Hadron Phenomenology and QCDs DSEs

Lecture 4: *Nucleon Form Factors*

Ian Cloët
University of Adelaide & Argonne National Laboratory

Collaborators

Wolfgang Bentz – Tokai University
Craig Roberts – Argonne National Laboratory
Anthony Thomas – University of Adelaide
David Wilson – Argonne National Laboratory

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Nucleon Static Properties

- If the proton was a point particle its electromagnetic properties would be characterized by two observables

charge: $e_p = +1$ & magnetic moment (μ_p)

- In 1933 Otto Stern measured the proton magnetic moment and found that it differed from one $\Longleftrightarrow$ anomalous magnetic moment

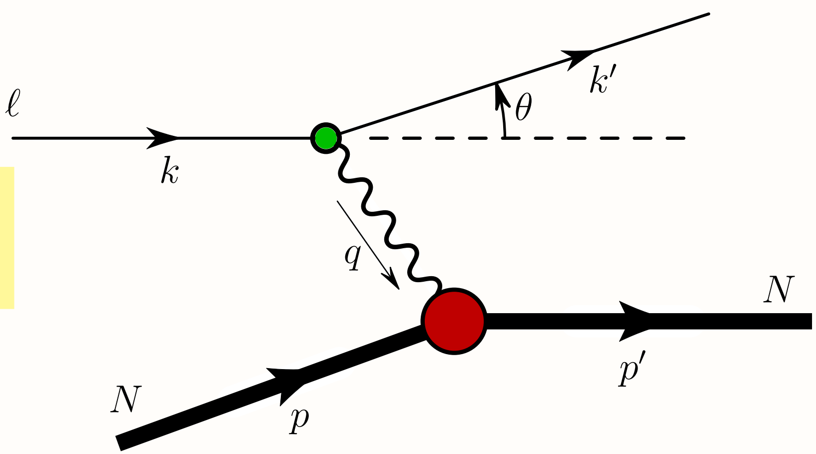
Dirac: $\mu_p = \dfrac{e_p \, \hbar}{2 \, M_P}$ Stern: $\mu_p = (1 + 1.79) \, \dfrac{e_p \, \hbar}{2 \, M_P}$

  - ✦ this was strong evidence that the proton was not a point particle
  - ✦ later of course quarks were discovered at SLAC in 1968 via deep inelastic experiments

- In 1943 Otto Stern would receive the Nobel Prize in part for this discovery

Nucleon electromagnetic form factors

- The electromagnetic structure of the nucleon is best determined by electron elastic scattering
- The electron makes a good probe because its interaction with the electromagnetic current is very well understood
  - the electron anomalous magnetic moment is known experimentally to 1 part in a 1 trillion $a = 0.00115965218085(76)$
  - theory agrees almost perfectly
- The interaction of the electromagnetic with the nucleon is characterized by two form factors



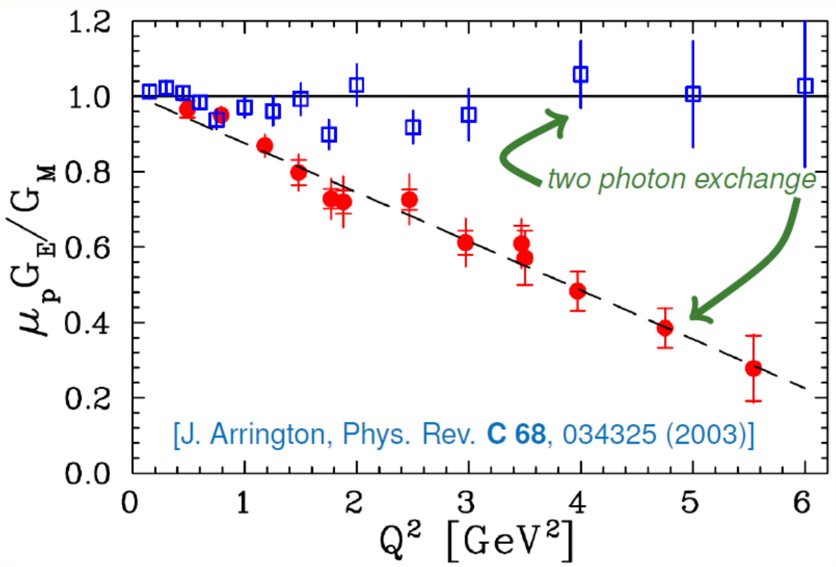
$$\langle J^\mu \rangle = u(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right] u(p)$$

Dirac (F_1) Pauli (F_2)

- Sachs form factors: $G_E = F_1 - \frac{Q^2}{4M^2} F_2, \quad G_M = F_1 + F_2$

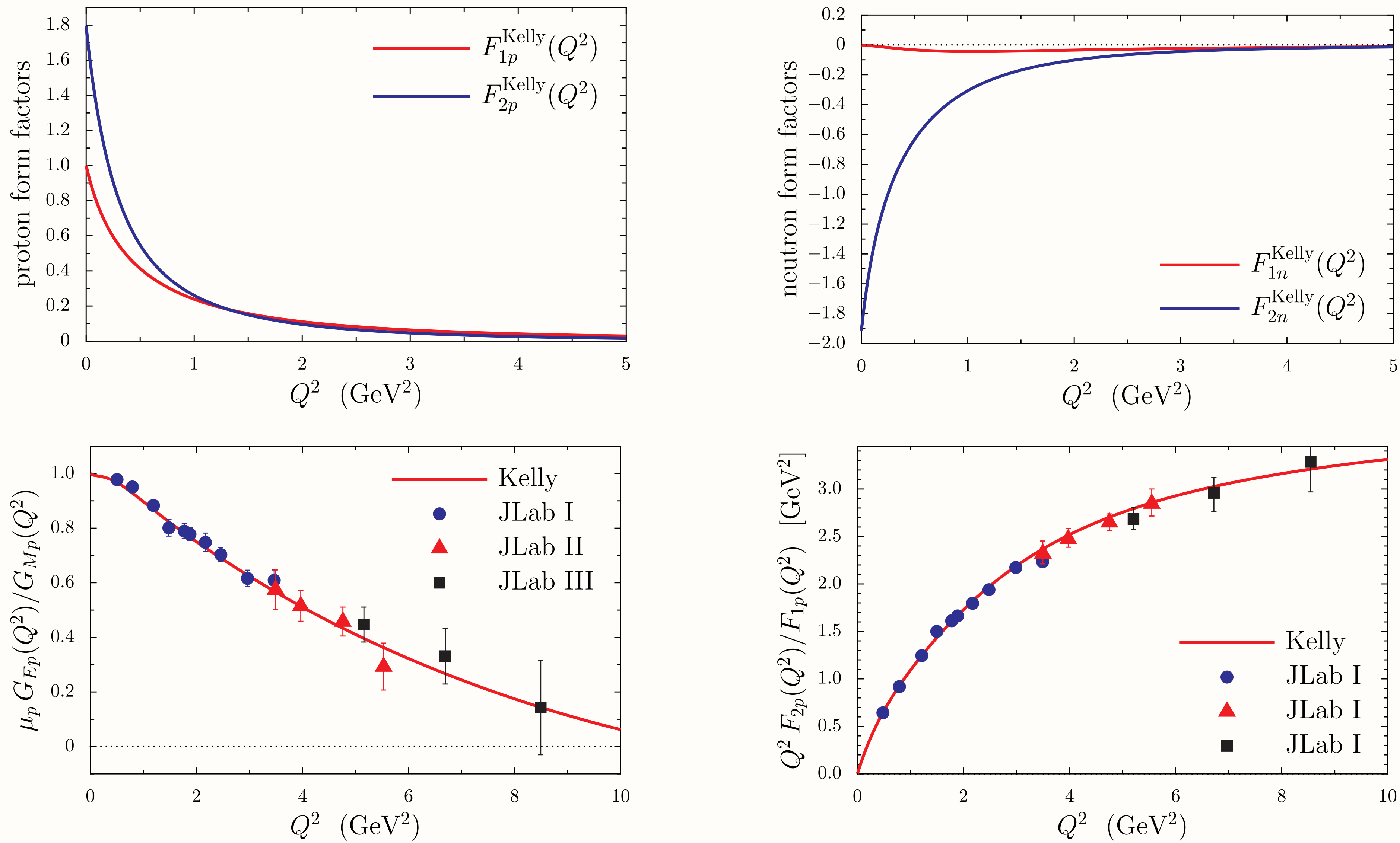
Experimental Status (1)

- Proton form factors were first measured by Hofstadter *et al.* in 1953
  - Deviation from constant gives information on nucleon structure e.g. radii
- Many new things are still being learnt about nucleon EM structure
- A recent atomic experiment discovered the "Proton Radius puzzle"
  - $r_{Ep} = 0.84184 \pm 0.00067$ fm muonic hydrogen [Pohl *et al.*]
  - $r_{Ep} = 0.8768 \pm 0.0069$ fm ep elastic scattering & hydrogen [PDG]
  - radius is defined by: $\langle r_E^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_E(Q^2)\Big|_{Q^2=0}$



- Until the late 90s Rosenbulth experiments found that the G_{Ep}/G_{Mp} ratio was flat
- However JLab polarization transfer experiments which are directly sensitive to this ratio, found a slope toward zero

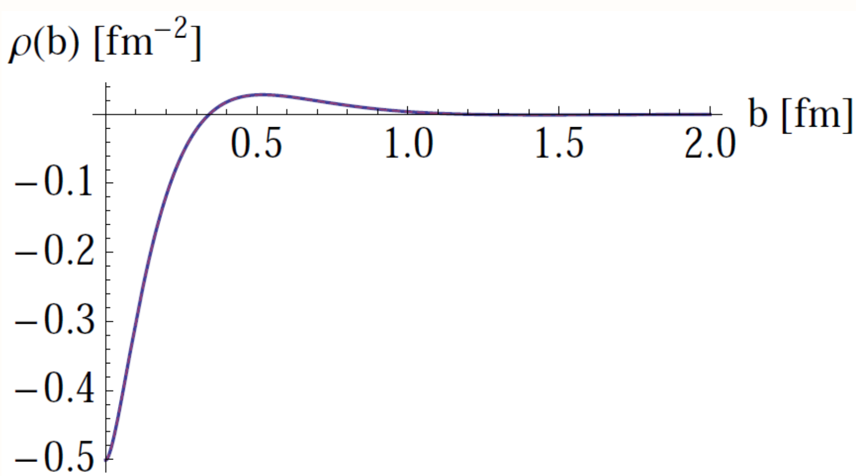
Experimental Status (2)



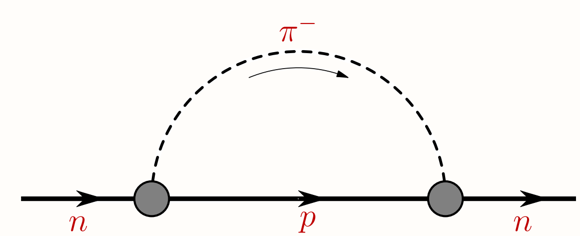
- pQCD $F_1 \sim 1/Q^4 \qquad F_2 \sim 1/Q^6 \quad \Longrightarrow \quad Q^2\, F_2/F_1 \sim$ constant
  - this behaviour is not seen in the data yet: $Q\, F_2/F_1 \sim$ constant

