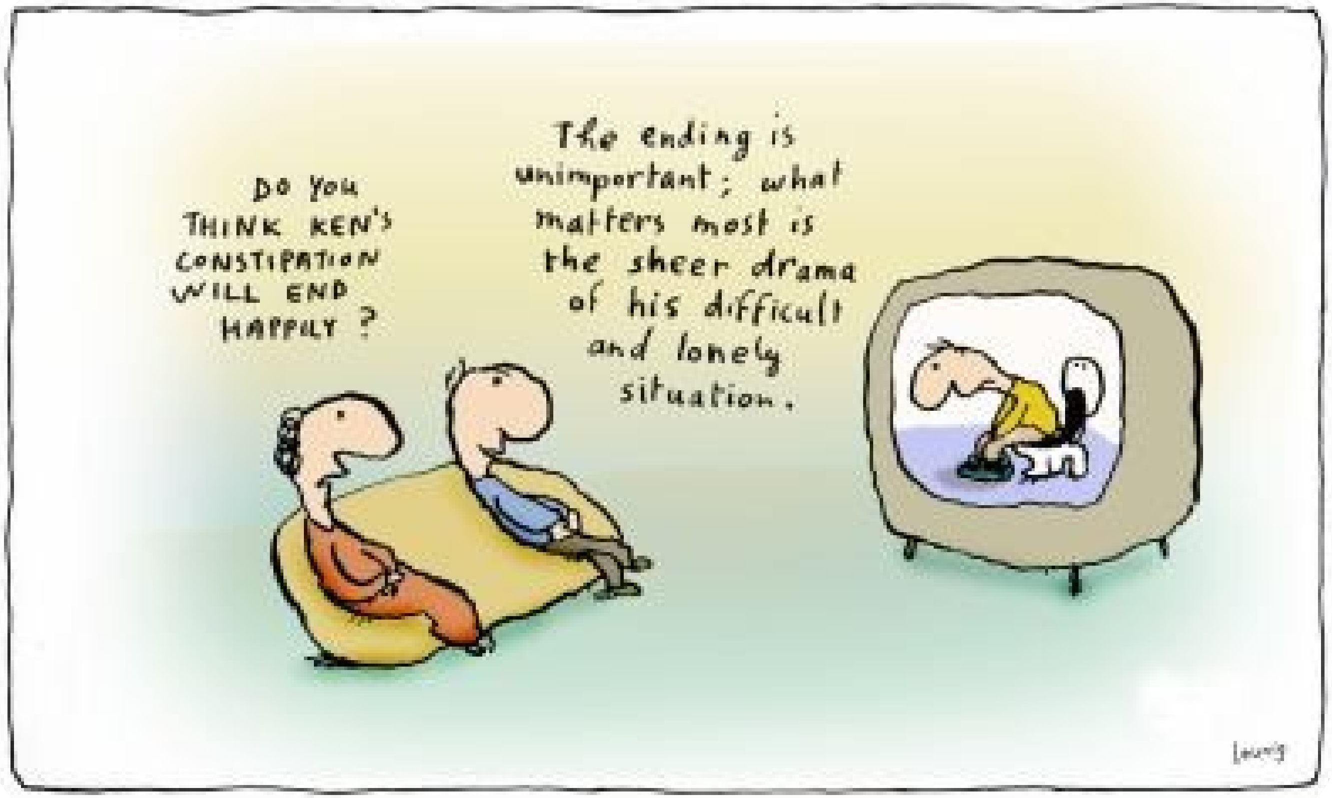

IF YOU SEE ANYTHING MYSTERIOUS OR UNUSUAL, JUST ENJOY IT WHILE YOU CAN
Leunig

Big Picture

Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that
- Confinement and DCSB are expressed in QCD's propagators and vertices
  - Nonperturbative modifications should have observable consequences
- Dyson-Schwinger Equations are a useful analytical and numerical tool for nonperturbative study of relativistic quantum field theory
- Simple models (NJL) can exhibit DCSB
  - DCSB $\not\Rightarrow$ Confinement
- Simple models (MN) can exhibit Confinement
  - Confinement $\Rightarrow$ DCSB

What's the story in QCD?



Confinement

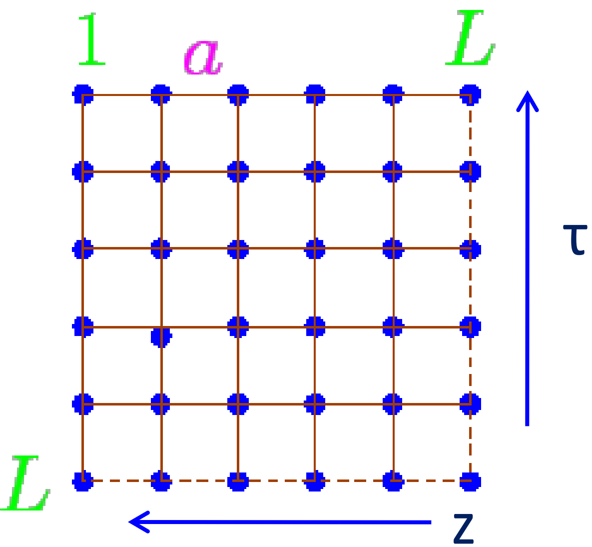
Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

Wilson Loop & the Area Law

$$W_C := \mathrm{Tr}\left(\mathcal{P}\exp i \oint_C A_\mu dx^\mu\right)$$



periodic lattice

- *C* is a closed curve in space,
  P is the path order operator
- Now, place static (infinitely heavy) fermionic sources of colour charge at positions

  $z_0=0$ & $z=½L$
- Then, evaluate $<W_C(z, \tau)>$ as a functional integral over gauge-field configurations
- In the strong-coupling limit, the result can be obtained algebraically; viz.,

  $<W_C(z, \tau)> = \exp(-V(z)\, \tau)$

  where $V(z)$ is the potential between the static sources, which behaves as $V(z) = \sigma z$

