# Hadron Phenomenology and QCDs DSEs

## Lecture 5: *Parton Distribution Functions*

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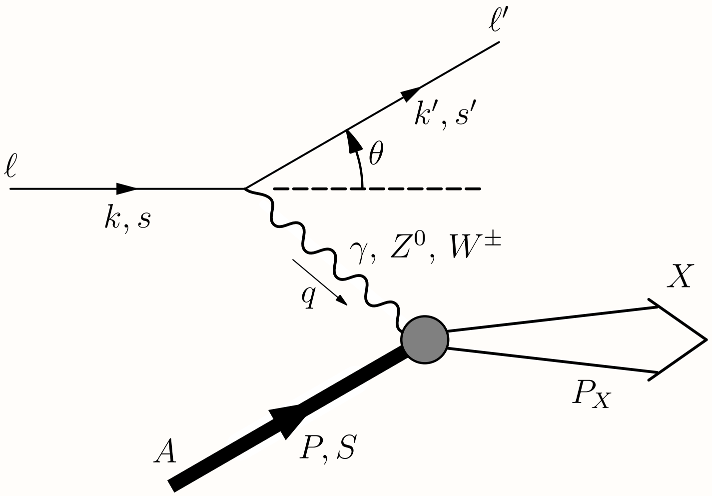
# Deep Inelastic Scattering




$$q^2 = (k - k')^2 = -Q^2 \le 0$$

$$x_A \equiv A\,\frac{Q^2}{2\,p\cdot q} = A\,\frac{Q^2}{2\,M_A\,\nu}, \qquad 0 < x_A \leqslant A$$

$$s = (\ell + P)^2, \qquad y = \frac{Q^2}{x\,s}$$

- Unpolarized cross-section for DIS with single photon exchange is

$$\frac{d\sigma^\gamma}{dx_A\,dQ^2} = \frac{2\pi\,\alpha_e^2}{x_A\,Q^4}\left[\left(1 + (1+y)^2\right)F_2^\gamma(x,Q^2) - y^2 F_L^\gamma(x,Q^2)\right]$$

✦ $F_L^\gamma(x,Q^2) = F_2^\gamma(x,Q^2) - 2\,x\,F_1^\gamma(x,Q^2)$

- The longitudinally polarized cross-section is

$$\frac{d\Delta_L\sigma^\gamma(\lambda)}{dx_A\,dQ^2} = \frac{4\pi\,\alpha_e^2}{x_A\,Q^4}\left[-2\lambda\left(1 - (1-y)^2\right)x\,g_1^\gamma(x,Q^2) + y^2 g_L^\gamma(x,Q^2)\right]$$

- Also structure functions for $\gamma Z$, $Z^0$ & $W^\pm$ exchange

# Bjorken Limit and Scaling

- The Bjorken limit is defined as:

$$Q^2,\ \nu \to \infty \ \mid \ x = \text{fixed}$$

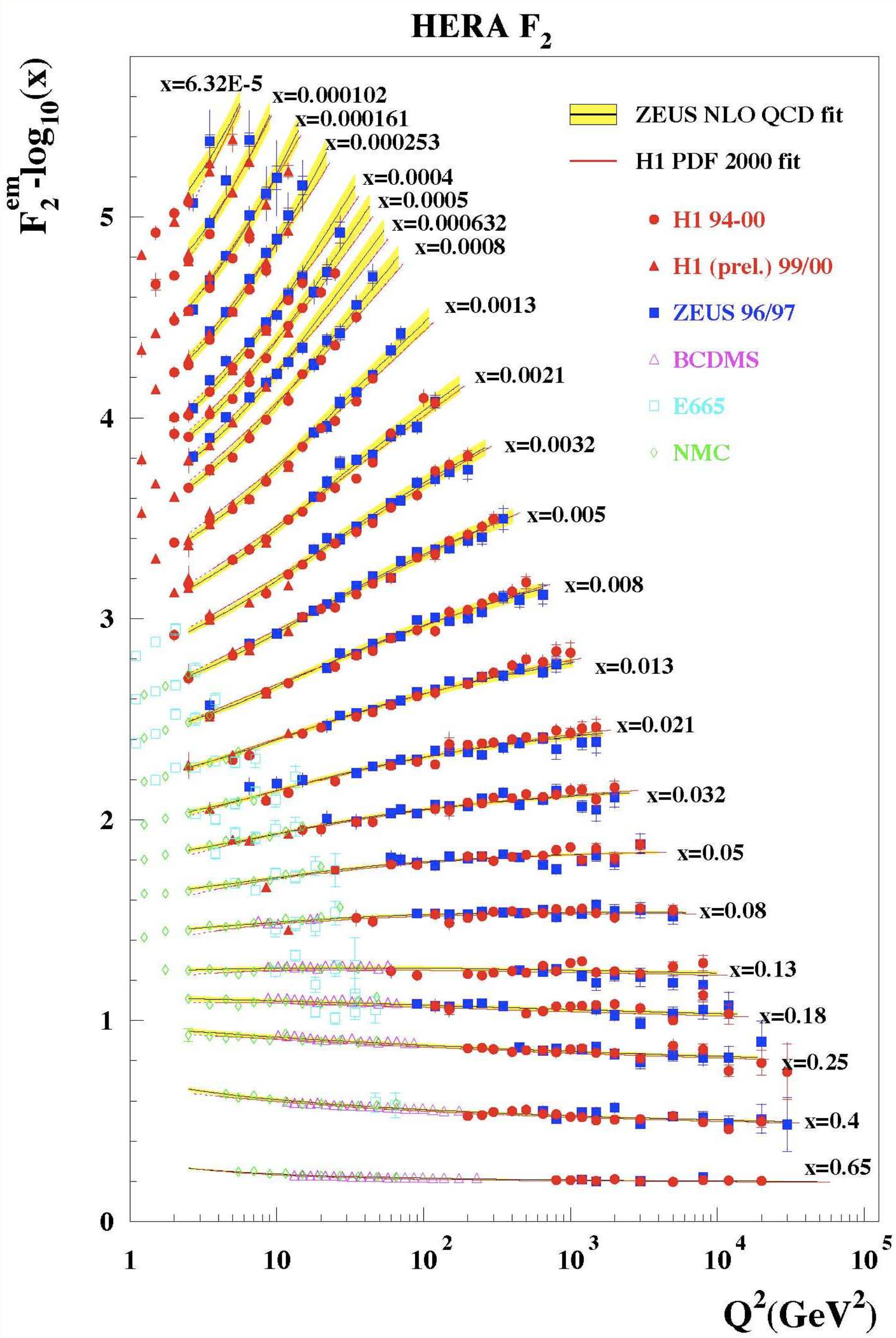
- In 1968 J. D. Bjorken argued that in this limit the photon interactions with the target constituents (partons) involves no dimensional scale, therefore

$$F_2^\gamma(x,Q^2) \to F_2^\gamma(x)$$
$$g_1^\gamma(x,Q^2) \to g_1^\gamma(x) \quad etc$$

  - ✦ Bjorken scaling

- Confirmation from SLAC in 1968 was the first evidence for pointlike constituents inside proton

- Scaling violation ⇔ perturbative QCD



# Physical meaning of Bjorken $x$

- Choose a frame where $\vec{q}_\perp = 0$ then photon moment is

$$q = \left[\nu, 0, 0, -\sqrt{\nu^2 + Q^2}\right] \xrightarrow{\text{Bjorken limit}} q = \left[\nu, 0, 0, -\nu - x\, M_N\right]$$

- Lightcone coordinates: $q^\pm = \frac{1}{\sqrt{2}}\left(q^0 \pm q^3\right) \Rightarrow a \cdot b = a^+ b^- + a^- b^+ - \vec{a}_\perp \cdot \vec{b}_\perp$
- Therefore in Bjorken limit: $q^- \to \infty \qquad q^+ \to -x\, M_N/\sqrt{2}$ and

$$x = \frac{Q^2}{2\, p \cdot q} = -\frac{q^+ q^-}{q^- p^+ + q^+ p^-} \to -\frac{q^+}{p^+}$$

- The lightcone dispersion relation reads: $k^- = \dfrac{m^2 + \vec{k}^{\,2}}{k^+}$
- Can only be satisfied for $k' = k + q$ if $k'^+ = 0$ which implies $k^+ = -q^+$
- Therefore $x$ has physical meaning of the lightcone momentum fraction carried by the struck quark before it is hit by photon

$$x = \frac{k^+}{p^+}$$

# Parton Distribution Functions

- Factorization theorems in QCD prove that the structure functions can be expressed in terms of *universal* parton distribution functions (PDFs)
  - that is, the cross-sections can be factorized into process depend perturbative pieces, determined by pQCD (Wilson coefficients) and the innately non-perturbative universal PDFs
- For example at LO and leading twist the structure functions are given by

$$F_2^{\gamma}(x,Q^2) = \sum_{q=u,d,s,\ldots} e_q^2\,\left[x\,q(x,Q^2) + x\,\bar{q}(x,Q^2)\right]$$