Physical Interpretation of Form Factors

- Most textbooks teach that the Fourier transform of the Sachs form factors, G_E and G_M, give the charge and magnetization densities
  - this is definitely true in the non-relativistic limit where the initial and final states are invariant under Galilean transformations
  - however relativistically the initial and final states are not invariant under Lorentz boosts and therefore a density cannot be defined
- A modern interpretation of the form factors is that they provide information on the transverse charge density, because the transverse structure is invariant under boosts in the z-direction
- The transverse charge densities are given by 2-dimensional Fourier transforms of the Dirac and Pauli form factors

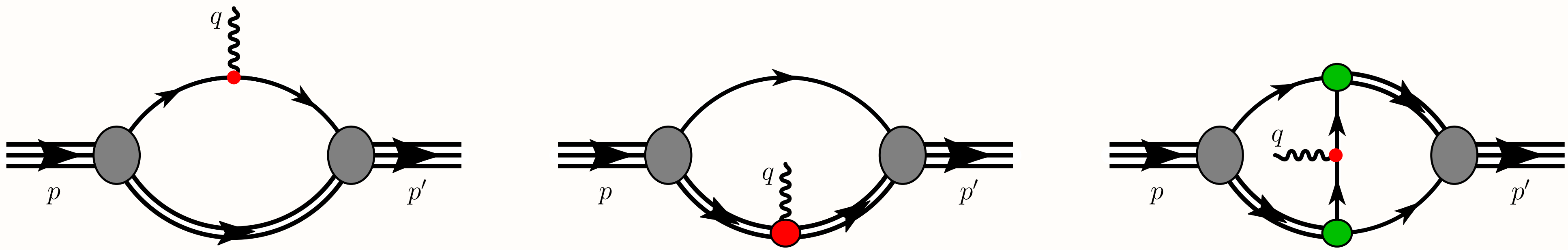


- Neutron negative charge density contradicts pion cloud picture



Nucleon Form Factors in the NJL model

- The Feynman diagrams that give the nucleon form factors in our NJL are



- Ingredients are:
  - ✦ nucleon Faddeev amplitude $\Longleftrightarrow$ Faddeev equation
  - ✦ diquark propagators $\Longleftrightarrow$ Bethe-Salpeter equation
  - ✦ diquark BS vertex $\Longleftrightarrow$ homogeneous Bethe-Salpeter equation
  - ✦ quark propagator $\Longleftrightarrow$ gap equation
  - ✦ quark photon vertex $\Longleftrightarrow$ inhomogeneous Bethe-Salpeter equation
- A separate calculation gives diquark form factors
- We also make the “static approximation” to the quark exchange kernel:

$$S(p) = \left[\not{p} - M + i\varepsilon\right]^{-1} \longrightarrow M^{-1}$$

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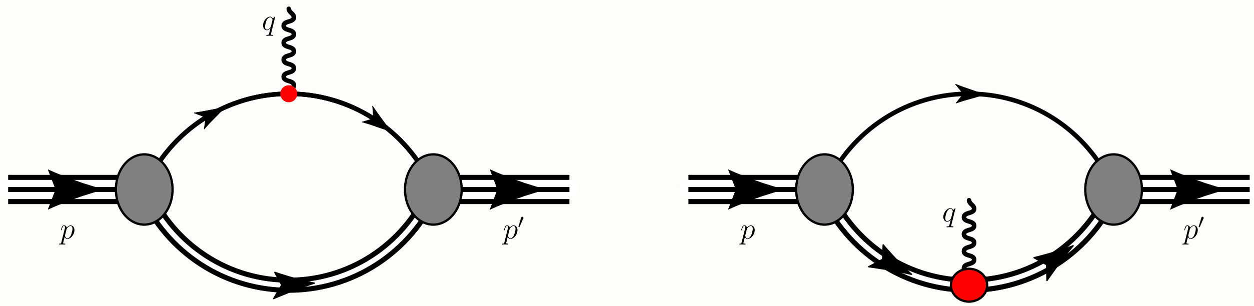

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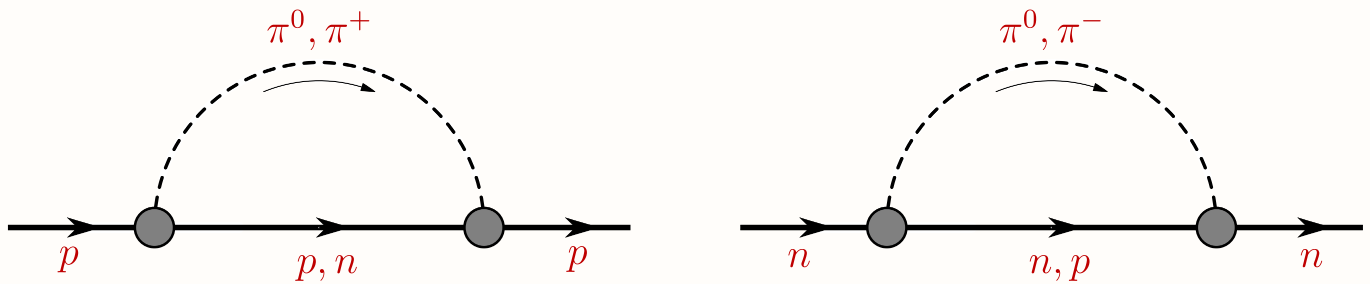
Model Parameters

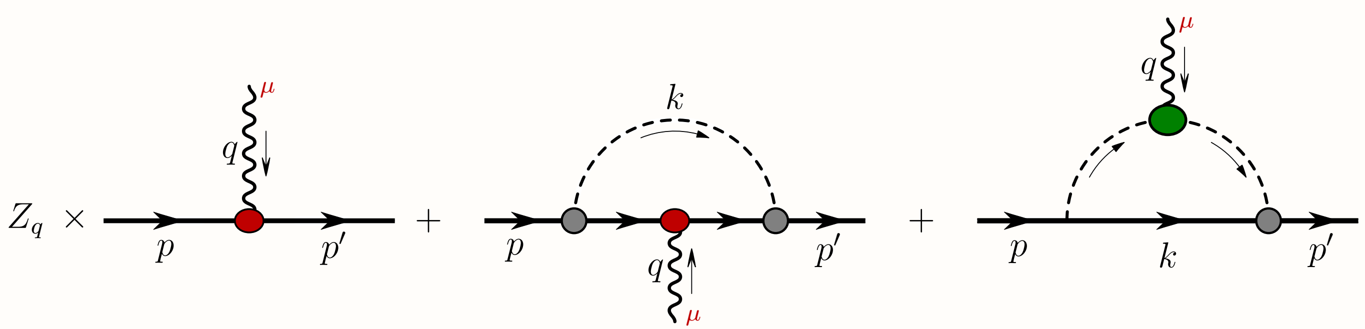
- Free Parameters:
  - $\Lambda_{IR},\ \Lambda_{UV},\ M_0,\ G_\pi,\ G_s,\ G_a,\ G_\omega,\ G_\rho$
- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV & $M_N = 940$ MeV
  - $g_A = 1.267$ [from Bjorken sum rule]
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\text{ MeV})$
  - $a_4 = 32$ MeV
  - $\Lambda_{IR} = 240$ MeV, $M_0 = 400$ MeV
- Obtain [MeV]:
  - $\Lambda_{UV} = 644$
  - $M_s = 690,\quad M_a = 990,\quad \ldots$
- Can now study a large array of observables:
  - e.g. meson and baryon quark distributions, form factors, GPDs, properties at finite temperature and density; neutron stars, etc

The role of Pions

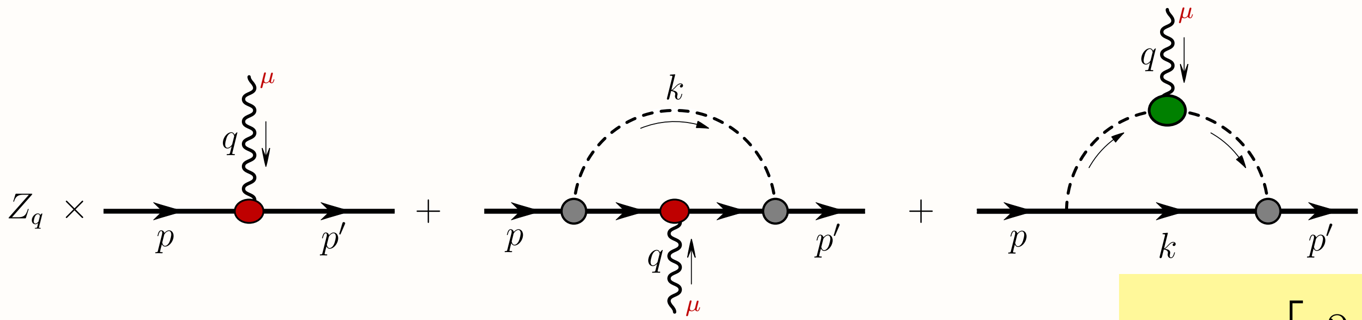
- Pions are the lightest hadrons, therefore, because of quantum fluctuations we expect them to play an important role in many observables



- Because the pion is light it is long range
  - ✦ expect proton and neutron charge and magnetic radii to be increased
  - ✦ the nucleon magnetic moments are also sensitive to pion cloud effects
- To include pions in NJL we dress the constituent quarks with a pion cloud