Linear potential
σ = String tension

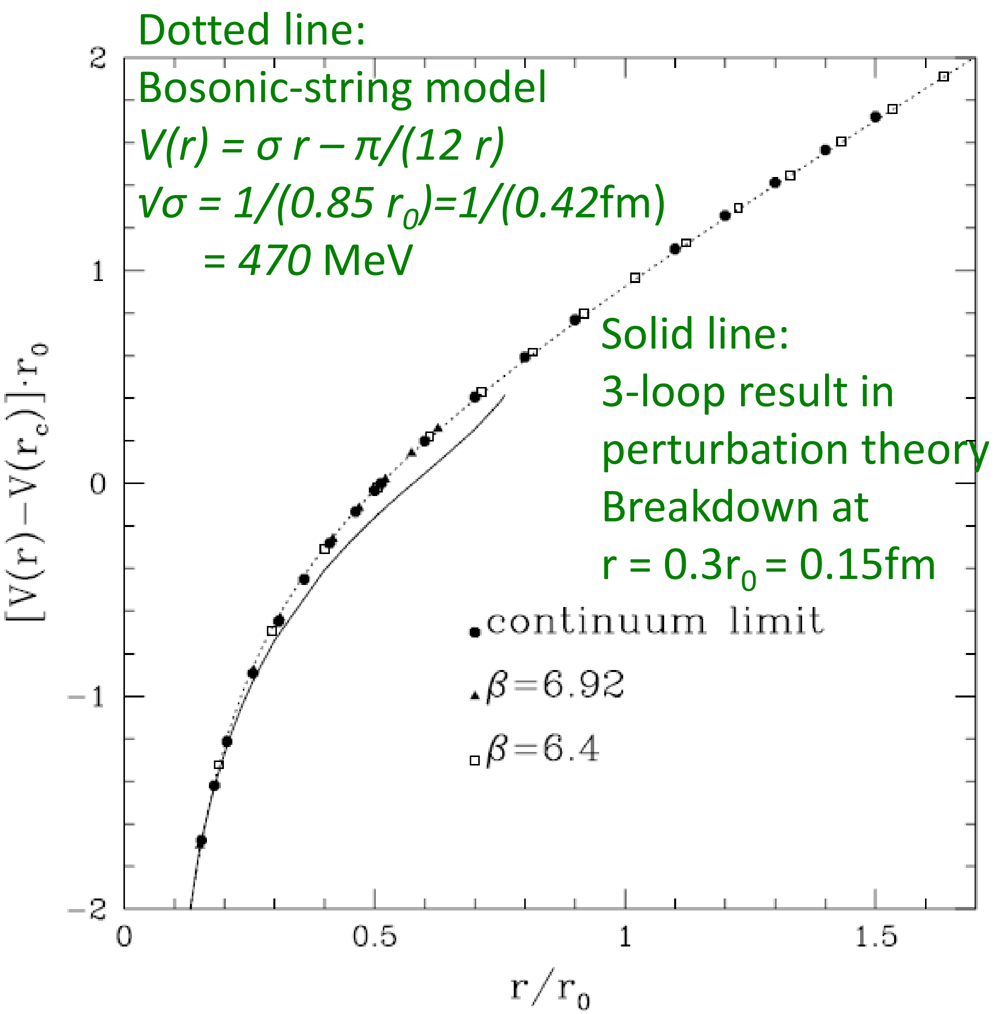
Wilson Loop & Area Law

- Typical result from a numerical simulation of pure-glue QCD (hep-lat/0108008)
- r_0 is the Sommer-parameter, which relates to the force between static quarks at intermediate distances.
- The requirement

  $$r_0^2\, F(r_0) = 1.65$$

  provides a connection between pure-glue QCD and potential models for mesons, and produces

  $$r_0 \approx 0.5 \text{ fm}$$



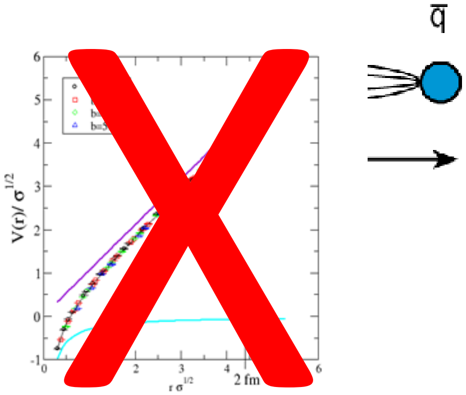
Flux Tube Models of Hadron Structure

- Illustration in terms of Action – density, which is analogous to plotting the force:

  $F(r) = \sigma - (\pi/12)(1/r^2)$

- It is pretty hard to overlook the flux tube between the static source and sink
- *Phenomenologists embedded in quantum mechanics and string theorists have been nourished by this result for many, many years.*

BUT … the Real World has light quarks … what then?!