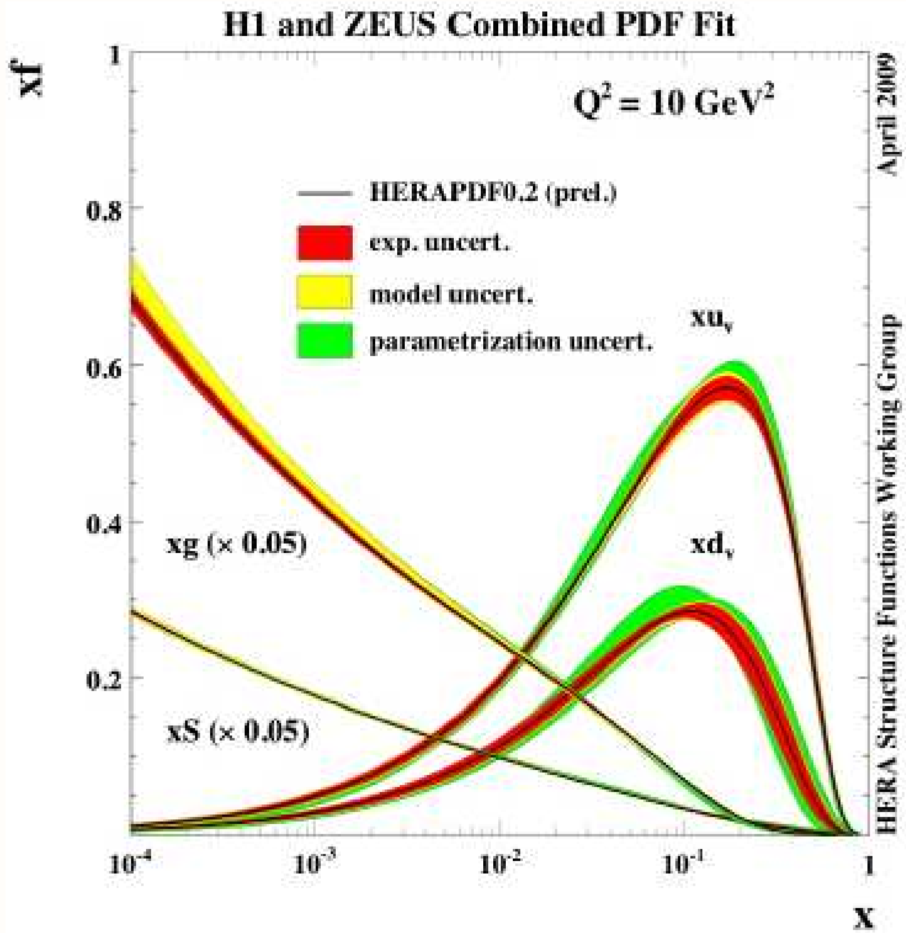
$$g_1^{\gamma}(x,Q^2) = \frac{1}{2}\sum_{q=u,d,s,\ldots} e_q^2\,\left[\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right]$$

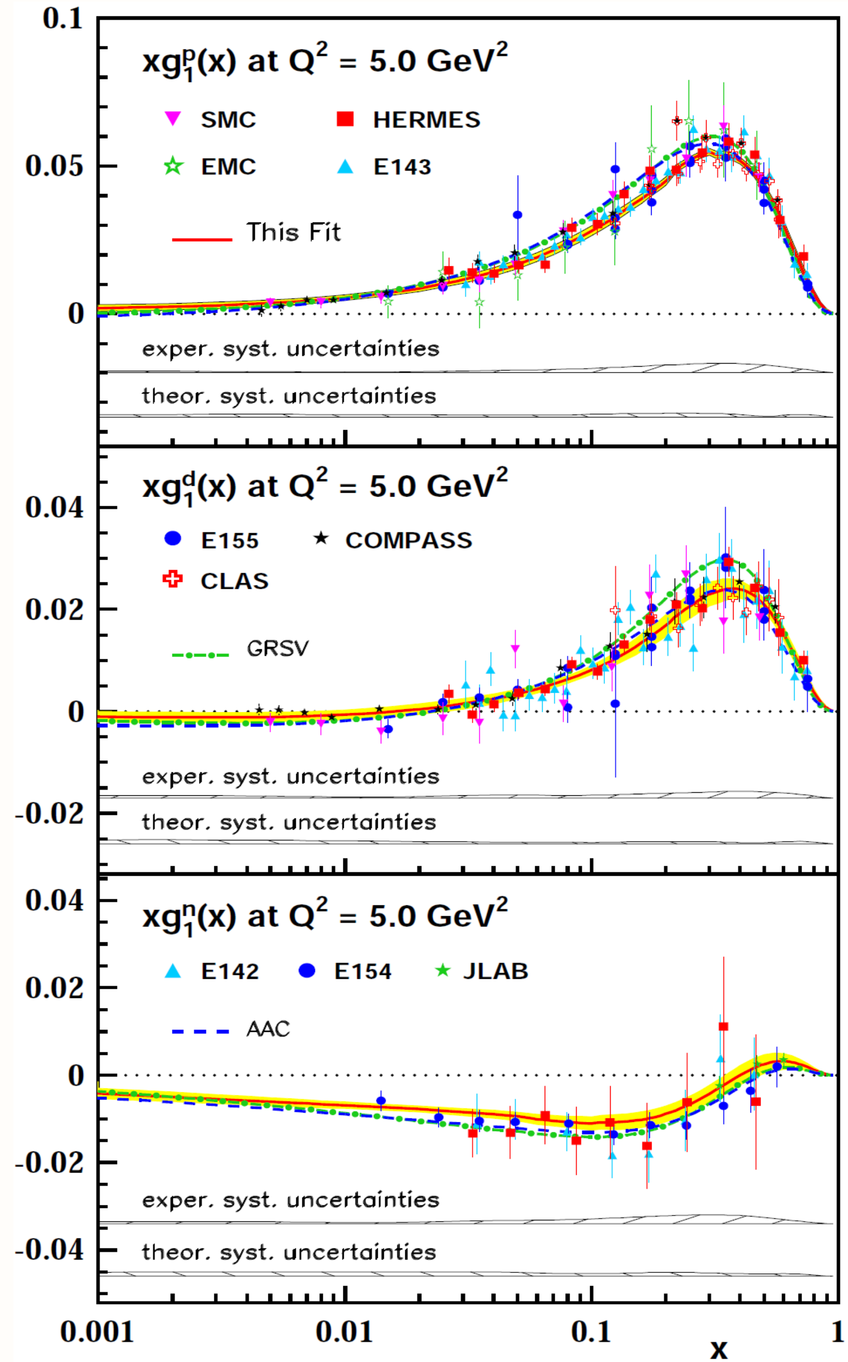
- These PDFs have a probability interpretation:

$q(x) = q_+(x) + q_-(x)$ [spin-independent PDF]
*"probability to strike a quark of flavour $q$ with lightcone momentum fraction $x$ of the target momentum"*

$\Delta q(x) = q_+(x) - q_-(x)$ [spin-dependent PDF]
*"helicity weighted probability to strike a quark of flavour $q$ with lightcone momentum fraction $x$ of the target momentum"*

# Experimental Status: Nucleon PDFs

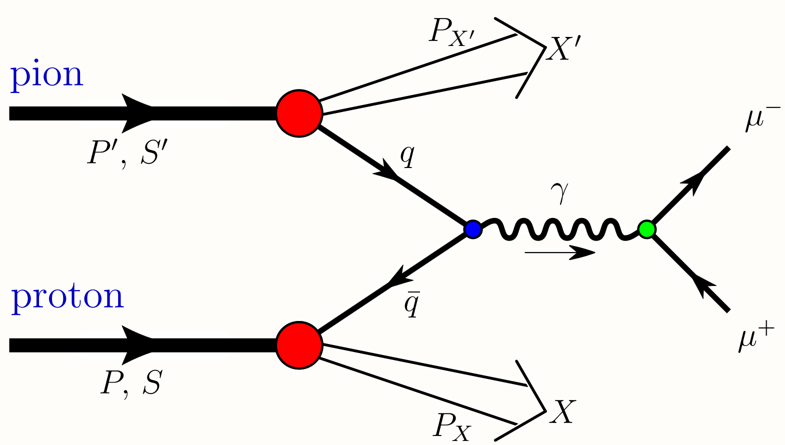


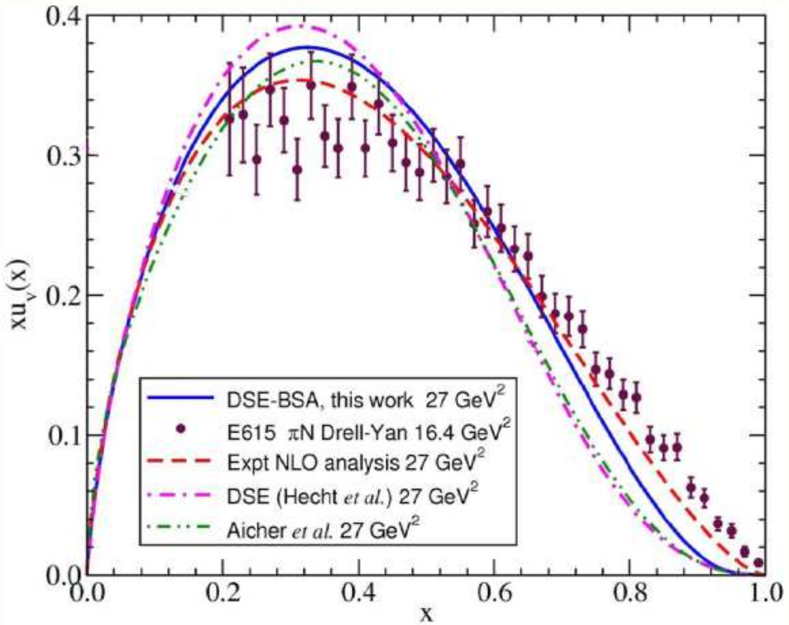




- The distance scales, $\xi$, probed in DIS are given by: $\xi \sim 1/(x\, M_N)$
  - $x = 0.5 \implies \xi = 0.4$ fm
  - $x = 0.05 \implies \xi = 4$ fm

# The Pion PDF

- In QCD alone the pion is a stable particle, however in the real world it decays via the electroweak interaction with a mean lifetime of $2.6 \times 10^{-8}$ s
- Therefore in nature there are no pion targets, however because of time dilation it is possible to create a beam of pions: e.g. $p + \text{Be} \to \pi^- + X$
- Can measure pion PDFs via a process called pion-induced Drell-Yan: $\pi\, p \to \mu^+\, \nu^-\, X$




- There have been three experiments: CERN 1983 & 1985, Fermilab 1989

$$q_\pi(x) \overset{x\to 1}{\longrightarrow} (1-x)^{1+\varepsilon} \qquad \text{pQCD} \implies q_\pi(x) \sim (1-x)^{2+\gamma}$$