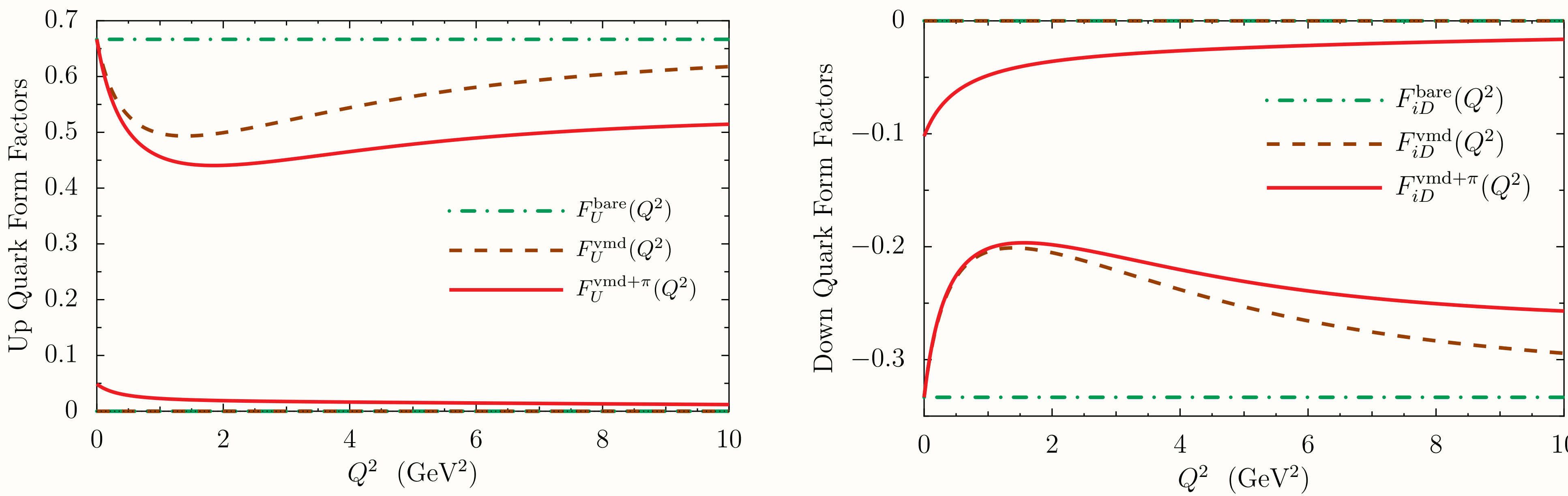
Quark Form Factors with Pion Cloud



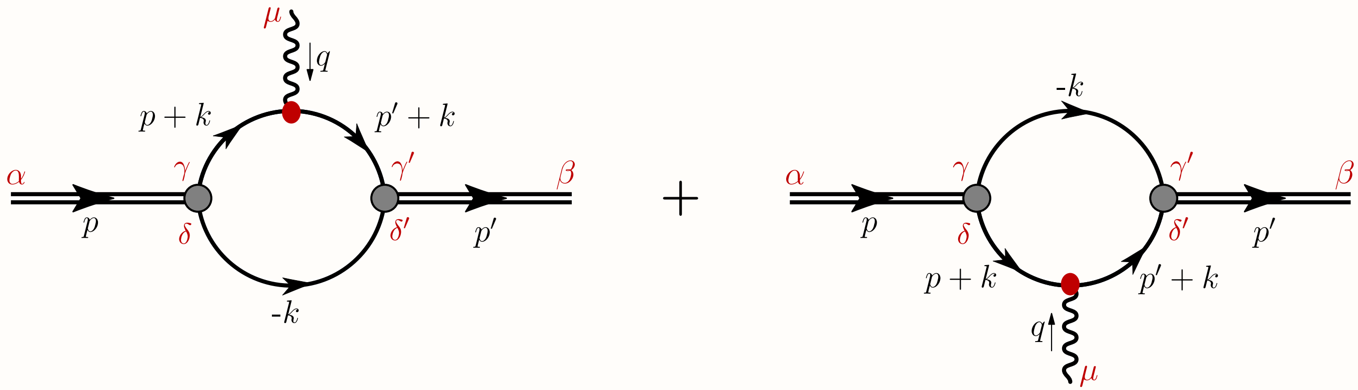
- Z_q is the probability to find a bare constituent quark: $Z_q = \left[\frac{\partial}{\partial \not{p}} S(p)\right]^{-1}_{\not{p}=M}$
- Pion cloud induces an anomalous magnetic moment for the quarks

$$F_{1q}(Q^2) = Z_q \left[\frac{1}{6} F_\omega(Q^2) + \frac{1}{2}\tau_3 F_\rho(Q^2)\right] + \left[F_\omega(Q^2) - \tau_3 F_\rho(Q^2)\right] F_{1q}^{(q)}(Q^2) + \tau_3 F_\rho\ F_{1q}^{(\pi)}(Q^2)$$

$$F_{2q}(Q^2) = \left[F_\omega(Q^2) - \tau_3 F_\rho(Q^2)\right] F_{2q}^{(q)}(Q^2) + \tau_3 F_\rho\ F_{2q}^{(\pi)}(Q^2)$$



Scalar Diquark Form Factor



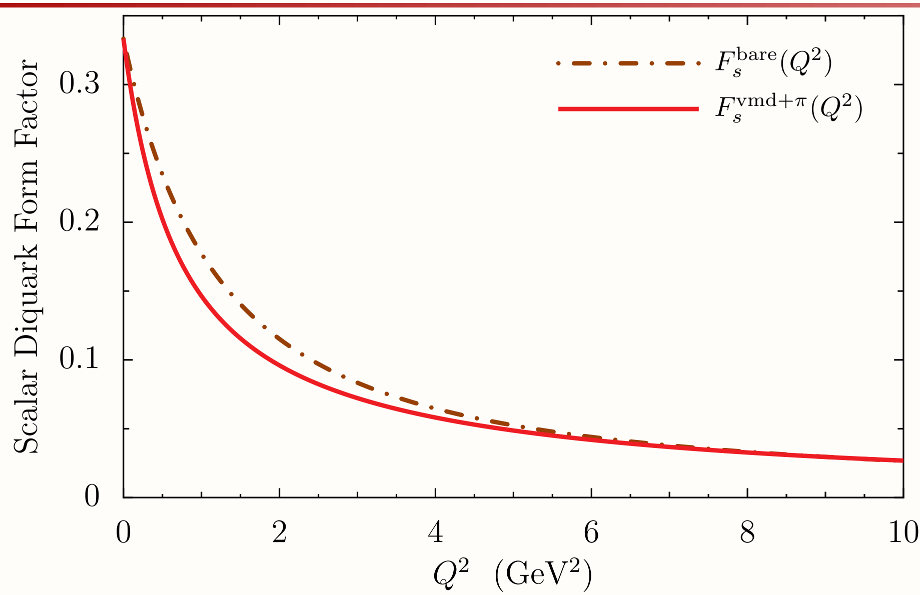
- Scalar diquark BS vertex: $\Omega_s = \sqrt{g_s}\,\gamma_5\,C\,\tau_2\,\beta^A$ (c.f. pion $\Omega_\pi = \sqrt{g_\pi}\,\gamma_5\,\tau_i$)
- Expressions for pion and scalar diquark form factors are very similar: $g_s \leftrightarrow g_\pi$ & $M_s \leftrightarrow m_\pi$
- In principle the scalar diquark is off its mass shell ($p^2 \neq M_s^2$), therefore like the off-shell pion case the electromagnetic vertex is of the form

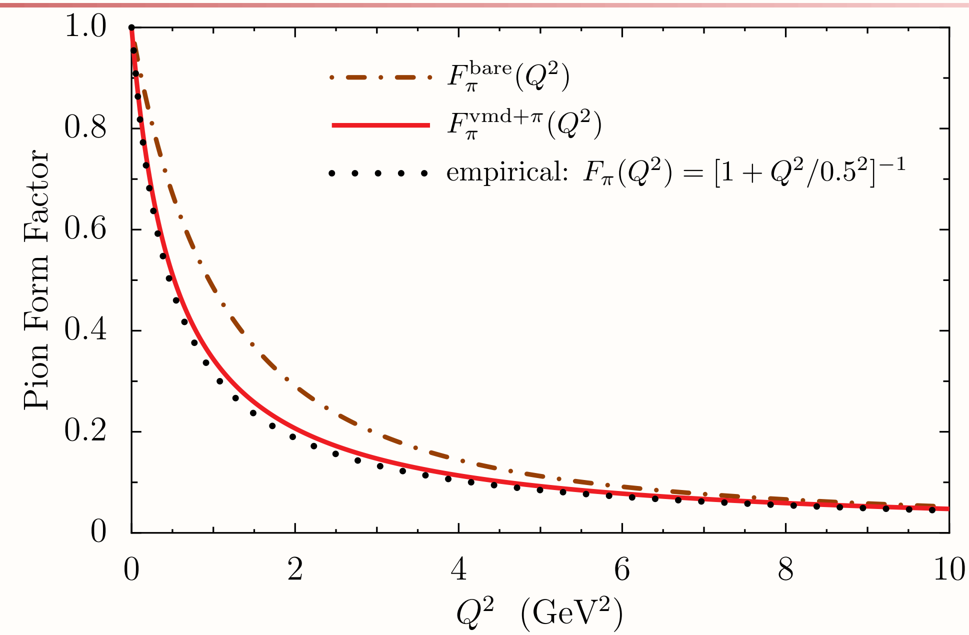
$$\langle \Gamma_s^\mu \rangle = (p' + p)^\mu\, F_{s1}(p'^2, p^2, Q^2) + (p' - p)^\mu\, F_{s2}(p'^2, p^2, Q^2) \to (p' + p)^\mu\, F_s(Q^2)$$

✦ therefore we make the on-shell approximation for this vertex

✦ comparison with experiment informs on veracity of this approximation

Scalar Diquark Form Factor Results





- Pion charge radius:

experiment: $\langle r_E^2\rangle_\pi = 0.45 \pm 0.01\,\text{fm}^2$; NJL model: $\langle r_E^2\rangle_\pi = 0.46\,\text{fm}^2$

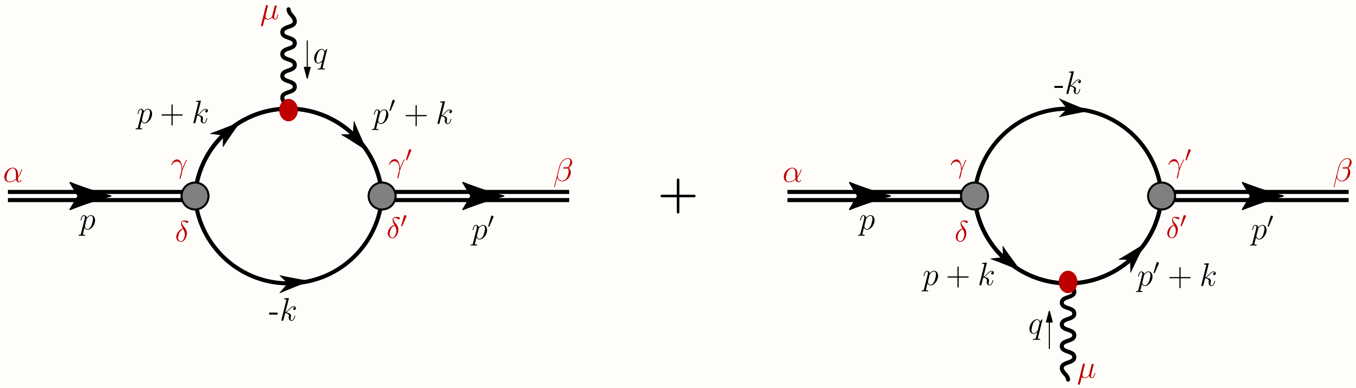
- Recall the perturbative QCD result for pion form factor at $Q^2 \to \infty$

$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2) \implies \alpha_{NJL} = 0.94 \implies Q^2 \sim 0.46\,\text{GeV}^2$$

  - ✦ in the NJL model $\alpha_s(Q^2) = $ constant
  - ✦ use above pQCD result to estimate effective strong coupling in NJL

- Evidence that our scalar diquark form factor is reliable

Axial–Vector Diquark Form Factors



- The axial-vector diquark and rho BS vertices are

$$\Omega_a^{\mu,i} = \sqrt{g_a}\,\gamma^\mu\, C\, \tau_i\, \tau_2\, \beta^A \qquad\qquad \Omega_\rho^{\mu,i} = \sqrt{g_\rho}\,\gamma^\mu\, \tau_i$$