Confinement

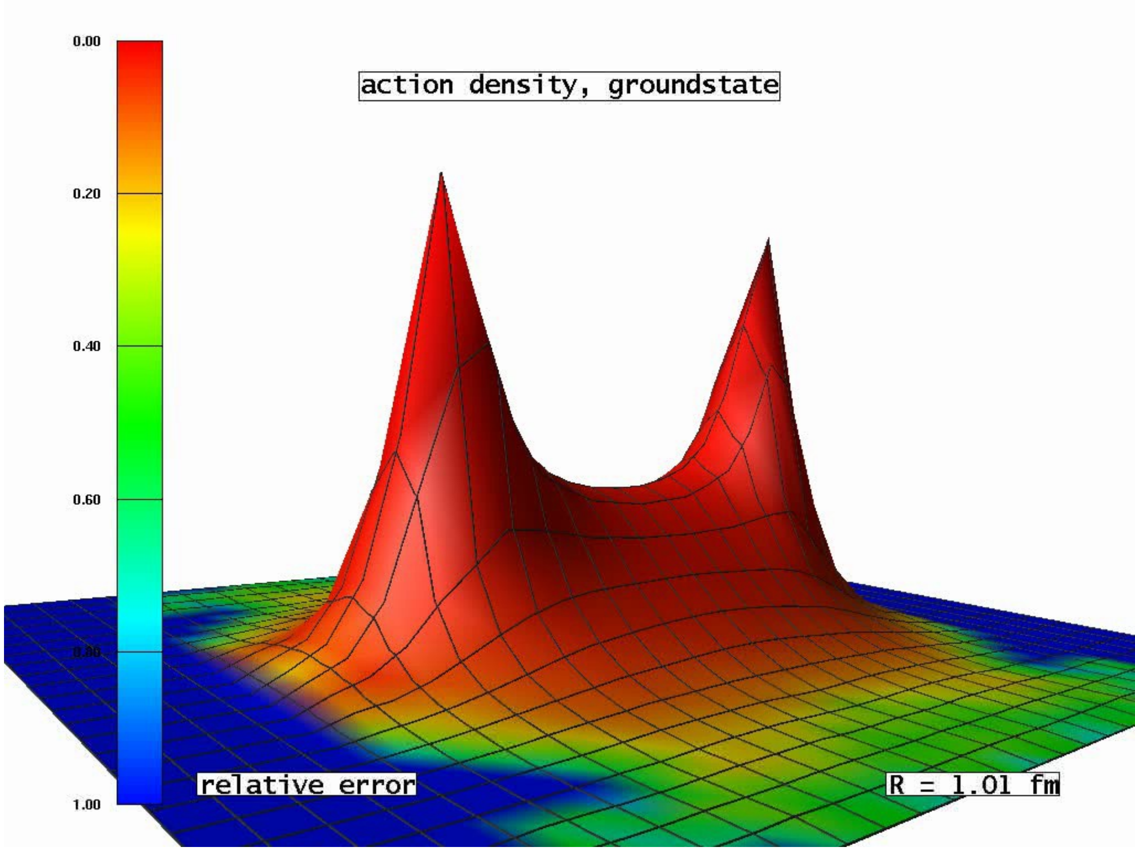
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, or any other hadron, one cannot liberate an individual quark or gluon
- Empirical fact. However
  - There is no agreed, theoretical definition of light-quark confinement
  - Static-quark confinement is irrelevant to real-world QCD
    - *There are no long-lived, very-massive quarks*
- Confinement entails *quark-hadron duality*; i.e., that all observable consequences of QCD can, in principle, be computed using an hadronic basis.

G. Bali et al., [*PoS LAT2005 (2006) 308*]

Confinement

"Note that the time is not a linear function of the distance but dilated within the string breaking region. On a linear time scale string breaking takes place rather rapidly. […] light pair creation seems to occur non-localized and instantaneously."

- Infinitely heavy-quarks *plus* 2 flavours with mass = m_s
  - Lattice spacing = 0.083fm
  - String collapses within one lattice time-step *R = 1.24 … 1.32* fm
  - Energy stored in string at collapse $E_c^{sb} = 2\, m_s$
  - (*mpg made via linear interpolation*)
- *No flux tube between light-quarks*



B_s anti-B_s

1993: *"for elucidating the quantum structure of electroweak interactions in physics"*

Regge Trajectories?

- Martinus Veltmann, "Facts and Mysteries in Elementary Particle Physics" (World Scientific, Singapore, 2003):

  In time the Regge trajectories thus became the cradle of string theory. Nowadays the Regge trajectories have largely disappeared, not in the least because these higher spin bound states are hard to find experimentally. At the peak of the Regge fashion (around 1970) theoretical physics produced many papers containing families of Regge trajectories, with the various (hypothetically straight) lines based on one or two points only!

Properties of Regge trajectories

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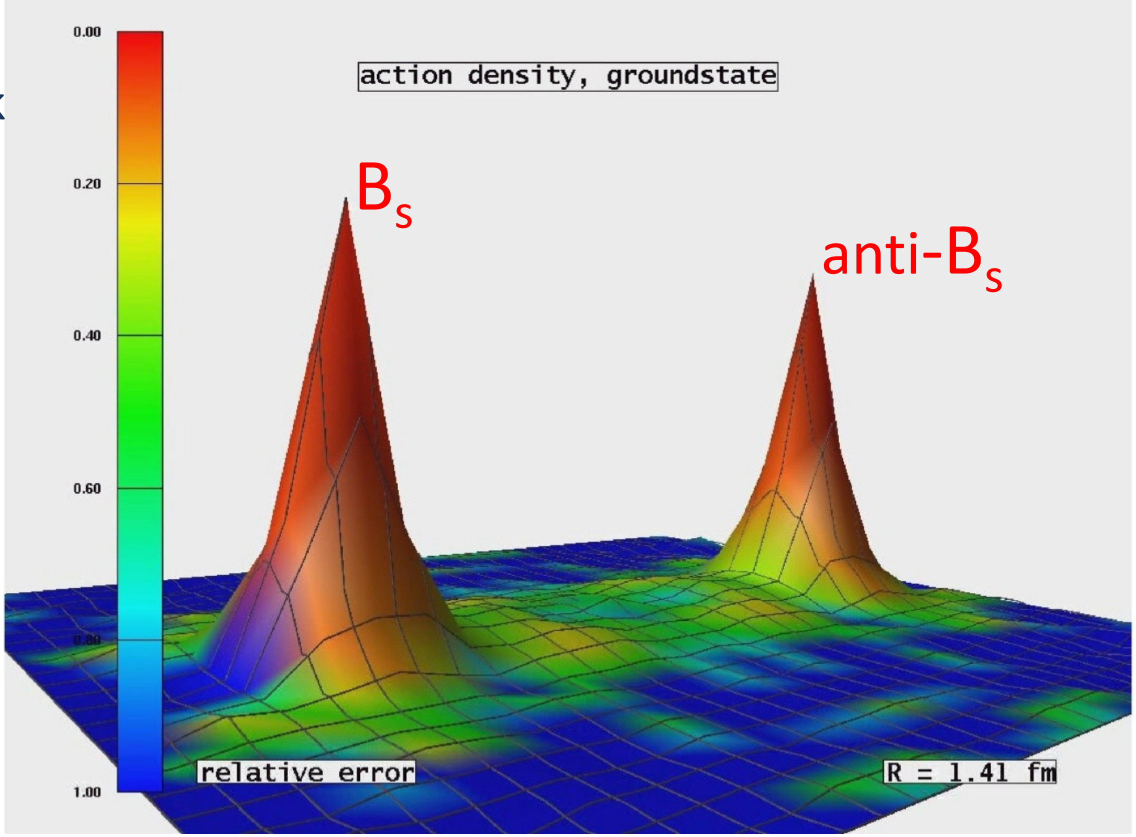
(Received 30 November 1999; published 8 June 2000)