# Theory Definition of Pion PDFs

- Pion is a spin zero particle $\Longrightarrow$ only has spin-independent PDFs: $q_\pi(x, Q^2)$
- The pion quark distribution function is defined by

$$q_\pi(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c,$$

- The *moments* of PDFs are defined by

$$\left\langle x^{n-1}\, q_\pi \right\rangle = \int_0^1 dx\; x^{n-1}\; q_\pi(x)$$

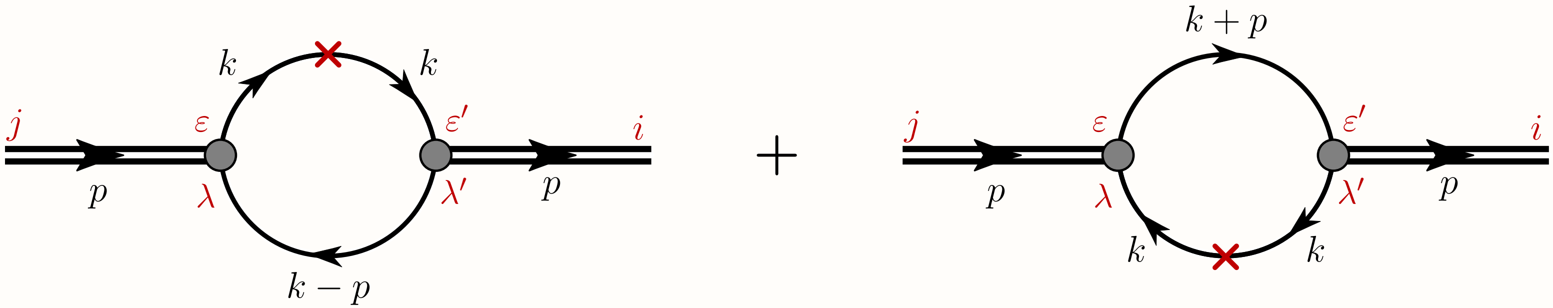
- The moments of these PDFs must satisfy the baryon number & momentum sum rules
- For example the $\pi^+ = u\bar{d}$ PDFs must satisfy

$$\langle u_\pi - \bar{u}_\pi \rangle = 1 \qquad \left\langle d_\pi - \bar{d}_\pi \right\rangle = -1 \qquad \left\langle x\, u_\pi + x\, \bar{d}_\pi + \ldots \right\rangle = 1$$

baryon number sum rules — momentum sum rule

✦ the baryon number sum rule is equivalent to charge conservation

# Pion PDF in the NJL Model




- The pion quark distribution functions can be obtains from a Feynman diagram calculation
- The needed ingredients are
  - ✦ the pion Bethe-Salpeter amplitude: $\Gamma_\pi = \sqrt{g_\pi}\,\gamma_5\,\tau_i$
  - ✦ dressed quark propagator: $S(p)^{-1} = \not{p} - M + i\varepsilon$
- The operator insertion is given by

$$\gamma^+\,\delta\!\left(x - \frac{k^+}{p^+}\right)\frac{1}{2}\,(1 \pm \tau_3)$$

  - ✦ plus sign projects out $u$-quarks and minus $d$-quarks
  - ✦ recall $x$ is the lightcone momentum fraction carried by struck quark

# Pion PDF Results in NJL

- PDFs are scale – $Q^2$ – dependent, however within the NJL model there is no way to determine the model scale $Q_0^2$
- Standard method is to fit the proton valence $u$-quark distribution to empirical results, best fit determines $Q_0^2$
- The NJL model result for $\pi^+$ PDFs at $Q^2 = Q_0^2 = 0.16\,\text{GeV}^2$

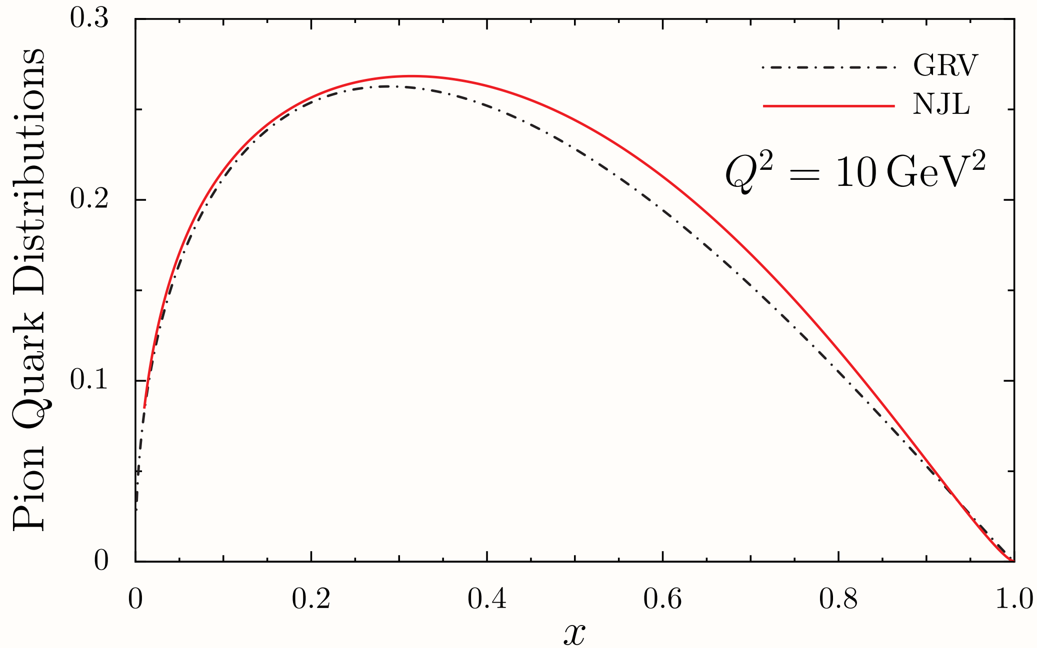
$$u_\pi(x) = \bar{d}_\pi(x) = \frac{3\,g_\pi}{4\pi^2} \int d\tau \left[\frac{1}{\tau} + x\,(1-x)\,m_\pi^2\right] e^{-\tau\left[x(x-1)m_\pi^2 + M^2\right]}.$$

- Agreement with data excellent
- At large $x$ NJL finds

$$u_\pi(x) \overset{x\to 1}{\simeq} (1-x)^1$$

- Disagrees with pQCD result

$$u_\pi(x) \overset{x\to 1}{\simeq} (1-x)^{2+\gamma}$$



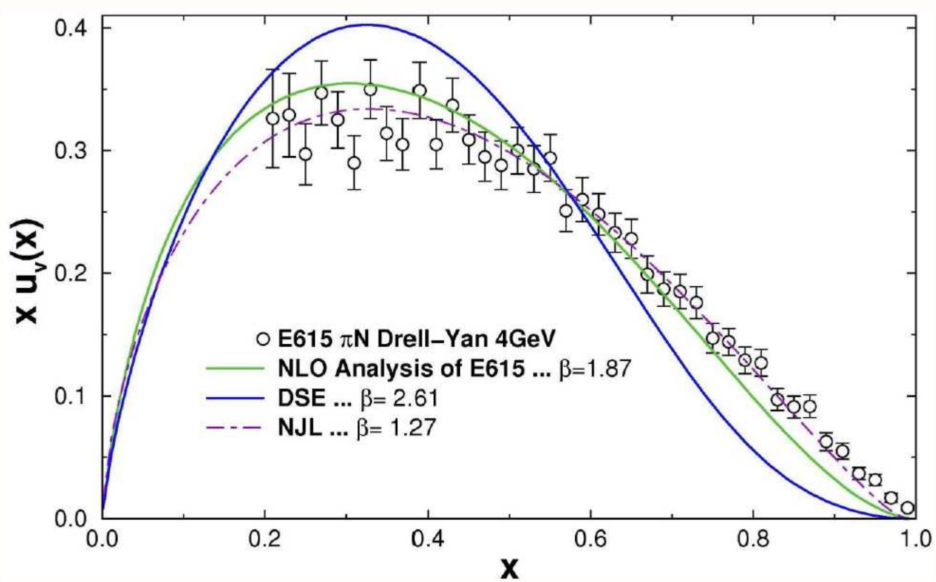
# *Pion PDF in DSEs*

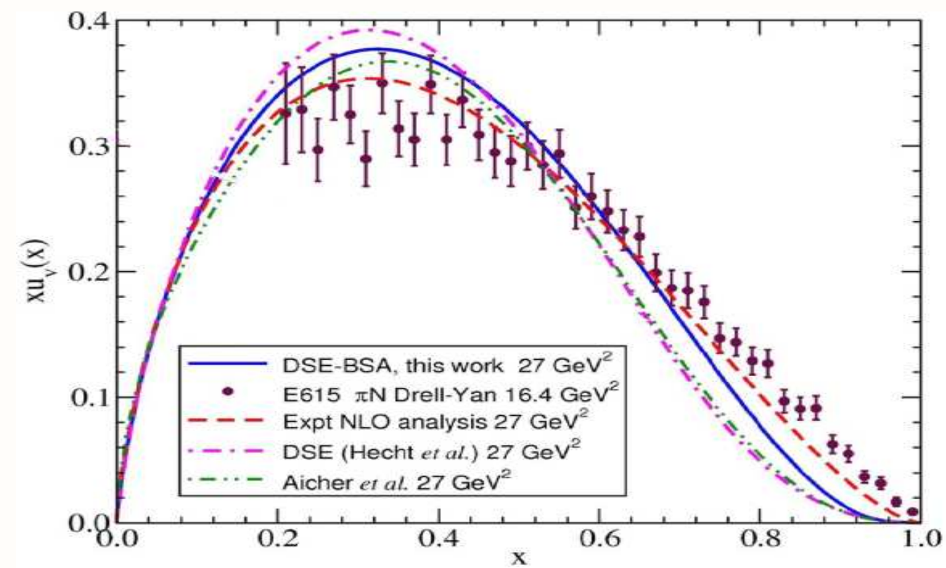
- DSE calculations – fully dressed quark propagator and BS vertex function

$$S(p)^{-1} = \not{p}\, A(p^2) + B(p^2)$$

$$\Gamma_\pi(p,k) = \gamma_5 \Big[ E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, \mathcal{G}(p,k) + \sigma^{\mu\nu} k_\mu p_\nu\, \mathcal{H}(p,k) \Big]$$

- At large $x$ DSE and pQCD results agree: $u_\pi(x) \overset{x\to 1}{\simeq} (1-x)^{2+\gamma}$
  - ✦ this 2001 result seemed to disagree with experiment for a decade
- Recent reanalysis of data by *Aicher et al.* now finds excellent agreement with DSEs!