- Results for the rho and axial-vector diquark form factors are very similar: $g_a \leftrightarrow g_\rho$ & $M_a \leftrightarrow m_\rho$
- Again, in principle the axial-vector diquark is off its mass shell ($p^2 \neq M_a^2$) & therefore the electromagnetic vertex has 14 form factors
- Make the on-shell approximation for photon–axial-vector diquark vertex

$$J^{\mu,\alpha\beta} = \left[g^{\alpha\beta}\, F_1(Q^2) - \frac{q^\alpha q^\beta}{2M_a^2}\, F_2(Q^2)\right](p+p')^\mu - \left(q^\alpha\, g^{\mu\beta} - q^\beta\, g^{\mu\alpha}\right)\, F_3(Q^2)$$

✦ an on-shell spin 1 particle has 3 electromagnetic form factors

Axial–Vector Diquark Form Factors (2)

- Sachs form factors can be obtained for a spin-1 particle & are given by

$$G_C(Q^2) = F_1(Q^2) + \frac{2}{3}\,\frac{Q^2}{4\,M^2}\,G_Q(Q^2)$$

$$G_M(Q^2) = F_3(Q^2)$$

$$G_Q(Q^2) = F_1(Q^2) + \left(1 + \frac{Q^2}{4\,M^2}\right) F_2(Q^2) - F_3(Q^2)$$

- The charge, magnetic moment & electric quadruple moment of a spin-1 particle are then given by

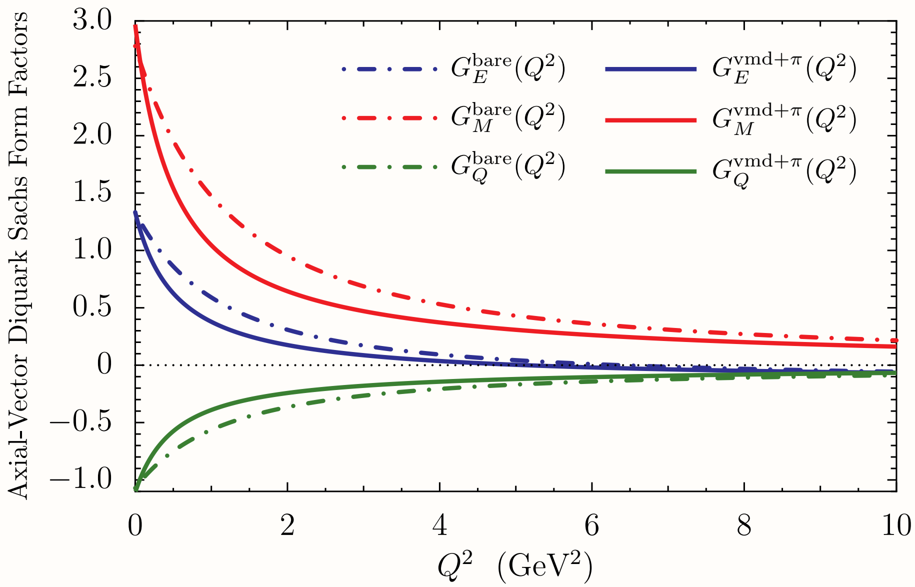
$$Q = G_C(0) \qquad \mu = G_M(0)\,\frac{e}{2\,M} \qquad \mathcal{Q} = G_Q(0)\,\frac{e}{M^2}$$

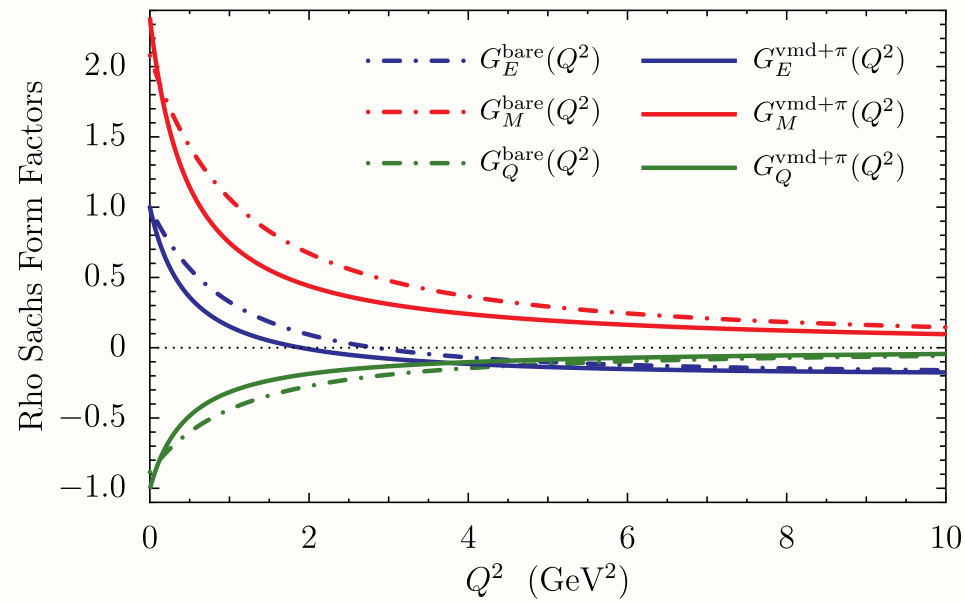
- A charge one structureless spin-1 particle has

$$Q \equiv 1, \qquad \mu = 2, \qquad \mathcal{Q} = -1$$

- Deviation from these canonical values gives information on internal structure of the particle

Axial–Vector Diquark Form Factors Results



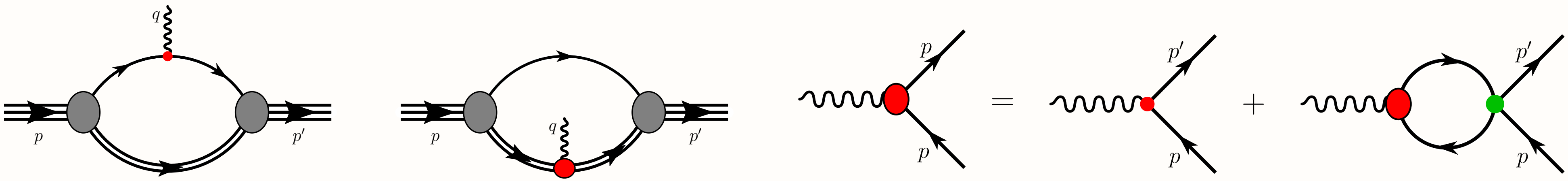


- There is no experimental information on the rho form factors
  - because of short lifetime probably never will be
- Can only compare our result with other calculations
- The NJL model gives: $\mu_\rho = 2.08, \quad \mathcal{Q}_\rho = -0.89, \quad \langle r_E^2 \rangle_\rho = 0.52\,\text{fm}^2$
- DSE results are: $\mu_\rho = 2.01, \quad \mathcal{Q}_\rho = -0.41, \quad \langle r_E^2 \rangle_\rho = 0.54\,\text{fm}^2$

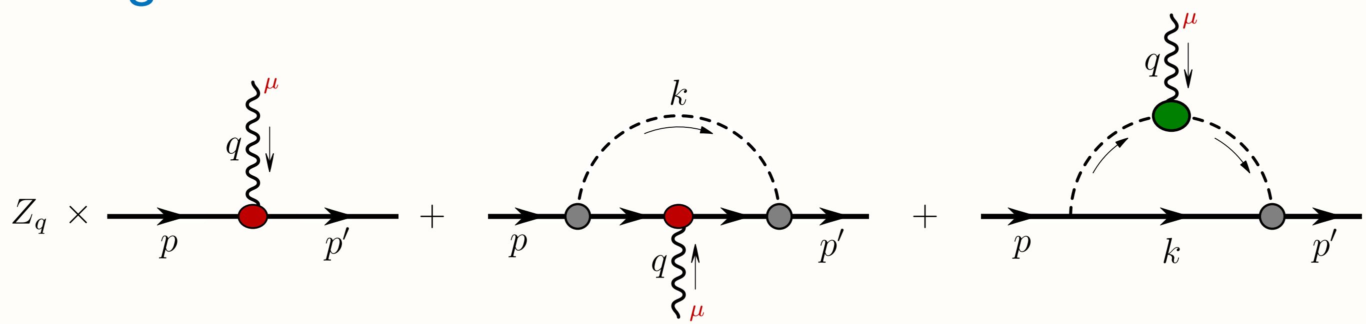
[M. S. Bhagwat and P. Maris, Phys. Rev. **C 77**, 025203 (2008)]

- Good agreement – except for $\mathcal{Q}$

Nucleon Electromagnetic Form Factors



- Now have all ingredients needed to determine NJL nucleon form factors



- The nucleon electromagnetic current is given by

$$\langle J^\mu \rangle = u(p') \left[\gamma^\mu\, F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M}\, F_2(Q^2)\right] u(p)$$

- Include both scalar and axial-vector diquarks

$$\tau_s(q) = \frac{-4i\, G_s}{1 + 2\, G_s\, \Pi_s(q^2)},$$

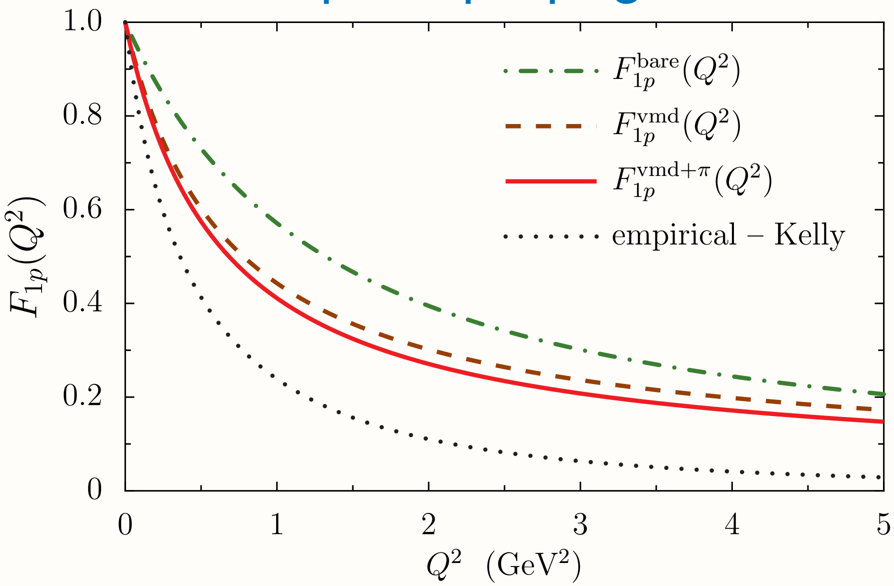
$$\tau_a^{\mu\nu}(q) = \frac{-4i\, G_a}{1 + 2\, G_a\, \Pi_a(q^2)} \left[g^{\mu\nu} + 2\, G_a\, \Pi_a(q^2)\, \frac{q^\mu q^\nu}{q^2}\right],$$