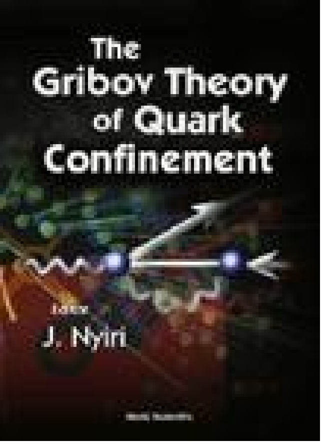
Early Chew-Frautschi plots show that meson and baryon Regge trajectories are approximately linear and non-intersecting. In this paper, we reconstruct all Regge trajectories from the most recent data. Our plots show that meson trajectories are non-linear and intersecting. We also show that all current meson Regge trajectories models are ruled out by data.

PACS number(s): 11.55.Jy, 12.40.Nn, 14.20.−c, 14.40.−n Phys.Rev. D **62** (2000) 016006 [9 pages]

Confinement

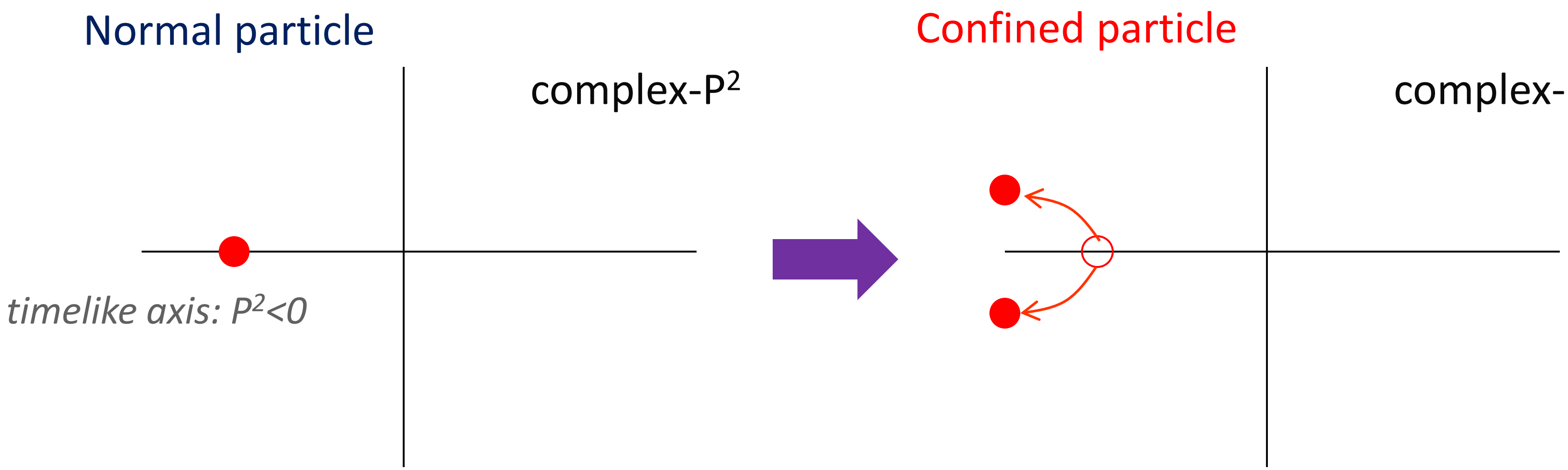
- Static-quark confinement is irrelevant to real-world QCD
  - *There are no long-lived, very-massive quarks*
- Indeed, potential models are irrelevant to light-quark physics, something which should have been plain from the start: *copious production of light particle-antiparticle pairs ensures that a potential model description is meaningless for light-quarks in QCD*





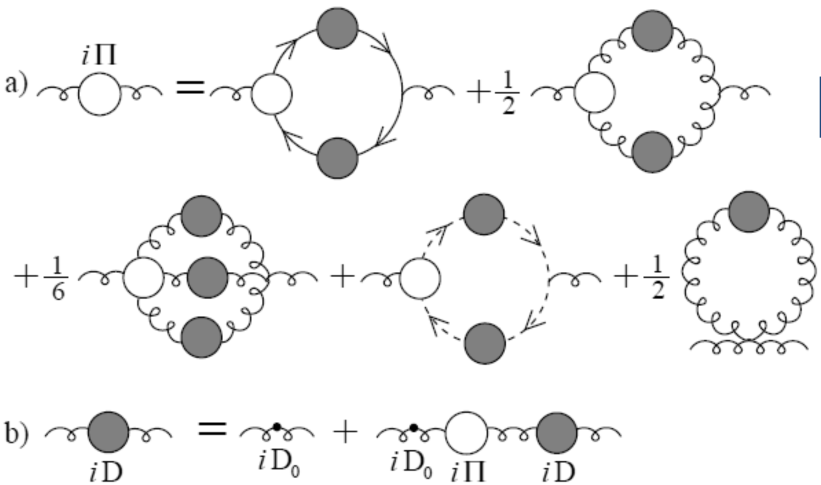
Confinement

- Confinement is expressed through a *violent* change in the analytic structure of propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator
  - Gribov (1978); Munczek (1983); Stingl (1984); Cahill (1989); Krein, Roberts & Williams (1992); Tandy (1994); …