# *QCD Evolution Equation*

- One of the greatest successes of perturbative QCD are the DGLAP evolution equations
  - DGLAP $\Longleftrightarrow$ Dokshitzer (1977), Gribov-Lipatov (1972), Altarelli-Parisi (1977)
- These QCD evolution equations relate the PDFs at one scale, $Q_0^2$, to another scale, $Q^2$, provided $Q_0^2,\, Q^2 \gg \Lambda_{QCD}$.
- The evolution equation for $q^- \equiv q - \bar{q}$ type PDFs is

$$\frac{\partial}{\partial \ln Q^2}\, q^-(x, Q^2) = \alpha_s(Q^2)\, P(z) \otimes q^-(y, Q^2) \qquad \text{[non-singlet]}$$

  - note that the gluon PDF does not contribute here
- Evolution equations for $q^+ \equiv q + \bar{q}$ and gluon, $g(x)$, PDFs are coupled

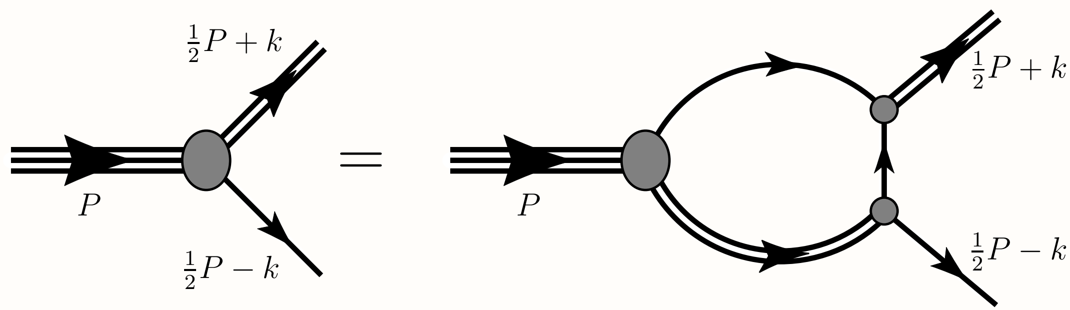
*The physics behind these equations is that a valence quark can radiate gluons and a gluon can become a quark–antiquark pair, therefore momentum can be shifted between the valence quarks, sea quarks and gluons. The probability of this radiation is scale, $Q^2$, dependent.*

# Nucleon PDFs in the NJL model

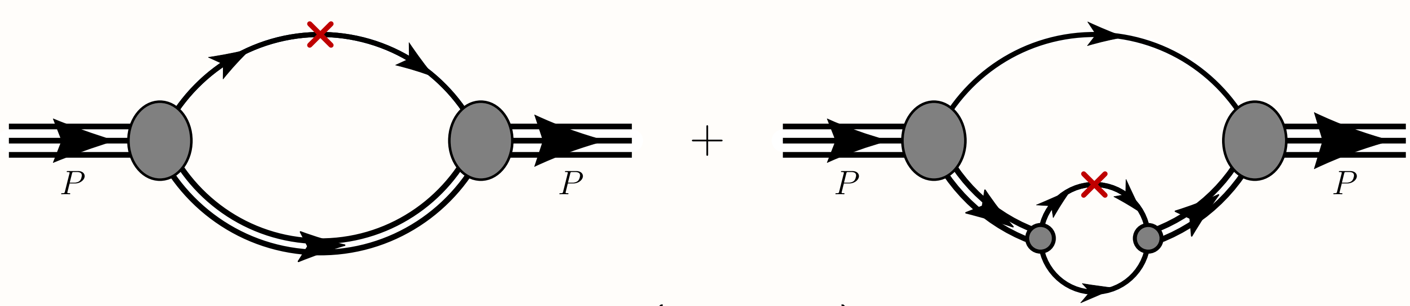
- Nucleon quark distributions are defined by

$$q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c, \qquad \Delta q(x) = \langle \gamma^+\gamma_5\rangle$$

- Nucleon bound state is obtained by solving the relativistic Faddeev equation in the quark-diquark approximation

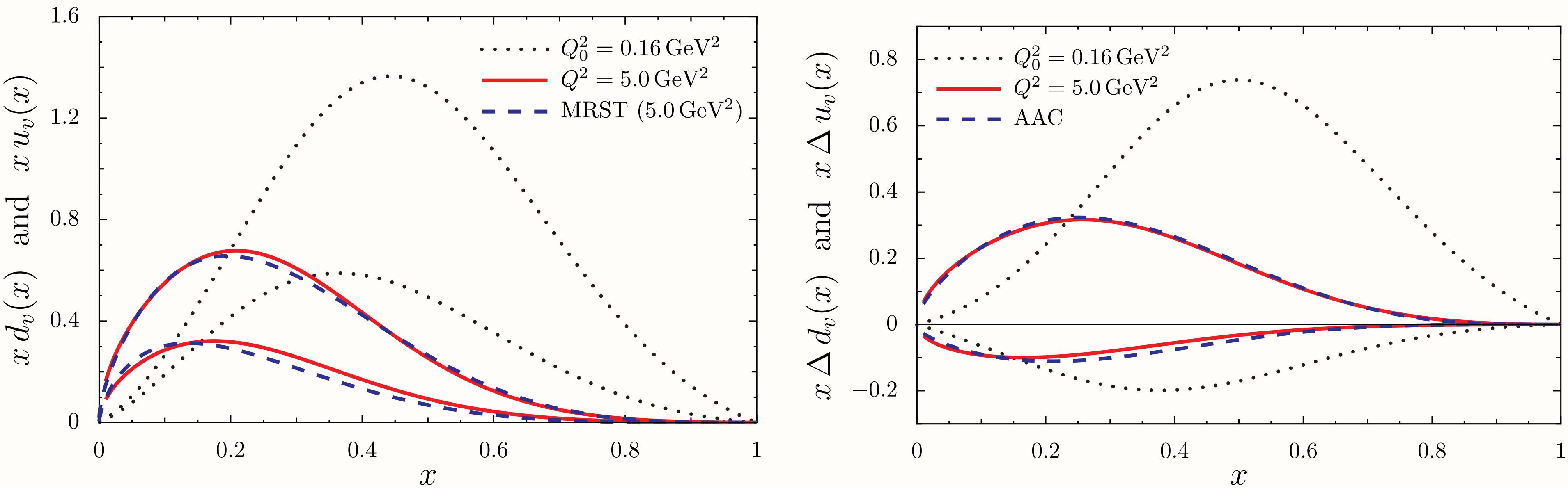



- PDFs are associated with the Feynman diagrams




✦ $[q(x), \Delta q(x), \Delta_T q(x)]$ ➞ X $= \delta\left(x - \frac{k^+}{p^+}\right)\left[\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5\right]$

# Results: proton quark distributions




- Covariant, correct support, satisfies baryon and momentum sum rules

$$\int dx\,[q(x) - \bar{q}(x)] = N_q, \qquad \int dx\,x\,[u(x) + d(x) + \ldots] = 1$$

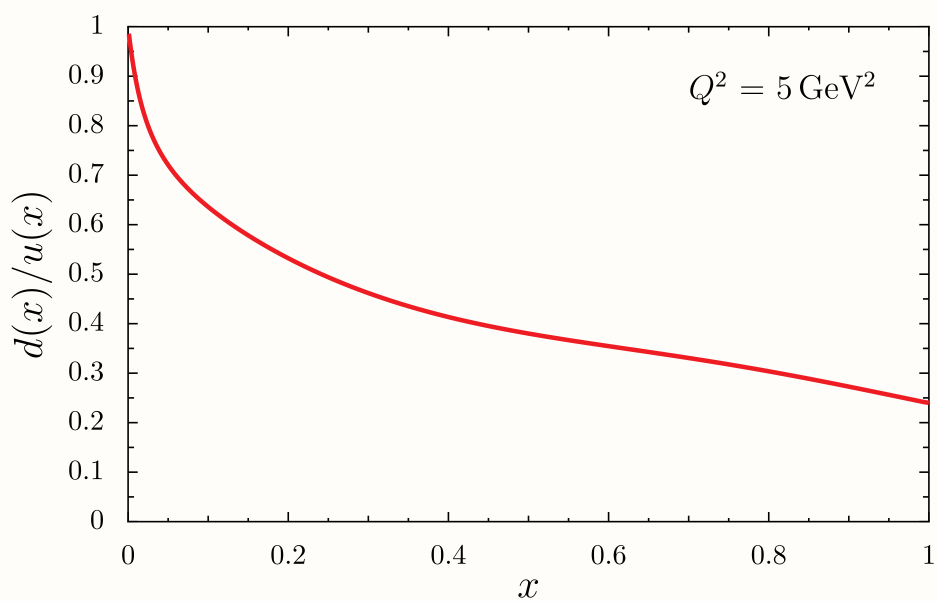
- Satisfies positivity constraints and Soffer bound