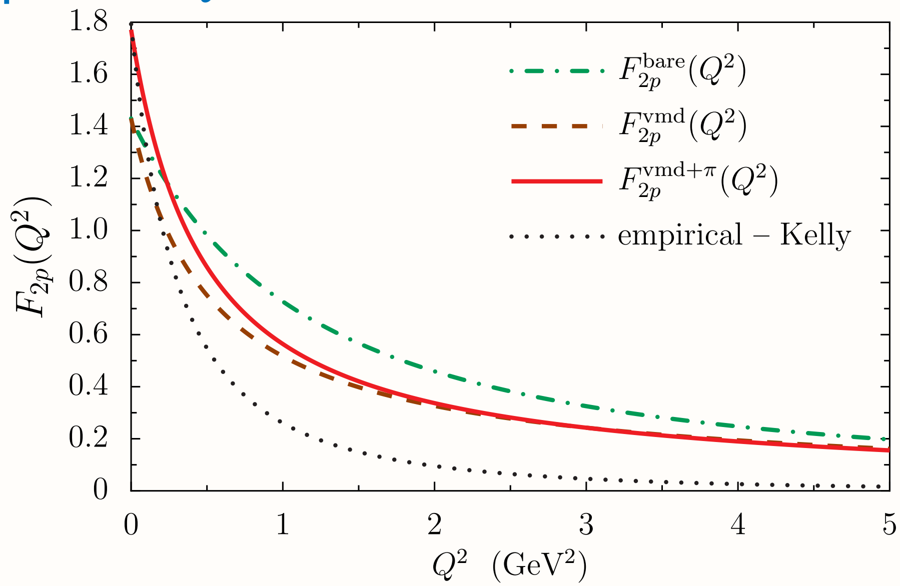
Proton Form Factor Results

- For the proton magnetic moment $(\mu = 1 + \kappa)$ find

$$\mu_p^{\text{bare}} = 2.43\,\mu_N, \qquad \mu_p^{\text{vmd}+\pi} = 2.78\,\mu_N, \qquad \mu_p^{\text{experiment}} = 2.79\,\mu_N$$

  - ✦ pion increases anomalous magnetic moment by $\sim 30\%$
  - ✦ NJL gives excellent results for nucleon static properties
- The NJL form factors fall off too slowing with Q^2, need an extra $1/Q^2$ factor
  - ✦ origin is that Faddeev amplitude has no relative momentum suppression
  - ✦ a self-consistent calculation relates this back to the quark propagator
  - ✦ NJL the quark propagator is too simple for $Q^2 > 0$ observables





Proton Form Factor Results

- For the proton charge and magnetic radii, where

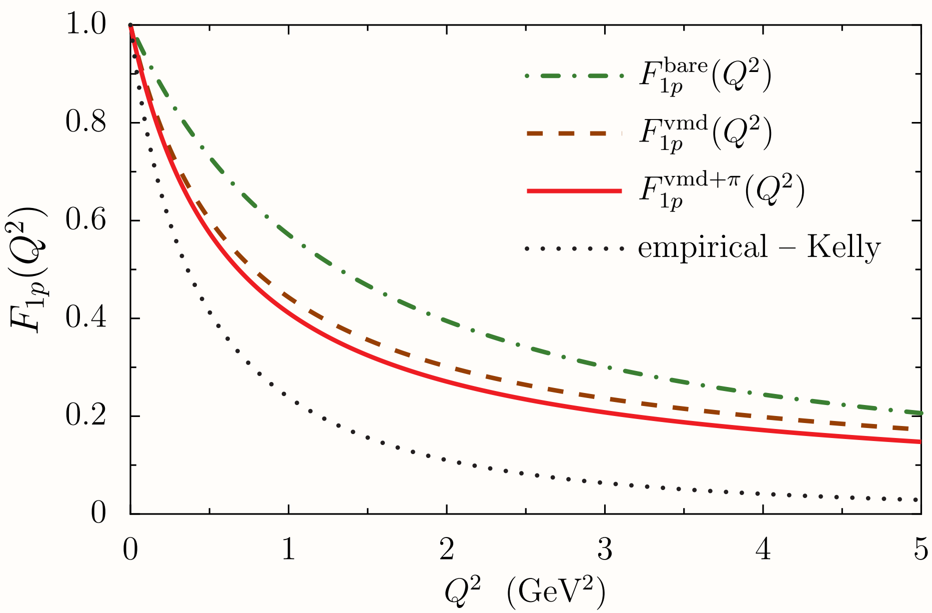
$$\langle r_E^2 \rangle = -6\, \frac{\partial\, G_E(Q^2)}{\partial Q^2}\Big|_{Q^2=0}, \qquad \langle r_M^2 \rangle = -\frac{6}{\mu}\, \frac{\partial\, G_M(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$$

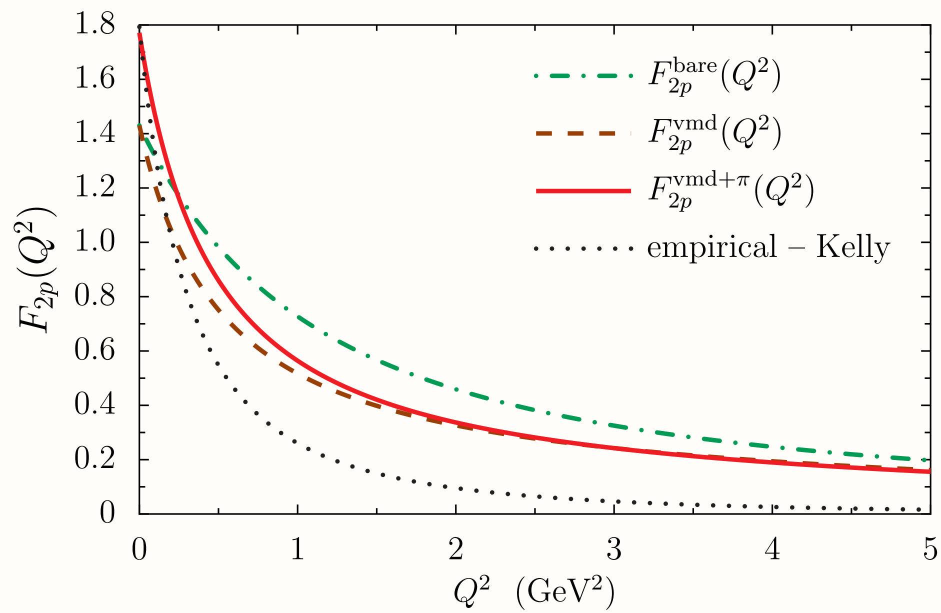
  - ✦ we find

$$\langle r_E^2 \rangle_p^{\text{vmd}} = 0.56\,\text{fm}^2 \quad \langle r_E^2 \rangle_p^{\text{vmd}+\pi} = 0.65\,\text{fm}^2 \quad \langle r_E^2 \rangle_p^{\text{experiment}} = 0.72\,\text{fm}^2$$

$$\langle r_M^2 \rangle_p^{\text{vmd}} = 0.46\,\text{fm}^2 \quad \langle r_M^2 \rangle_p^{\text{vmd}+\pi} = 0.58\,\text{fm}^2 \quad \langle r_M^2 \rangle_p^{\text{experiment}} = 0.71\,\text{fm}^2$$

- Pion cloud is important – however NJL is not sufficient to get good radii





Neutron Form Factor Results

- For the neutron magnetic moment $(\mu = \kappa)$ find

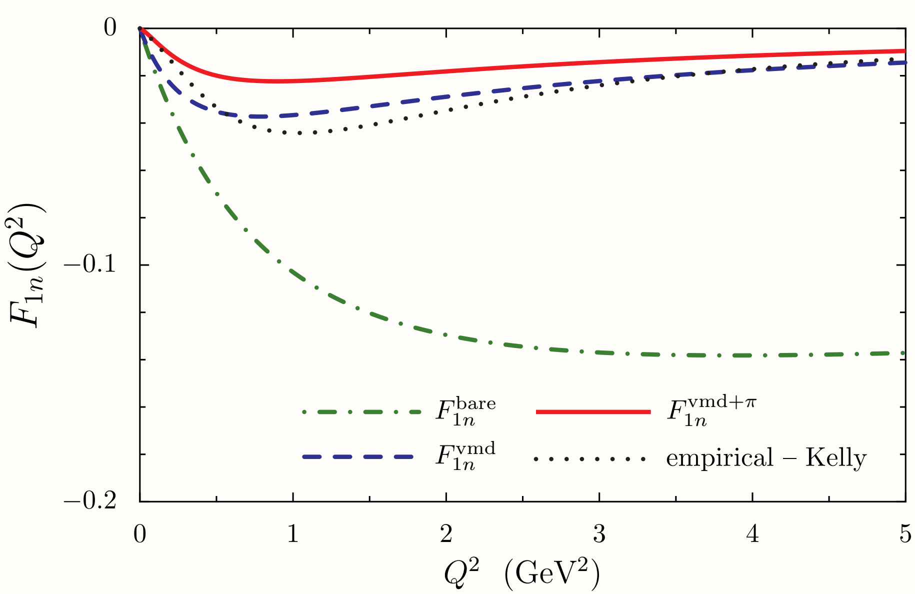
$$\mu_n^{\text{bare}} = 1.25\,\mu_N, \qquad \mu_n^{\text{vmd}+\pi} = 1.81\,\mu_N, \qquad \mu_n^{\text{experiment}} = 1.91\,\mu_N$$

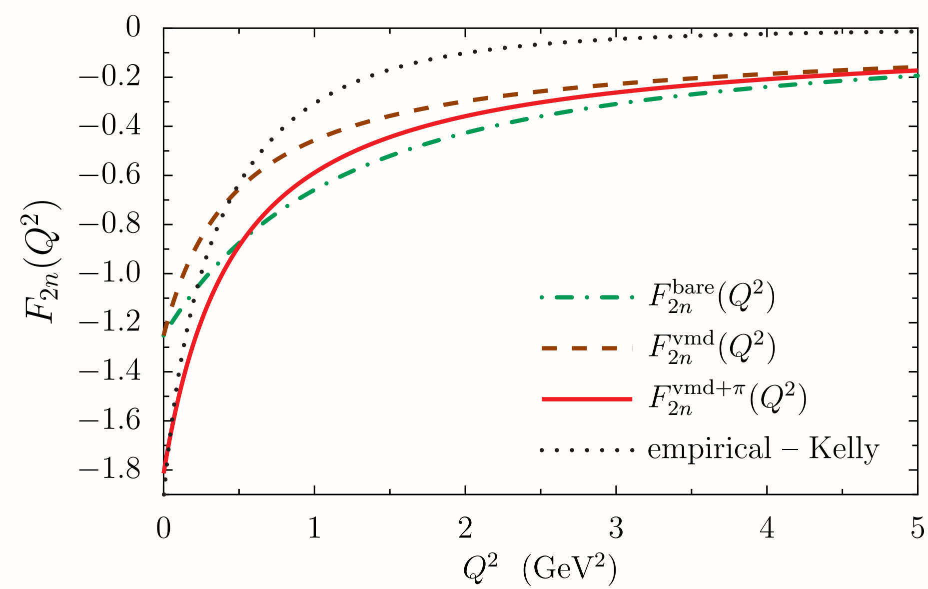
  - ✦ pion increases anomalous magnetic moment by $\sim 45\%$
  - ✦ NJL gives excellent results for nucleon static properties