- *Real-axis mass-pole splits, moving into pair(s) of complex conjugate poles or branch points*
- *Spectral density no longer positive semidefinite*

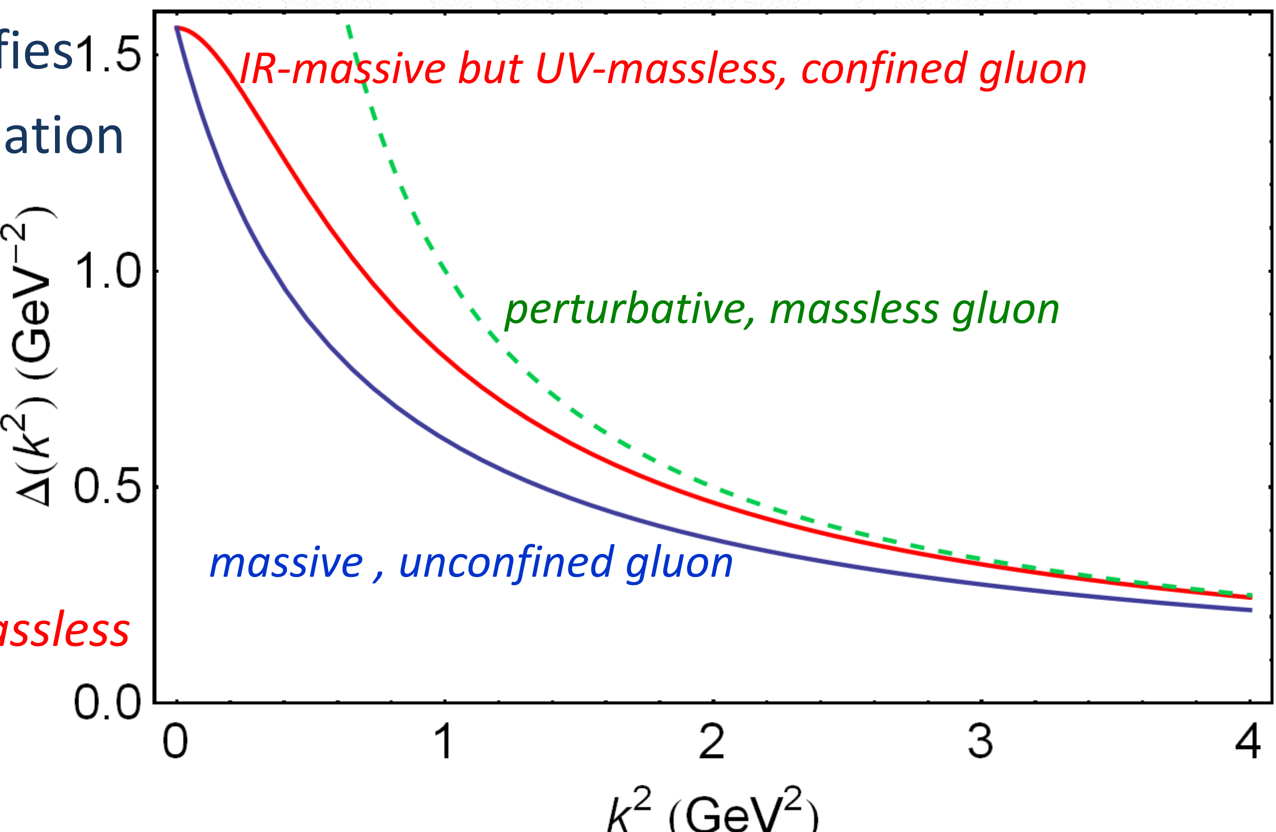
  & hence state cannot exist in observable spectrum



Dressed-gluon propagator

A.C. Aguilar et al., [*Phys.Rev. D80 (2009) 085018*]

- Gluon propagator satisfies a Dyson-Schwinger Equation
- Plausible possibilities for the solution
- DSE and lattice-QCD agree on the result
  - *Confined gluon*
  - *IR-massive but UV-massless*
  - *$m_G \approx$ 2-4 Λ_{QCD}*



Charting the interaction between light-quarks

This is a well-posed problem whose solution is an elemental goal of modern hadron physics. The answer provides QCD's running coupling.

- Confinement can be related to the analytic properties of QCD's Schwinger functions.
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
  - Of course, the behaviour of the β-function on the perturbative domain is well known.

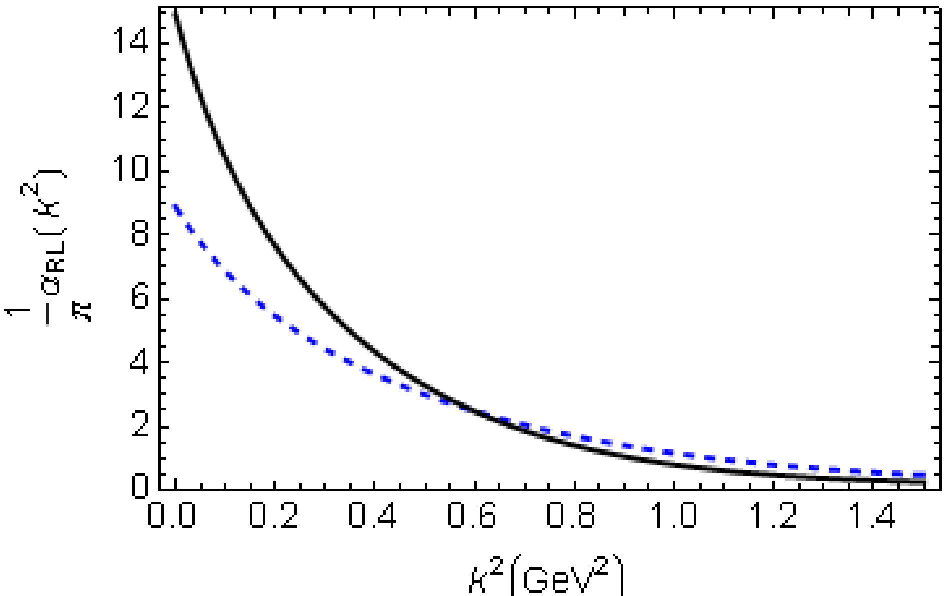
Charting the interaction between light-quarks