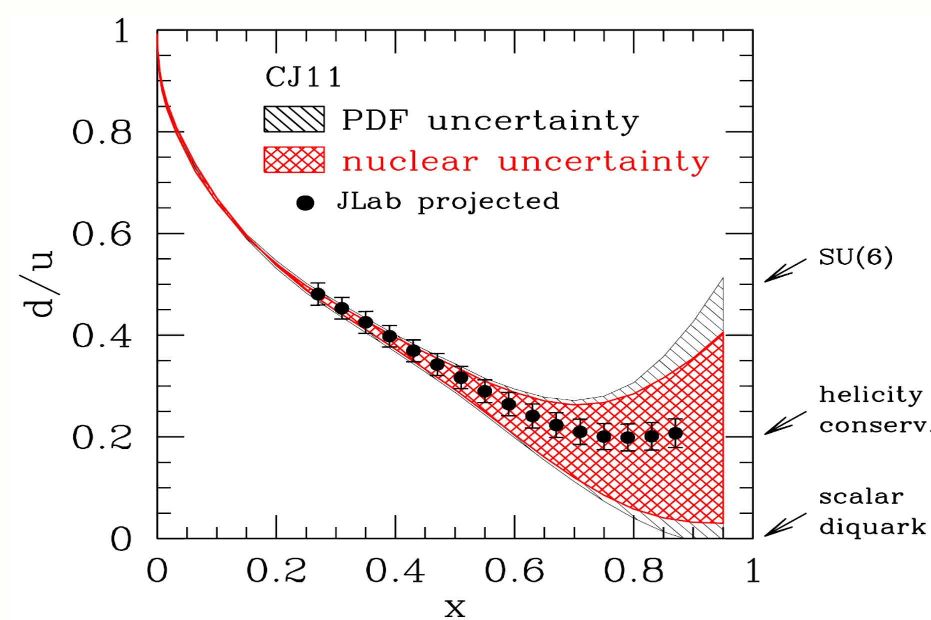
$$|\Delta q(x)|\,,\ |\Delta_T q(x)| \leqslant q(x), \qquad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$


- Martin, Roberts, Stirling and Thorne, Phys. Lett. B **531**, 216 (2002).
- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

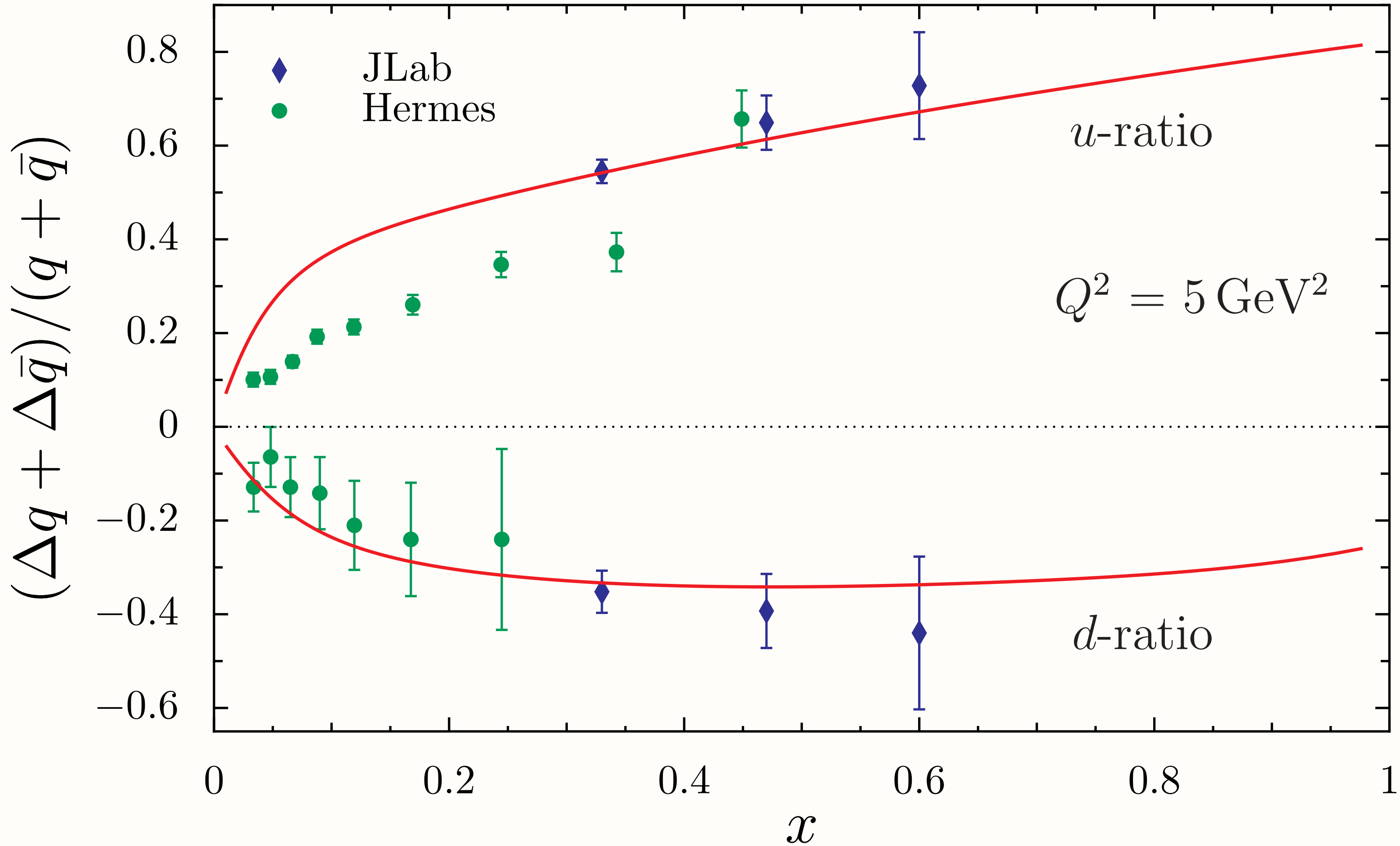
# Proton $d/u$ ratio

- The $d(x)/u(x)$ ratio as $x \to 1$ is of great interest because if does not change with QCD evolution and pQCD can make predictions
- There are three classes of predictions for this ratio
  - $SU(6)$ spin-flavour symmetry $\Longrightarrow$ $d(x)/u(x) \stackrel{x\to 1}{=} 1/2$
  - pQCD and helicity conservation $\Longrightarrow$ $d(x)/u(x) \stackrel{x\to 1}{=} 1/5$
  - scalar diquark dominance $\Longrightarrow$ $d(x)/u(x) \stackrel{x\to 1}{=} 0$
- Using DSEs with running mass: $d(x)/u(x) \stackrel{x\to 1}{=} 0.23 \pm 0.09$



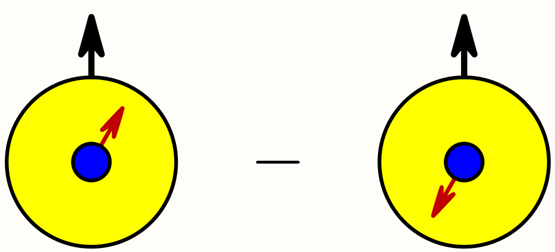
# *Perturbative QCD Predictions* $x \to 1$

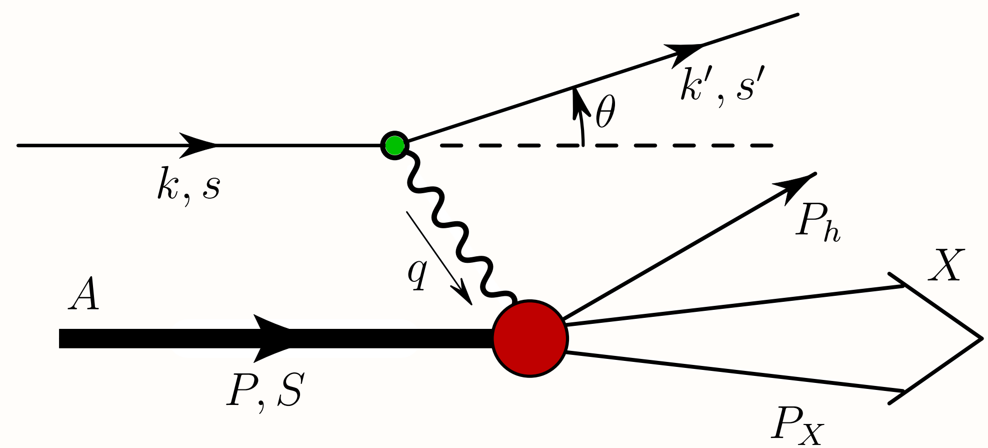



- The pQCD predictions for $\Delta q(x)/q(x)$ as $x \to 1$ are the most robust
- The pQCD prediction is: $\Delta q(x)/q(x) \overset{x\to 1}{=} 1$ for $q \in u,\, d$
- Realization would require a dramatic change in sign of $\Delta d(x)/d(x)$

# Transversity PDFs

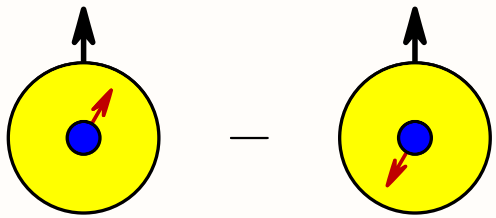
- At leading twist there are three collinear PDFs
  - $q(x)$ – spin-independent
  - $\Delta q(x)$ – spin-dependent
  - $\Delta_T q(x)$ – transversity
- However transversity PDFs are chiral odd and therefore do not appear in deep inelastic scattering
- Can be measured using semi-inclusive DIS on a transversely polarized target or certain Drell-Yan experiments

$\Delta_T q(x) =$ 




- Quarks in eigenstates of $\gamma^{\perp}\gamma_5$

# Why is Transversity Interesting?

$\Delta_T q(x) =$ 

- Quarks in eigenstates of $\gamma^{\perp}\gamma_5$
- Tensor charge [c.f. Bjorken sum rule for $g_A$]