- For the neutron charge and magnetic radii find

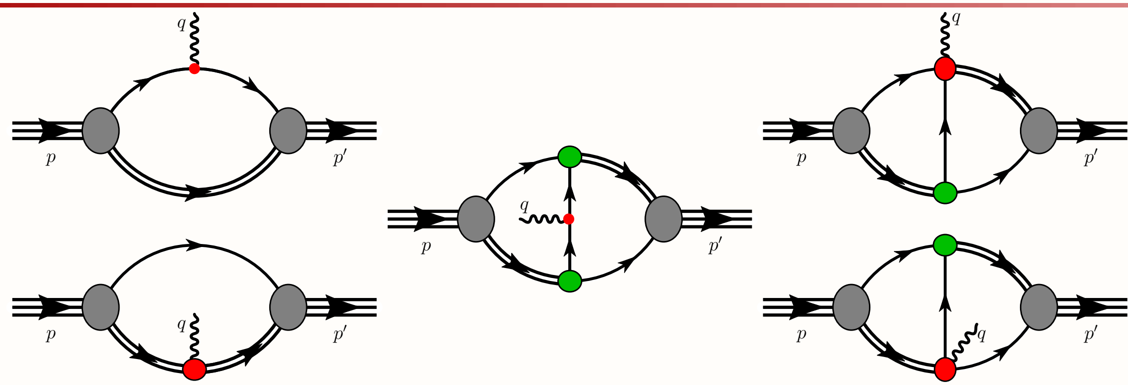
$$\langle r_E^2\rangle_n^{\text{vmd}} = -0.04\,\text{fm}^2 \quad \langle r_E^2\rangle_n^{\text{vmd}+\pi} = -0.08\,\text{fm}^2 \quad \langle r_E^2\rangle_n^{\text{experiment}} = -0.12\,\text{fm}^2$$

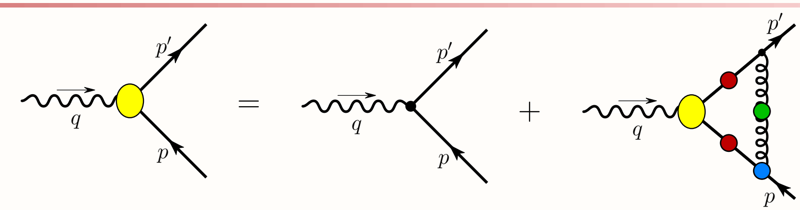
$$\langle r_M^2\rangle_n^{\text{vmd}} = 0.43\,\text{fm}^2 \quad \langle r_M^2\rangle_n^{\text{vmd}+\pi} = 0.61\,\text{fm}^2 \quad \langle r_M^2\rangle_n^{\text{experiment}} = 0.79\,\text{fm}^2$$





Nucleon electromagnetic current using DSEs

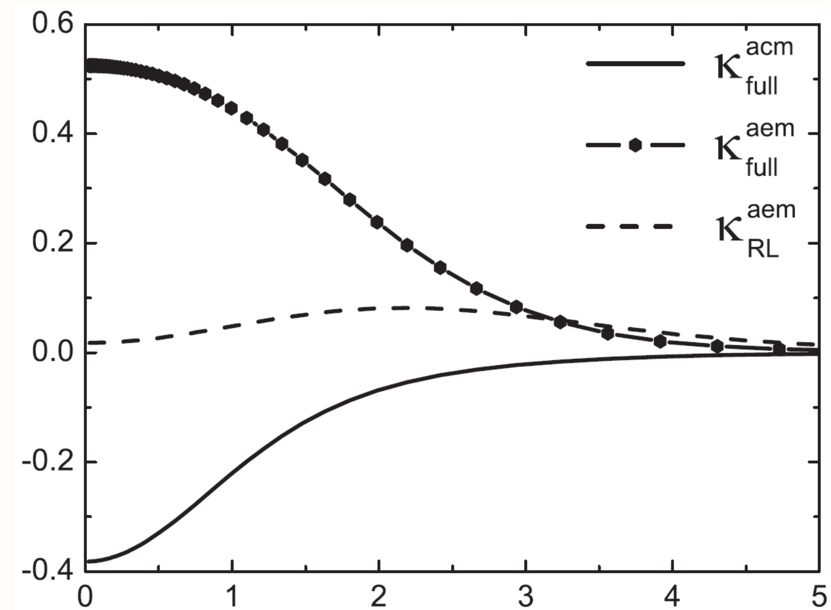




$$\Gamma^\mu = \Gamma^\mu_L + \Gamma^\mu_T; \quad q_\mu \, \Gamma^\mu_T = 0$$

$$q_\mu \, \Gamma^\mu_L = \hat{Q} \, \left[S^{-1}(p') - S^{-1}(p) \right]$$

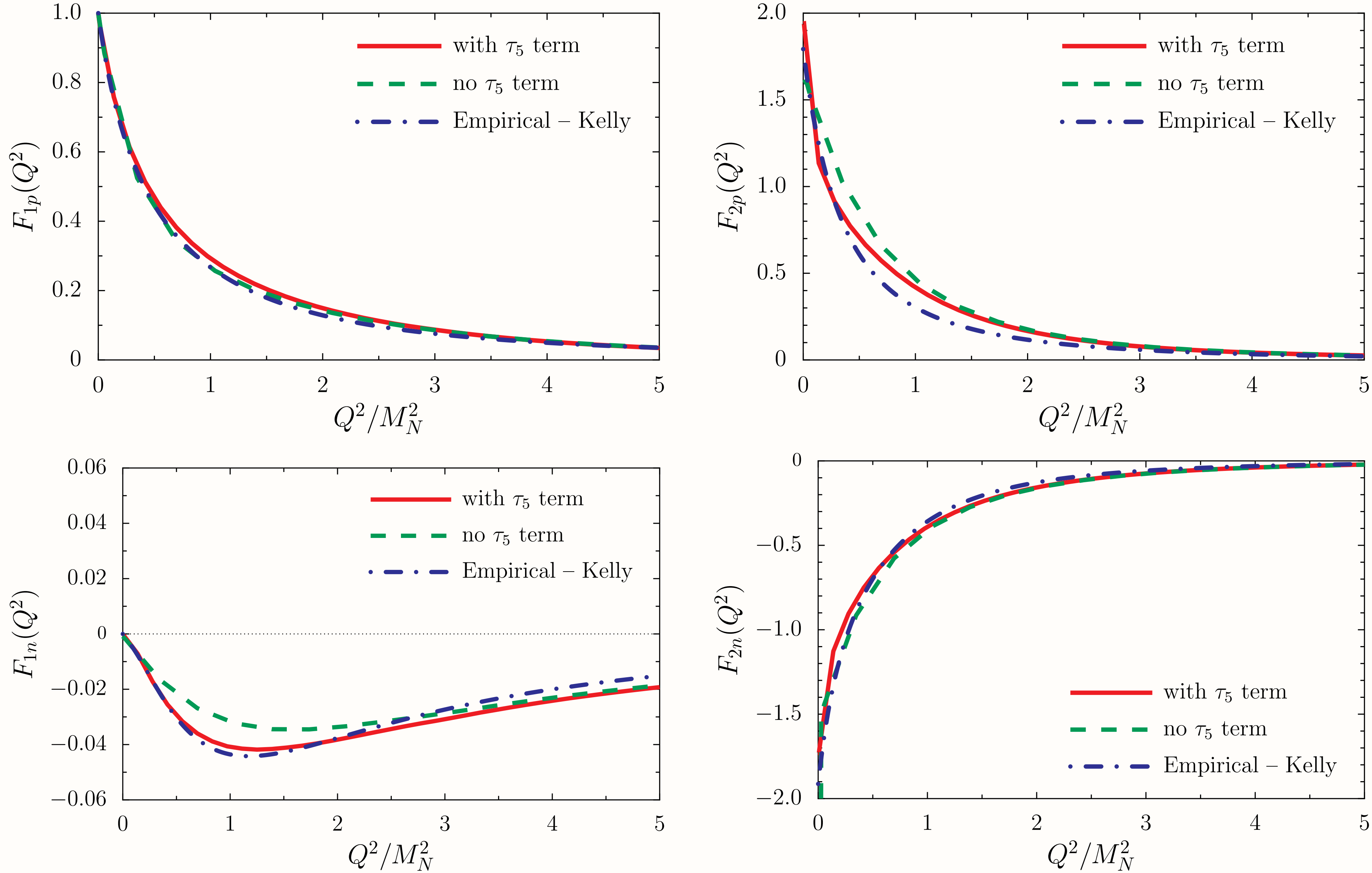
- Feedback with experiment can constrain DSE quark–gluon vertex
- Knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs
  - ✦ also gives β-function and may shed light on confinement
- Add anomalous chromomagnetic term, $i\sigma^{\mu\nu} q_\nu \, \tau_5(p', p)$, to quark–gluon vertex
- Generates anomalous electromagnetic term in quark–photon vertex
- Quarks are strongly dressed by gluons, except sizeable an'lous mag'tic moment



[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]

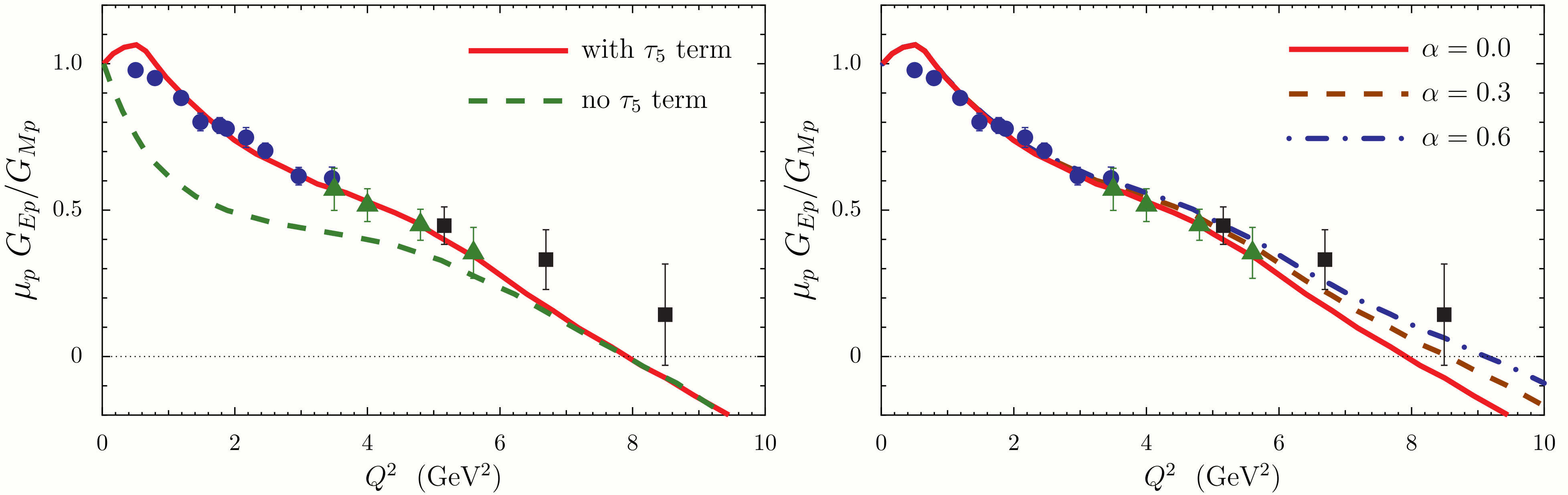
DSE nucleon form factors

[ICC, G. Eichmann, B. El-Bennich, T. Klahn and C. D. Roberts,, Few Body Syst. **46**, 1 (2009)]