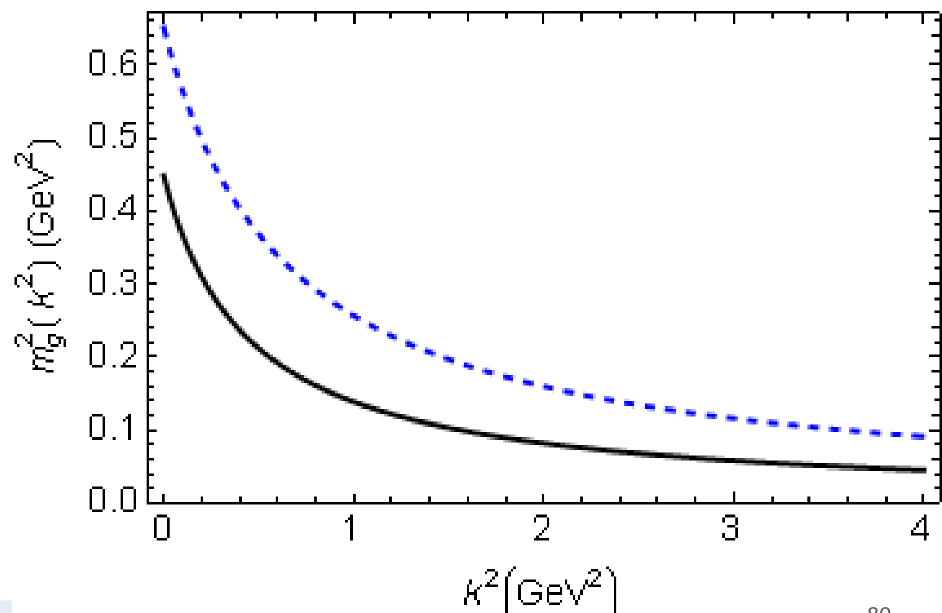
- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines the pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors
  - Parton distribution functions

  can be used to chart β-function’s long-range behaviour.
- Extant studies show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of the β-function than those of the ground states.

DSE Studies - Phenomenology of gluon

- Wide-ranging study of π & ρ properties
- Effective coupling
  - Agrees with pQCD in ultraviolet
  - Saturates in infrared
    - $\alpha(0)/\pi$ = 8-15
    - $\alpha(m_G^2)/\pi$ = 2-4
- Running gluon mass
  - Gluon is massless in ultraviolet in agreement with pQCD
  - Massive in infrared
    - $m_G(0)$ = 0.67-0.81 GeV
    - $m_G(m_G^2)$ = 0.53-0.64 GeV



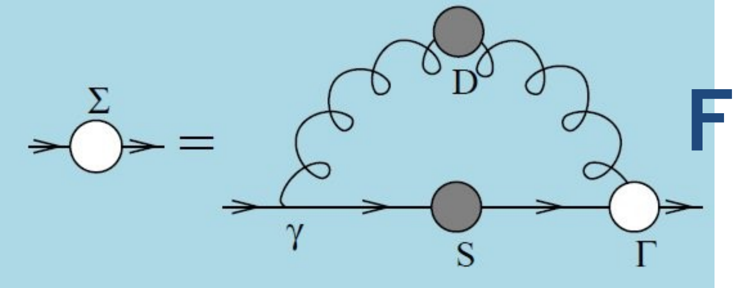


Dynamical Chiral Symmetry Breaking
Mass Gap

Dynamical Chiral Symmetry Breaking

- Strong-interaction: QCD
- Confinement
  - Empirical feature
  - Modern theory and lattice-QCD support conjecture
    - that light-quark confinement is a fact
    - associated with violation of reflection positivity; i.e., novel analytic structure for propagators and vertices
  - Still circumstantial, no proof yet of confinement
- On the other hand, *DCSB is a fact in QCD*
  - It is the most important mass generating mechanism for visible matter in the Universe.