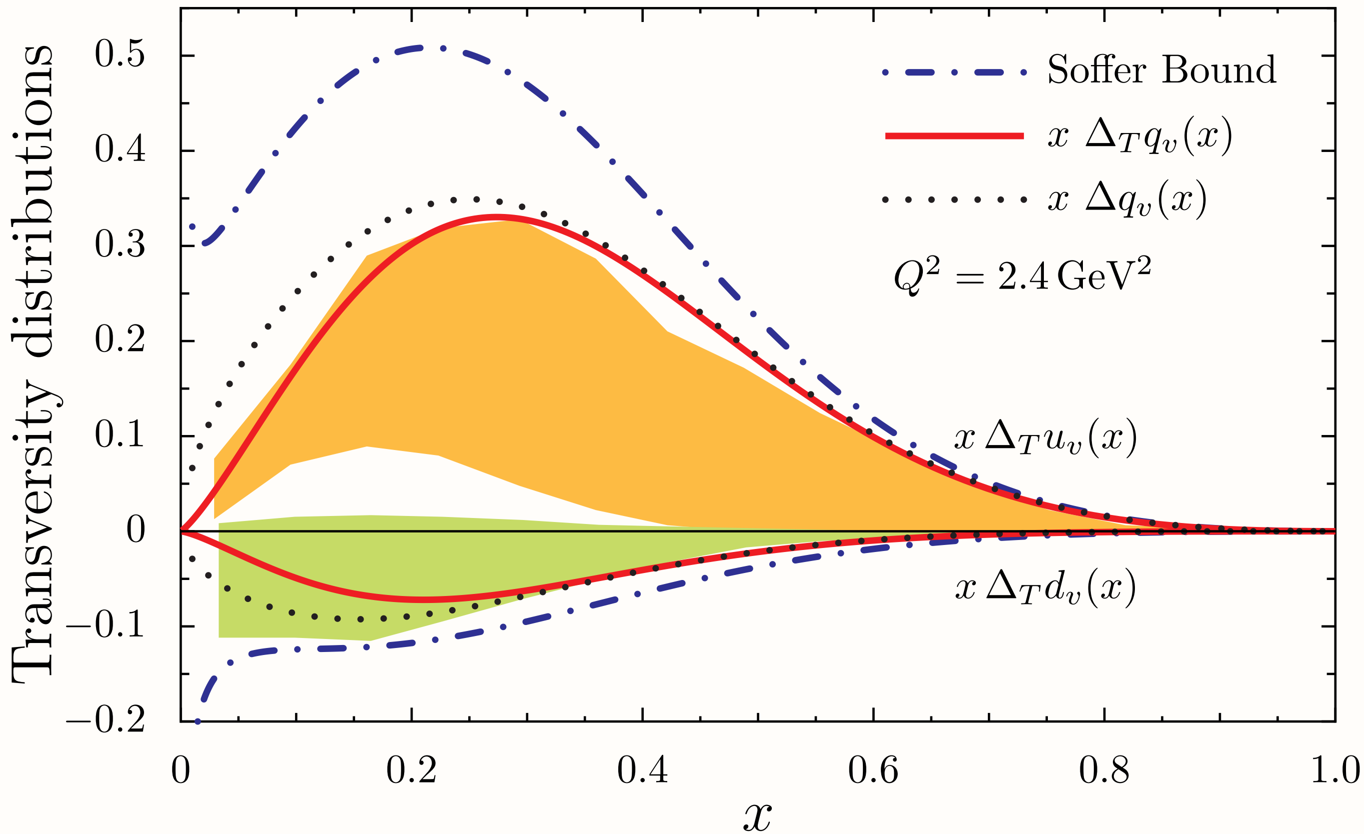
$$g_T = \int dx\, [\Delta_T u(x) - \Delta_T d(x)] \qquad g_A = \int dx\, [\Delta u(x) - \Delta d(x)]$$

- In non-relativistic limit: $\Delta_T q(x) = \Delta q(x)$
  - ✦ therefore $\Delta_T q(x)$ is a measure of relativistic effects
- Helicity conservation $\Longrightarrow$ no mixing between $\Delta_T q$ & $\Delta_T g$
- For $J \leqslant \frac{1}{2}$ we have $\Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated
- *Important!!* A very common mistake – transverse spin sum:

$$\int dx\, \Delta_T q(x) = \langle \overline{\psi}_q \gamma^+ \gamma^1 \gamma_5 \psi_q \rangle \neq \langle \psi_q^\dagger \gamma^0 \gamma^1 \gamma_5 \psi_q \rangle = \Sigma_T^q$$

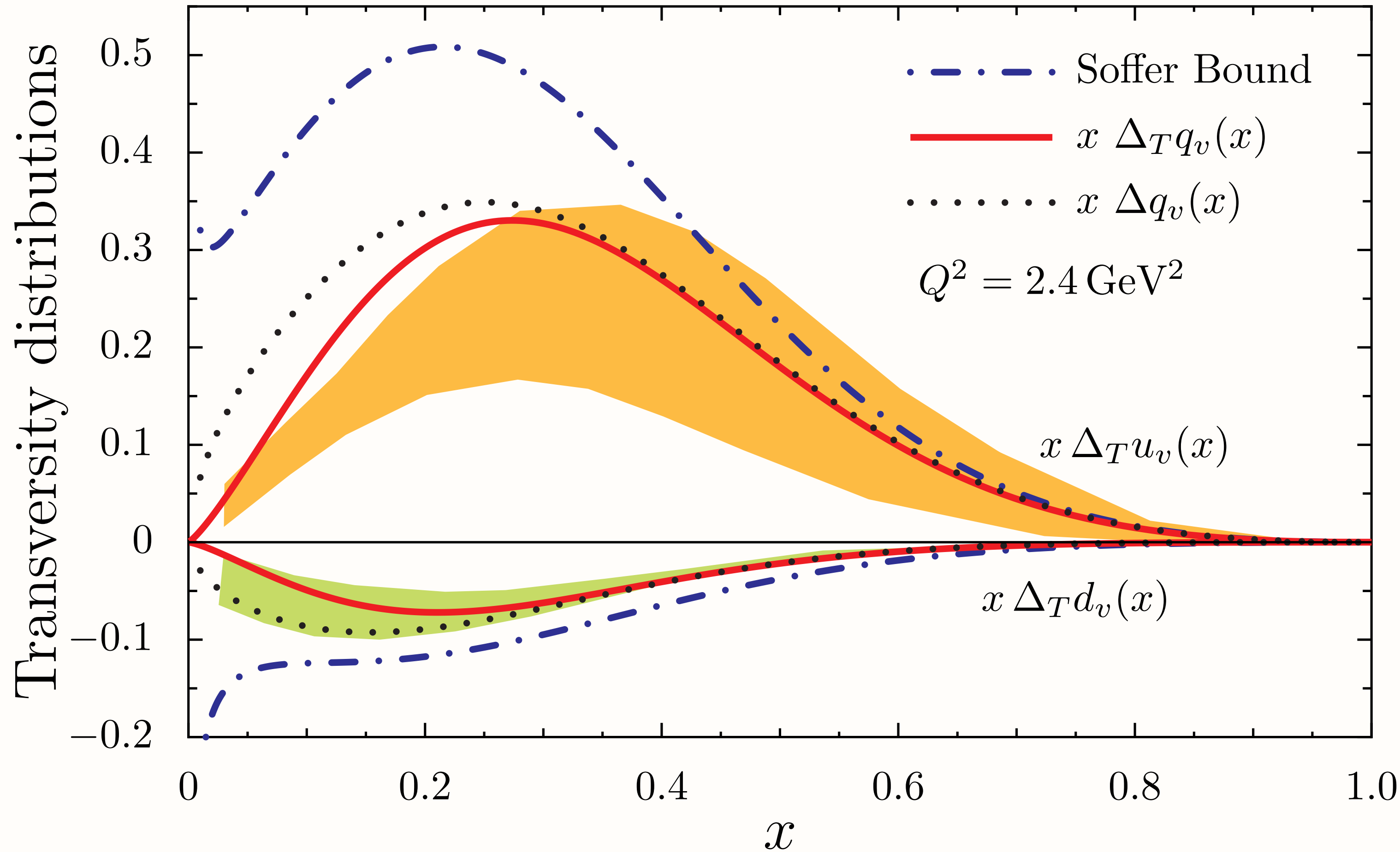
  - ✦ transversity moment $\neq$ spin quarks in transverse direction [c.f. $g_T(x)$]