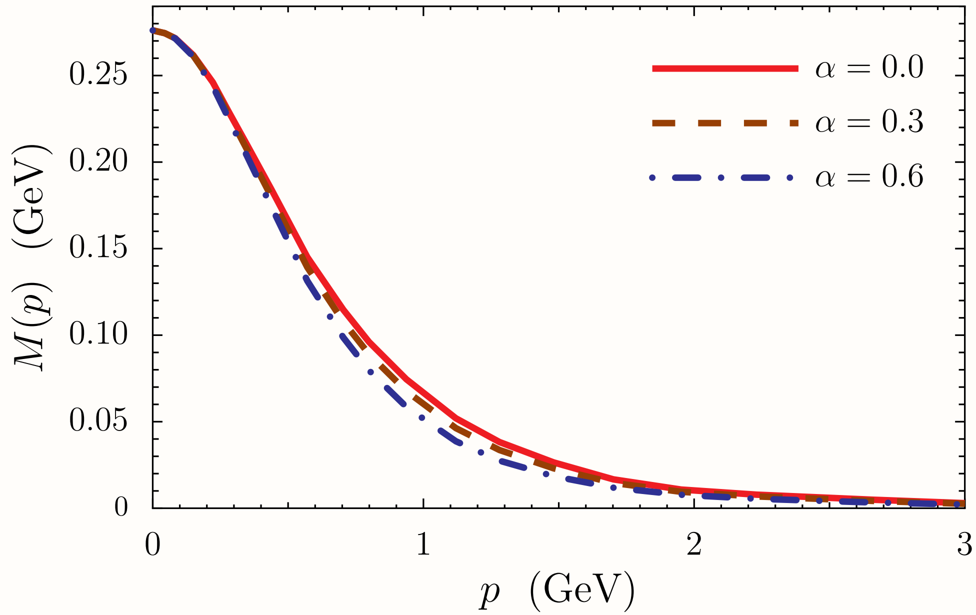
- $\tau_5(p', p)$ is the anomalous magnetic moment term in quark–photon vertex

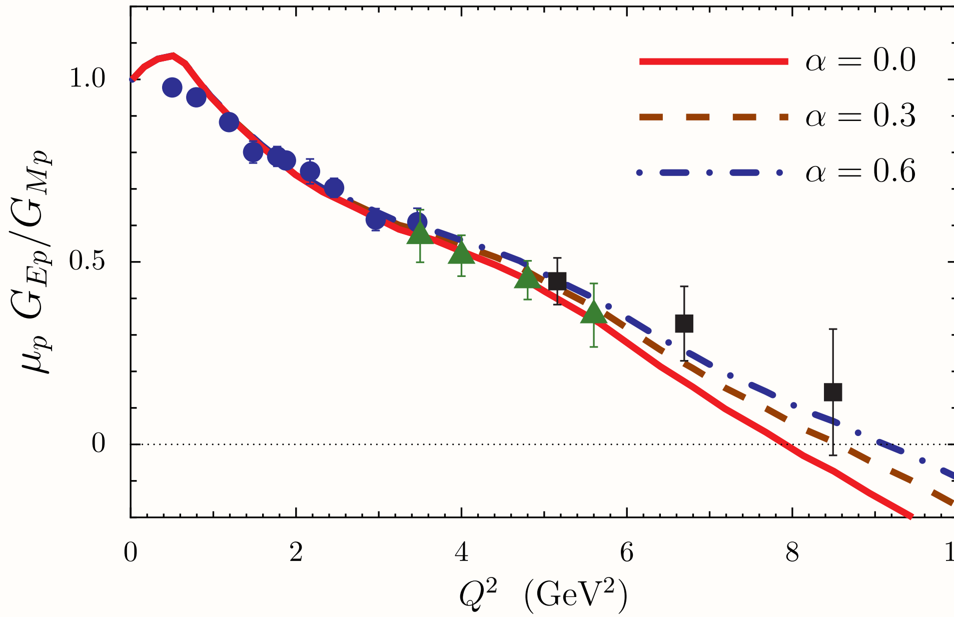
Proton form factors ratios



- Quark anomalous magnetic moment gives good agreement with data
  - important for low to moderate Q^2
- Low Q^2 discrepancy will be alleviated by including ρ and ω contribution to quark-photon vertex
- Zero crossing sensitive to nature of transition from non-perturbative to perturbative regime
  - if perturbative regime is reached quicker $\Longrightarrow$ zero-crossing moves to larger Q^2, since $\tau_5(p',p)$ vanishes quicker: $G_E = F_1 - \frac{Q^2}{4\,M^2}\,F_2$

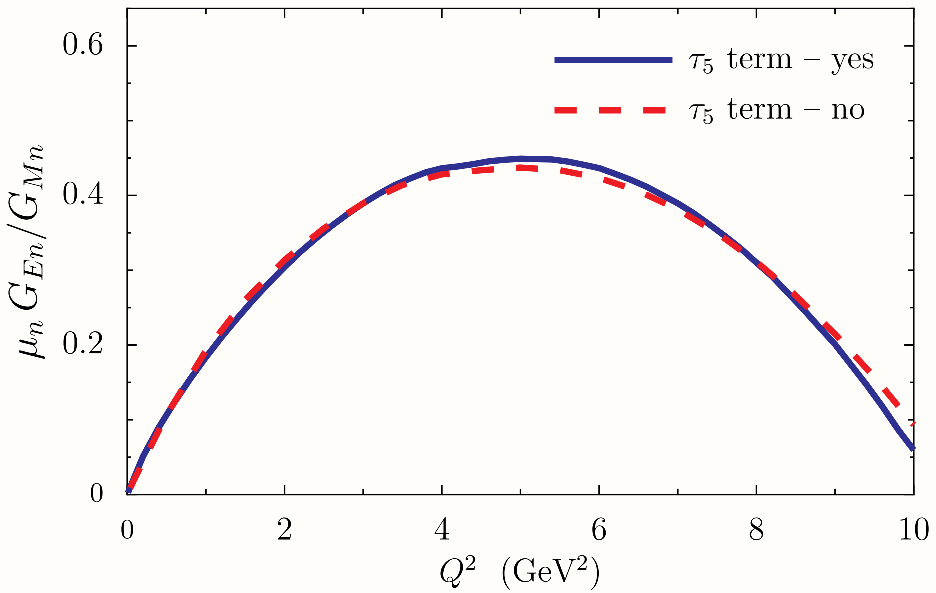
Proton form factors ratios



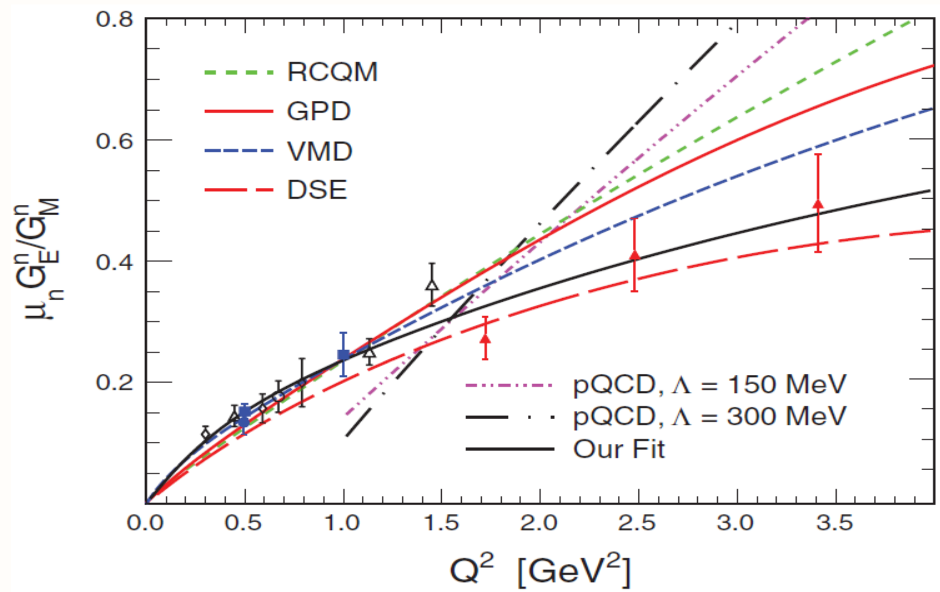


- Quark anomalous magnetic moment gives good agreement with data
  - important for low to moderate Q^2
- Low Q^2 discrepancy will be alleviated by including ρ and ω contribution to quark-photon vertex
- Zero crossing sensitive to nature of transition from non-perturbative to perturbative regime
  - if perturbative regime is reached quicker $\Longrightarrow$ zero-crossing moves to larger Q^2, since $\tau_5(p', p)$ vanishes quicker: $G_E = F_1 - \frac{Q^2}{4\,M^2}\,F_2$

Neutron Form Factors Ratios



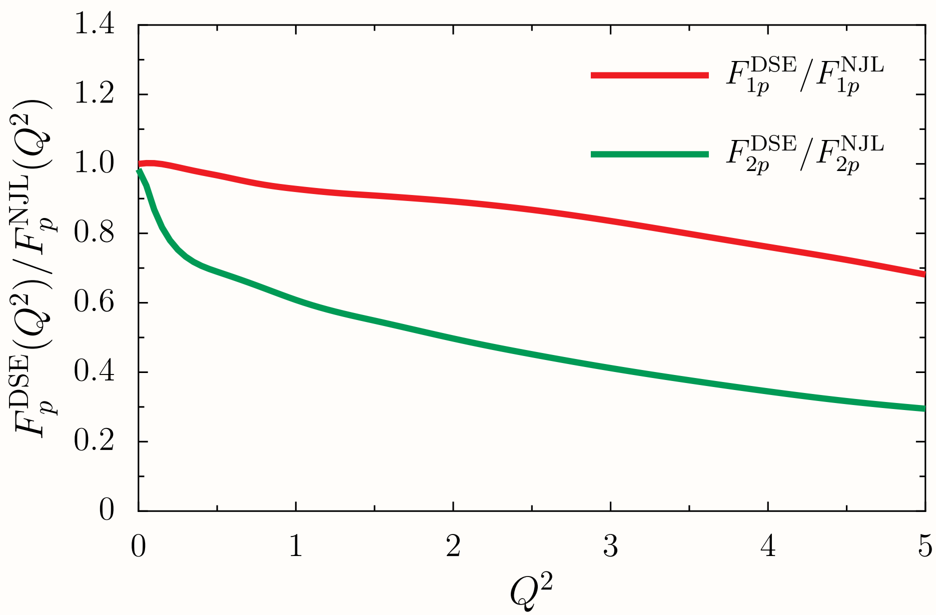
[I. C. Cloët *et al*, Few Body Syst. **46**, 1 (2009)]