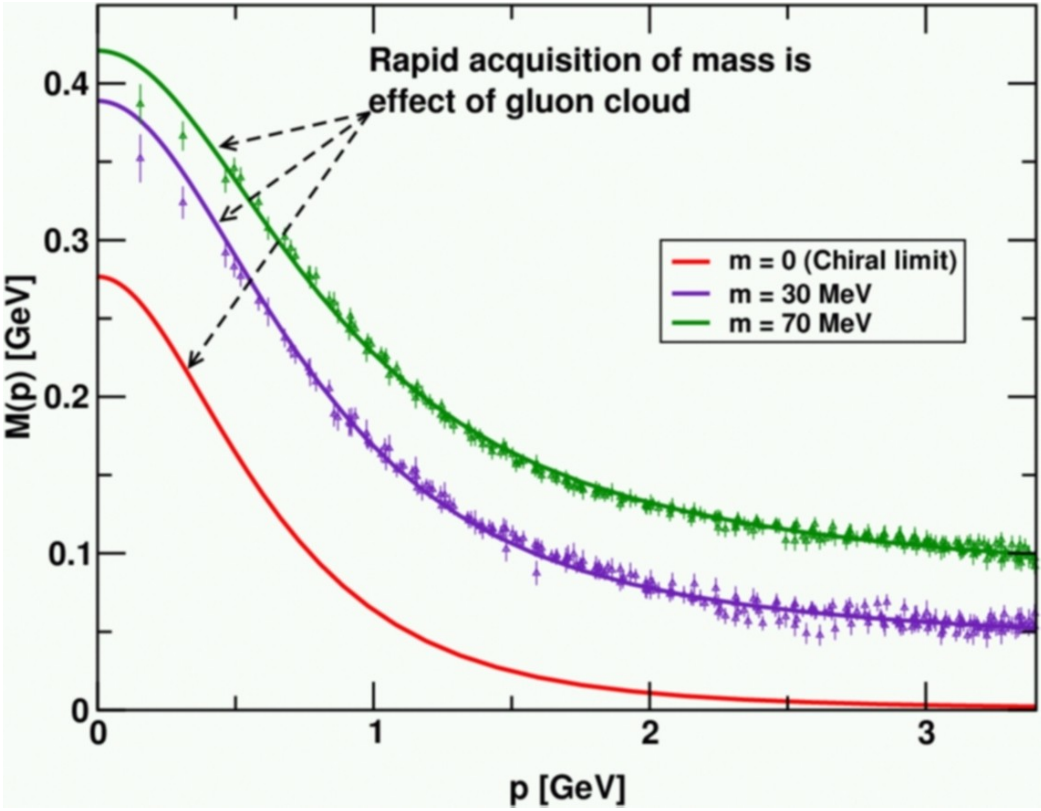
Responsible for approximately 98% of the proton's mass.
Higgs mechanism is (*almost*) irrelevant to light-quarks.



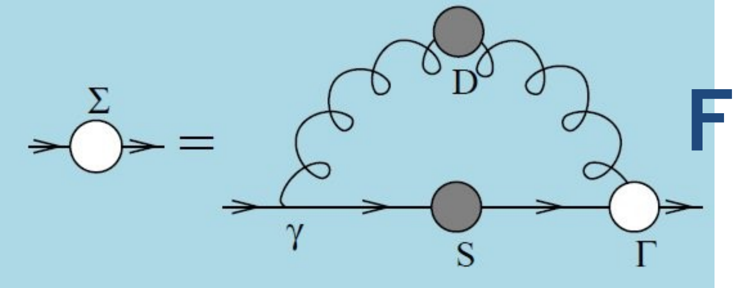
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50
M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227



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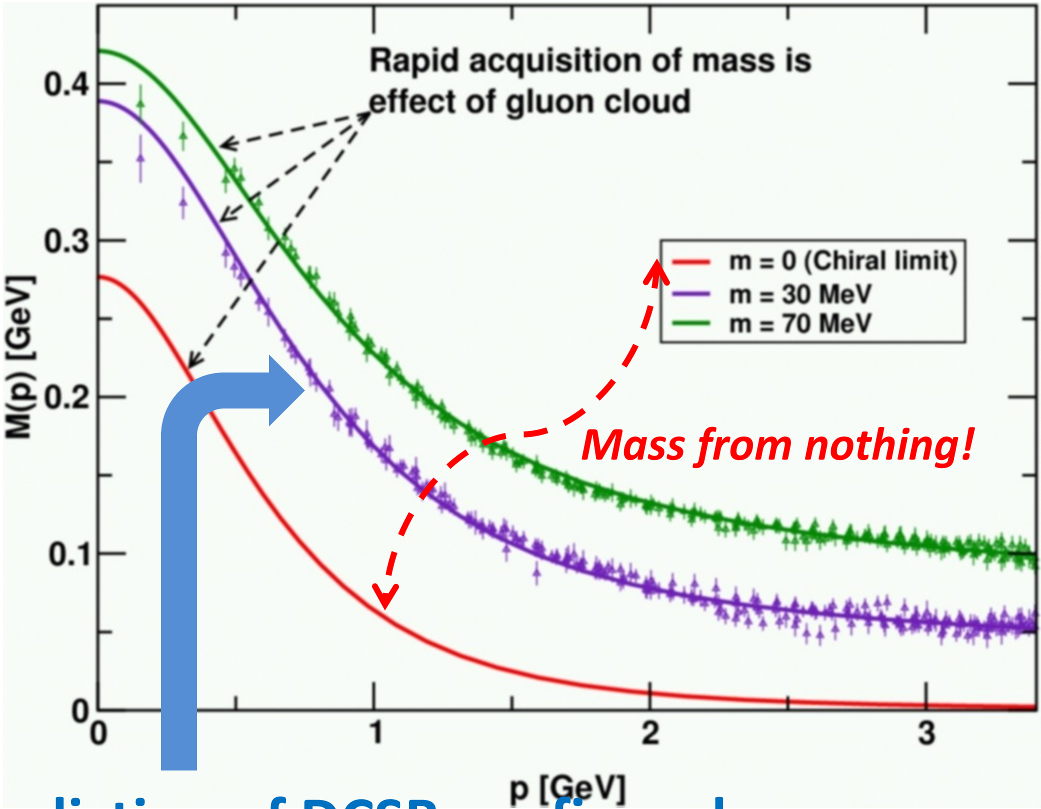
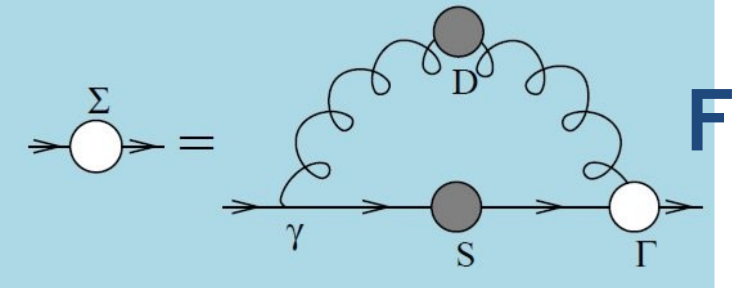