# $\Delta_T u_v(x)$ and $\Delta_T d_v(x)$ distributions



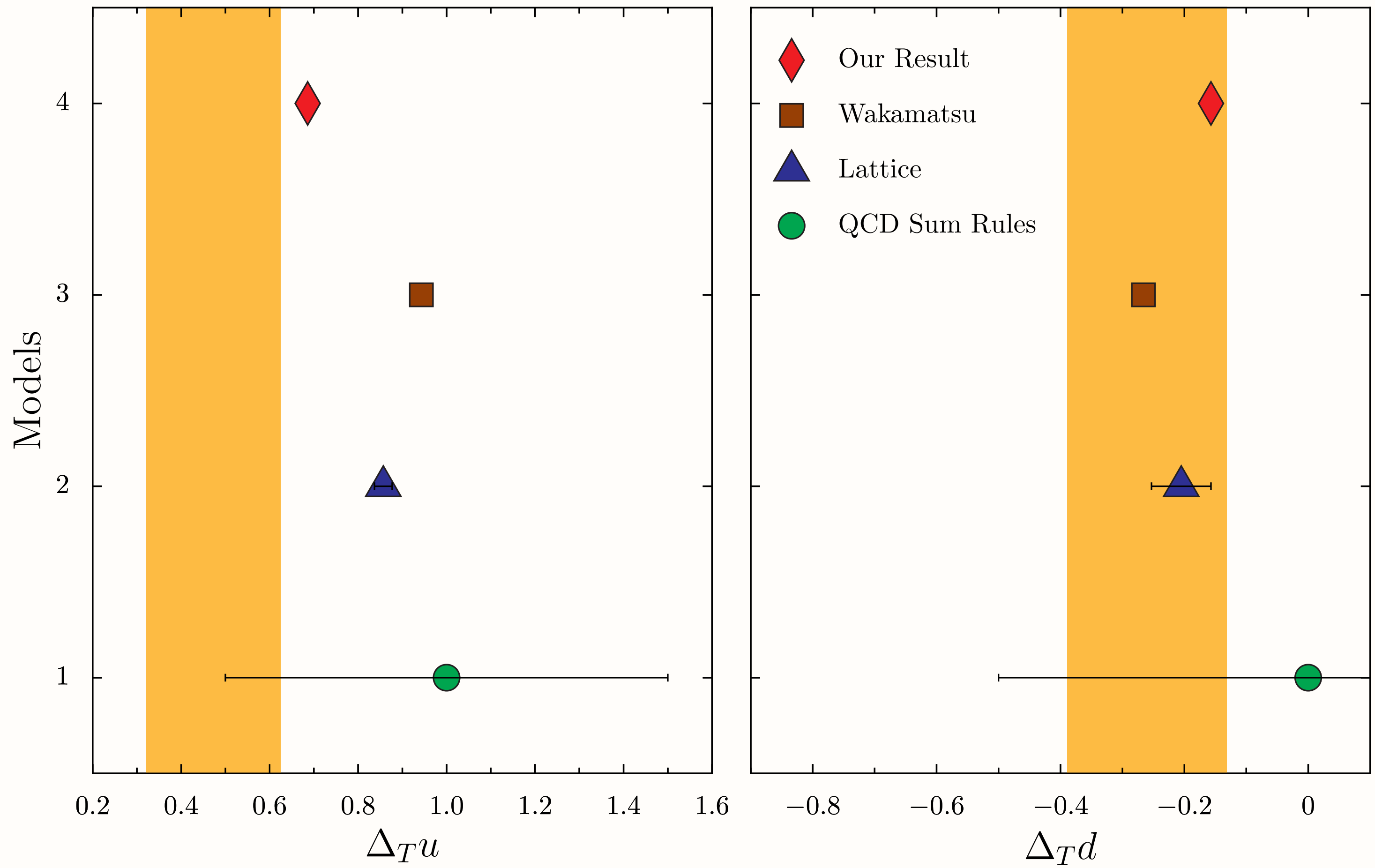

- Predict small relativistic corrections
- Empirical analysis *potentially* found large relativistic corrections
  - M. Anselmino *et. al.*,Phys. Rev. D **75**, 054032 (2007).
- Large effects difficult to support with quark mass $\sim 0.4$ GeV
  - maybe running quark mass is needed

# Transversity: Reanalysis




- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- Our results are now in better agreement updated distributions
- Concept of constituent quark models safe . . . for now

# Transversity Moments

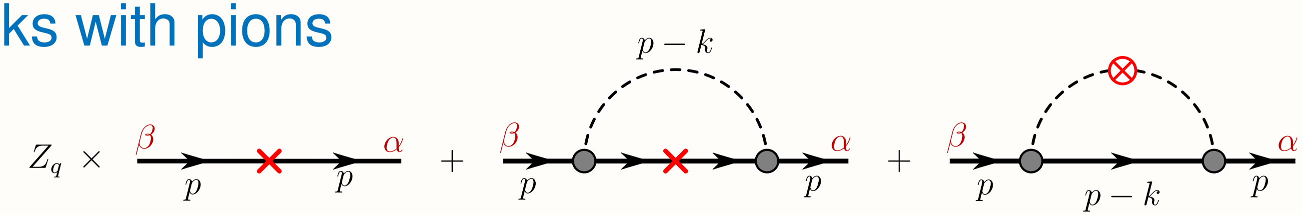


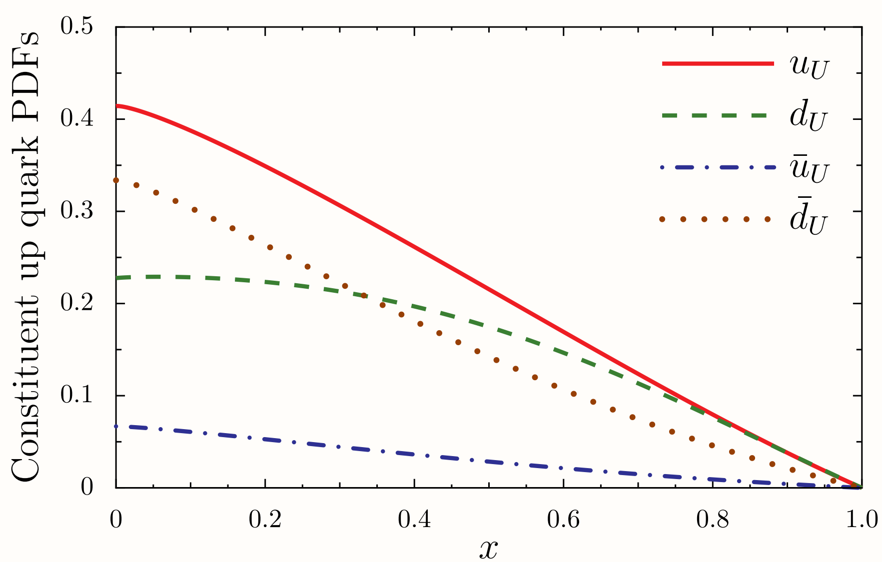

- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- At model scale we find tensor charge

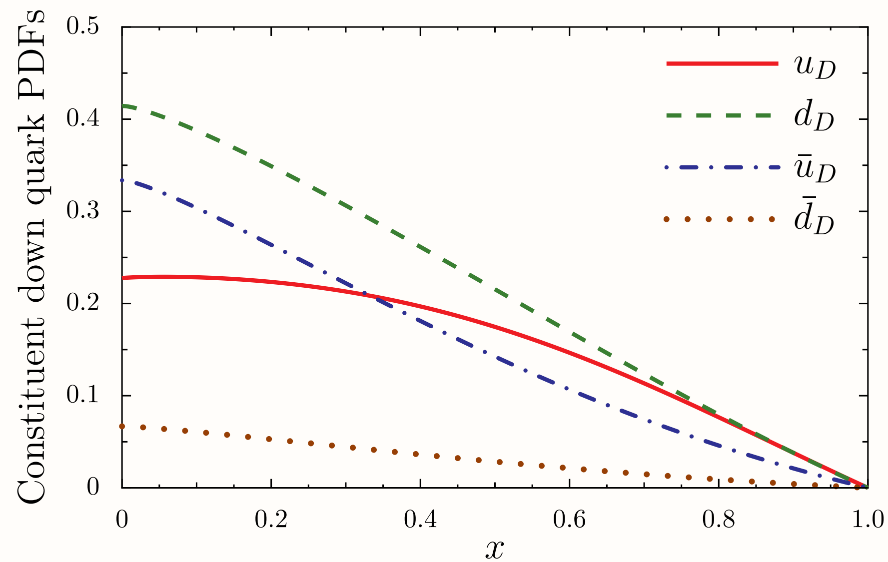
$g_T = 1.28$ compared with $g_A = 1.267$

# Including Anti-quarks

- Dress quarks with pions




- Gottfried Sum Rule: NMC 1994: $S_G = 0.258 \pm 0.017\ [Q^2 = 4\,\text{GeV}^2]$

$$S_G = \int_0^1 \frac{dx}{x}\left[F_{2p}(x) - F_{2n}(x)\right] = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,\left[\bar{d}(x) - \bar{u}(x)\right]$$

- We find: $S_G = \frac{1}{3} - \frac{4}{9}\left(1 - Z_q\right) = 0.252 \qquad [Z_q = 0.817]$

# Extracting Proton Spin Content

- Ellis–Jaffe sum rule

$$\left[\tfrac{1}{2} = \tfrac{1}{2}\,\Delta\Sigma + L_q + J_g\right]$$

$$\int dx\, g_{1p}^{\gamma}(x, Q^2) = \frac{1}{36}\,[3\,\Delta q_3 + \Delta q_8] + \frac{1}{9}\,\Delta q_0,$$

$$\Delta\Sigma = \Delta q_0 = \Delta u^+ + \Delta d^+ + \Delta s^+ \qquad \text{[singlet]}$$

$$g_A = \Delta q_3 = \Delta u^+ - \Delta d^+ \qquad \text{[triplet]}$$

$$\Delta q_8 = \Delta u^+ + \Delta d^+ - 2\,\Delta s^+ \qquad \text{[octet]}$$

- To help extract $\Delta\Sigma$ usual to use semi-leptonic hyperon decays and assume $SU(3)$ flavour symmetry to relate $\Delta q_3$ and $\Delta q_8$

$$\Delta q_3 = F + D \qquad \Delta q_8 = 3\,F - D$$

$$n\,p \to F + D, \qquad \Lambda\,p \to F + \tfrac{1}{3}\,D, \qquad \Sigma\,n \to F - D, \quad \text{etc}$$

- Solve for quark polarizations

$$\Delta u^+ = \tfrac{1}{3}\,\Delta q_0 + \tfrac{1}{2}\,\Delta q_3 + \tfrac{1}{6}\,\Delta q_8$$

$$\Delta d^+ = \tfrac{1}{3}\,\Delta q_0 - \tfrac{1}{2}\,\Delta q_3 + \tfrac{1}{6}\,\Delta q_8$$

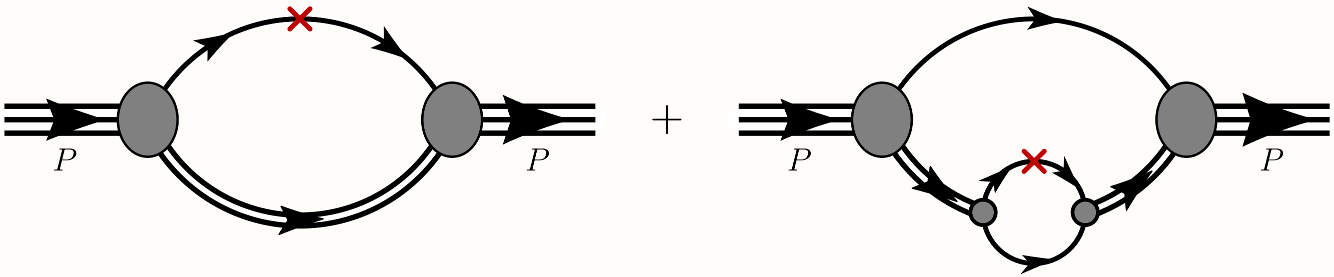
$$\Delta s^+ = \tfrac{1}{3}\,\Delta q_0 - \tfrac{1}{3}\,\Delta q_8$$