DSE prediction of DCSB confirmed



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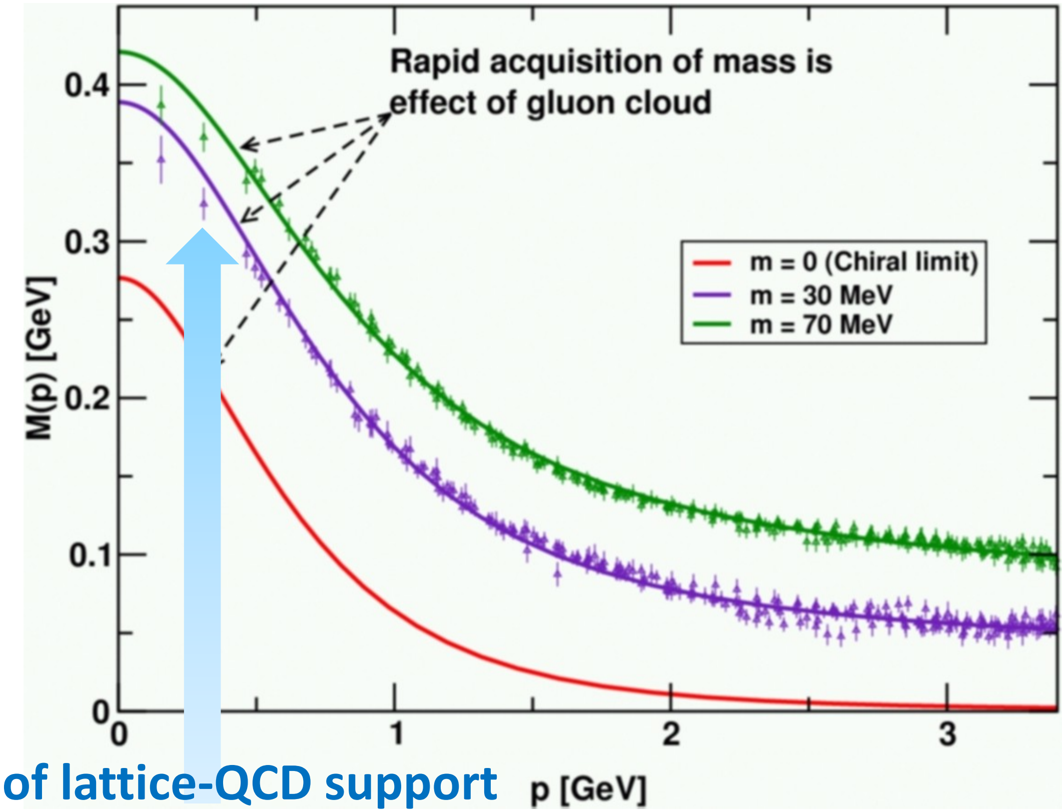
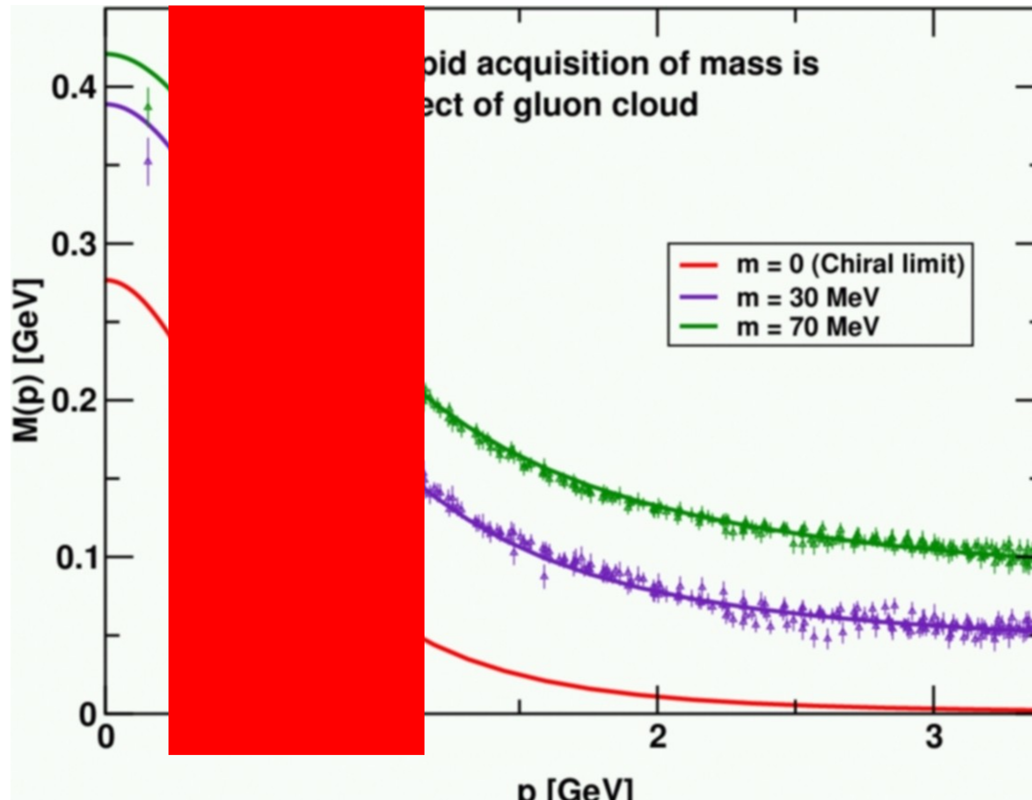


Hint of lattice-QCD support for DSE prediction of violation of reflection positivity

12GeV
The Future of JLab

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.

Universal Truths

- Hadron spectrum, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.
  - Higgs mechanism is (*almost*) irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.
  - Covariance + $M(p^2)$ require existence of quark orbital angular momentum in hadron's rest-frame wave function.
- Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator.
  - It is intimately connected with DCSB.