# *Spin Sum in NJL Model*

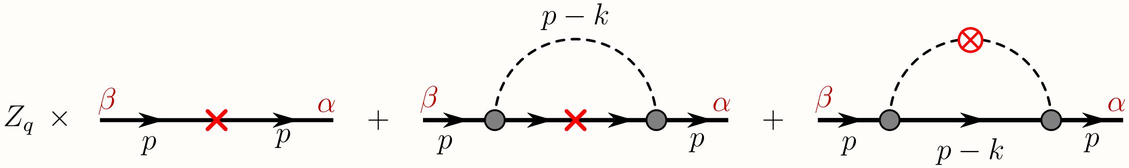
- Nucleon angular momentum must satisfy: $J = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

$\Delta\Sigma = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$ [COMPASS & HERMES]

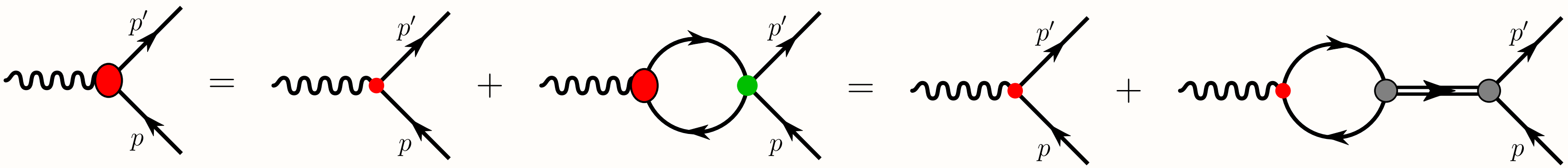
- Result from Faddeev calculation: $\Delta\Sigma = 0.66$




- Correction from pion cloud: $\Delta\Sigma = 0.79 \times 0.66 = 0.52$

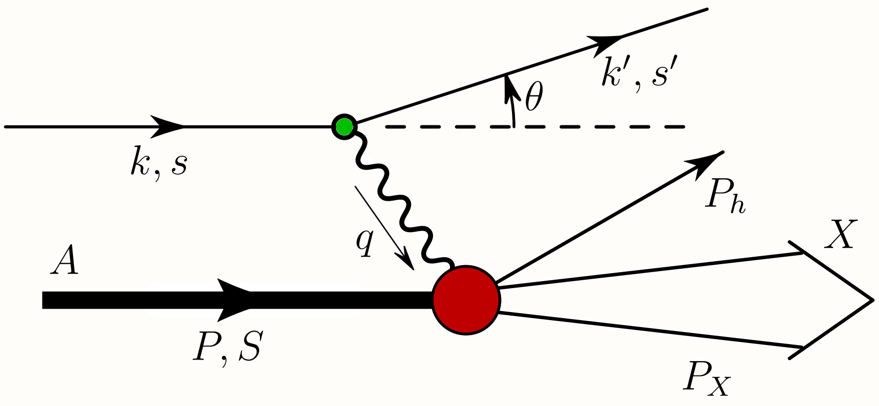



- Bare operator $\gamma^\mu\gamma_5$ gets renormalized: $\Delta\Sigma = 0.91 \times 0.52 = 0.47$



# A 3D image of the nucleon – TMD PDFs

- Measured in semi-inclusive DIS



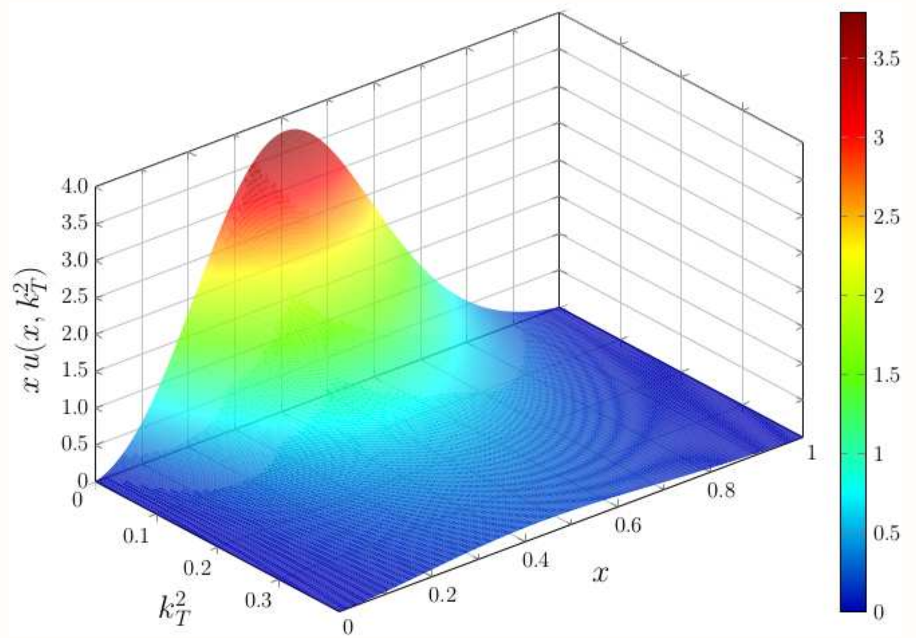

- Leading twist 6 $T$-even TMD PDFs

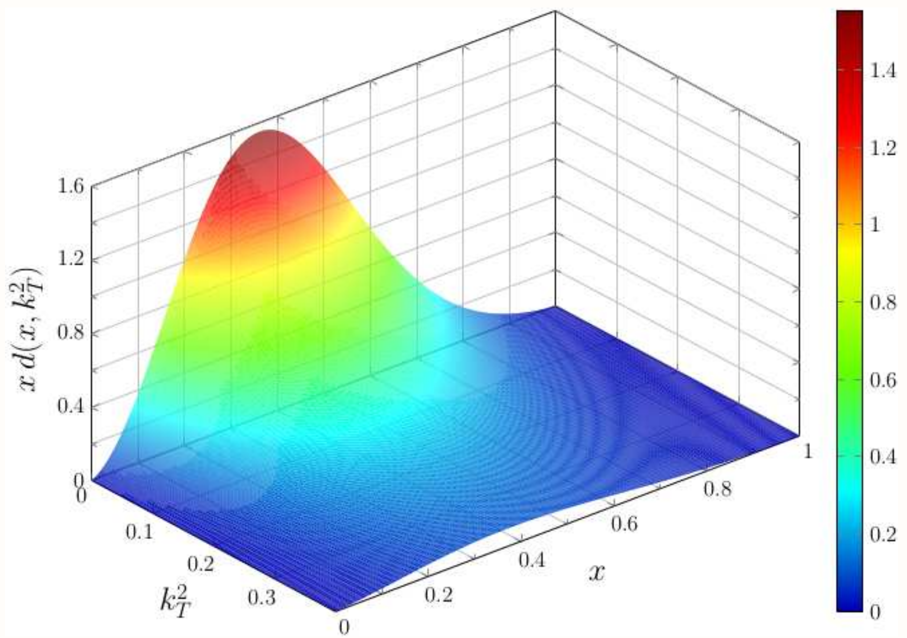
$$q(x, k_\perp^2), \quad \Delta q(x, k_\perp^2), \quad \Delta_T q(x, k_\perp^2)$$
$$g_{1T}^q(x, k_\perp^2), \quad h_{1L}^{\perp q}(x, k_\perp^2), \quad h_{1T}^{\perp q}(x, k_\perp^2)$$

- $$\langle p_T \rangle (x) \equiv \frac{\int d\vec{k}_\perp \, k_\perp \, q(x,k_\perp^2)}{\int d\vec{k}_\perp \, q(x,k_\perp^2)}$$

[H. Avakian, *et al.*, Phy. Rev. D81, 074035 (2010).]

- $\langle p_T \rangle^{Q^2=Q_0^2} = 0.36$ GeV c.f. $\langle p_T \rangle_{\text{Gauss}} = 0.56$ GeV [HERMES], $0.64$ GeV [EMC]



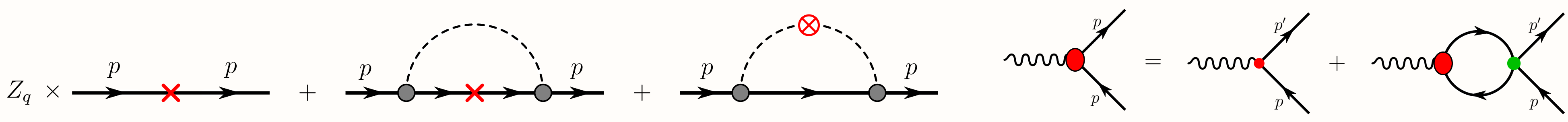




[H. H. Matevosyan, ICC *et al.*, Phys. Rev. D **85**, 014021 (2012).]

# NJL robust conclusions

- Diquark correlations in nucleon are very important
  - ✦ $d(x)$ is softer than $u(x)$ $\iff$ scalar diquark $(ud)_{0+}$
  - ✦ $d(x)/u(x) \overset{x\to 1}{\simeq} 0.2$ – sensitive to strength of axial-vector diquark
- Using DSEs with running mass: $d(x)/u(x) \overset{x\to 1}{=} 0.23 \pm 0.09$
- Can *almost* reproduce measured spin sum: $\Delta\Sigma = 0.366^{+0.042}_{-0.062}$ [DSSV]
  - ✦ relativistic effects + pions + vertex renormalization $\Longrightarrow \Delta\Sigma = 0.47$
  - ✦ perturbative gluon dressing on quarks will reduce $\Delta\Sigma$ further
- Perturbative pions $\Rightarrow$ Gottfried Sum Rule: [$S_G = 0.258 \pm 0.017\ (Q^2 = 4$ GeV$^2$) – NMC 1994]

$$S_G = \int_0^1 \frac{dx}{x}\left[F_{2p}(x) - F_{2n}(x)\right] = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\left[\bar{d}(x) - \bar{u}(x)\right] \overset{NJL}{\to} \frac{1}{3} - \frac{4}{9}\left(1 - Z_q\right) = 0.252$$



# Table of Contents