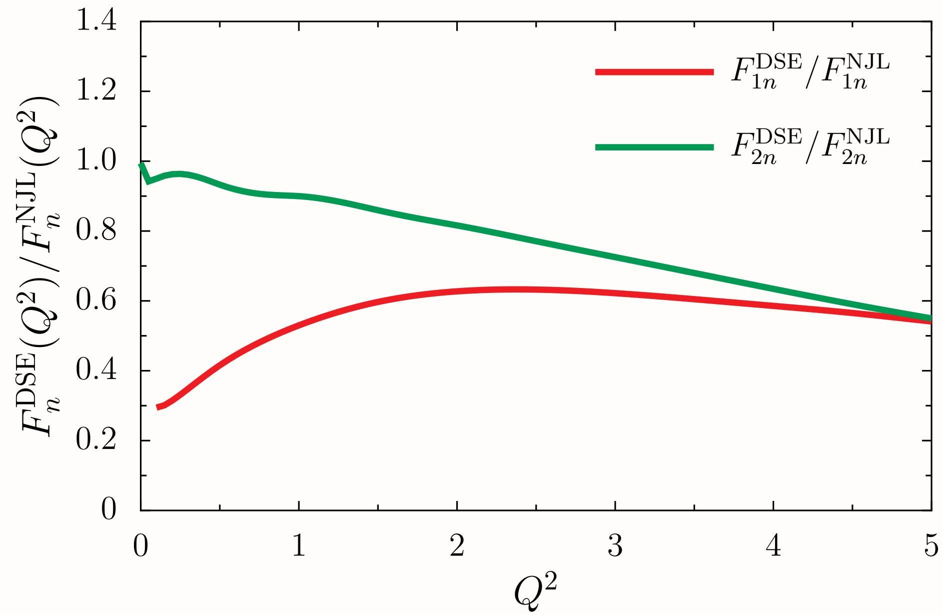


[S. Riordan *et al*, Phys. Rev. Lett. **105**, 262302 (2010)]

- Quark anomalous magnetic moment only has minor impact on neutron form factor ratio
- Predict a zero-crossing in G_{En}/G_{Mn} at $Q^2 \simeq 11\,\text{GeV}^2$
- DSE predictions were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5\,\text{GeV}^2$
- Agreement at low Q^2 requires the pion as a explicit degree of freedom

Comparison with NJL model results

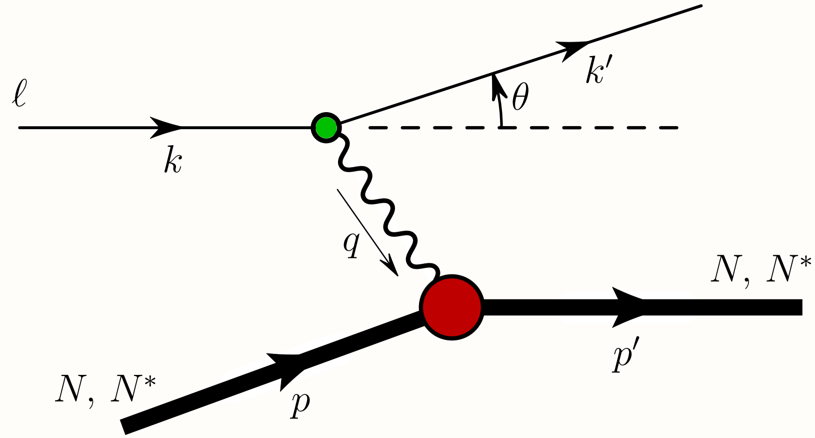




- Find that at $Q^2 = 0$ two results agree rather well
- Reinforces the notion that a constant constituent mass is a reasonable approximation to low energy QCD
  - provided symmetries are preserved
  - good for calculating static properties: magnetic moments, PDFs, etc
- However for $Q^2 \neq 0$ operators – running mass is important

Nucleon, N^* & $N \to N^*$ Electromagnetic Form Factors

- Recall that the nucleon electromagnetic current has the form



$$\langle J^\mu \rangle = u_N(p') \left[\gamma^\mu F_{1N} + \frac{i\sigma^{\mu\nu} q_\nu}{2\, M_N} F_{2N} \right] u_N(p)$$

- The Roper [$N^*(1440)$] is thought to be the first excited of the nucleon and has the same quantum numbers

- Therefore the Roper electromagnetic current has the form

$$\langle J^\mu \rangle = u_{N^*}(p') \left[\gamma^\mu F_{1N^*}(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, M_N^*} F_{2N^*}(Q^2) \right] u_{N^*}(p)$$

- EM current cause transition between the nucleon and Roper $[N \to N^*]$

- Gauge invariance implies this transition current must satisfy: $q_\mu\, J^\mu = 0$

$$\langle J^\mu \rangle = u_{N^*}(p') \left[\left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2} \right) F_{1N\to N^*} + \frac{i\sigma^{\mu\nu} q_\nu}{M_N + M_{N^*}} F_{2N\to N^*} \right] u_N(p)$$

The N^* (Roper) Resonance

- N^* manifests as second pole in Faddeev equation kernel
  - $M_N = 0.940$ GeV and $M_{N^*} = 1.8$ GeV
  - Agrees very well with EBAC value for quark core mass

- "Wavefunction" is given by eigenvector at pole: $p^2 = m_i^2$

- For NJL model N, N^* "wavefunction" has the simple form

$$\Gamma(p) = \begin{bmatrix} \alpha_1 \\ \alpha_2 \frac{p^\mu}{M}\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5 \end{bmatrix} u(p)$$

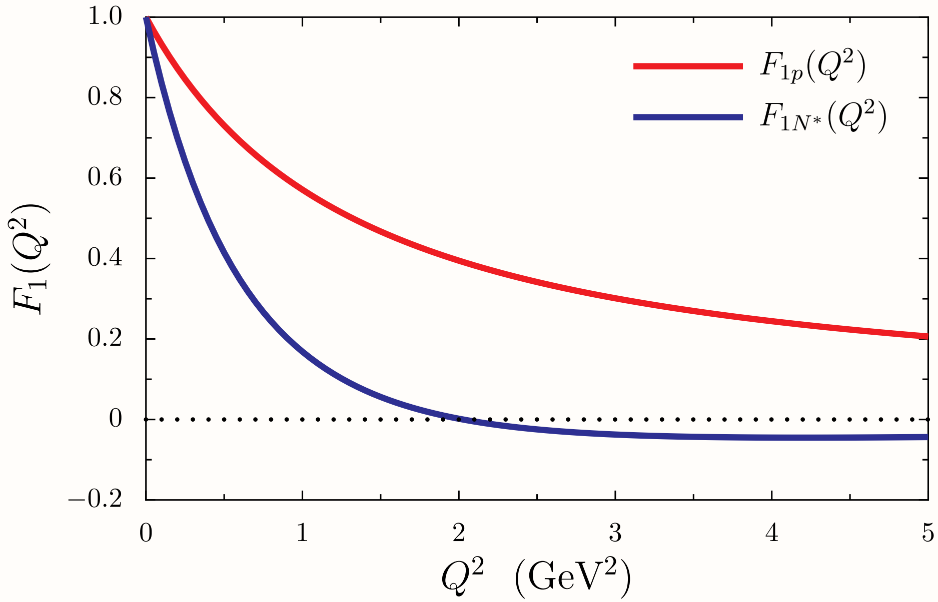
- For the nucleon: $\alpha_1 = 0.43, \quad \alpha_2 = 0.024, \quad \alpha_3 = -0.45$

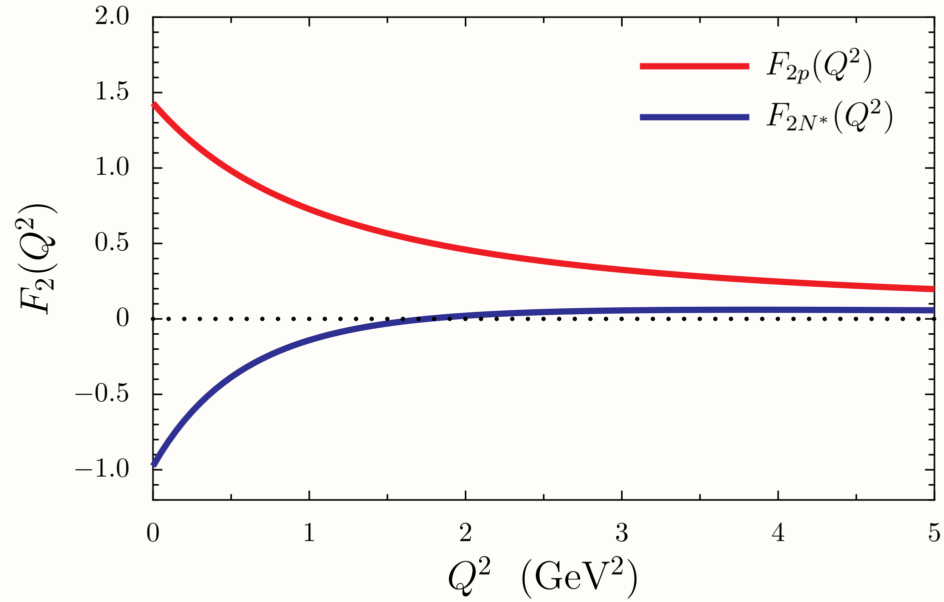
- For the Roper: $\alpha_1 = 0.0011, \quad \alpha_2 = 0.94, \quad \alpha_3 = -0.051$

- For nucleon scalar and axial–vector diquarks equally dominant

- However, N^* is complete dominated by the axial–vector diquark

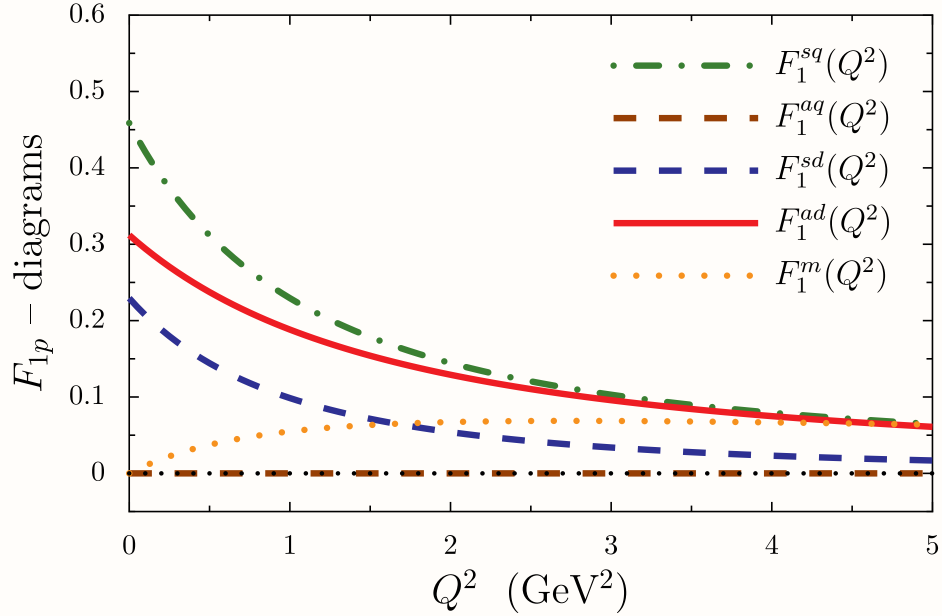
Nucleon and N^* Form Factors

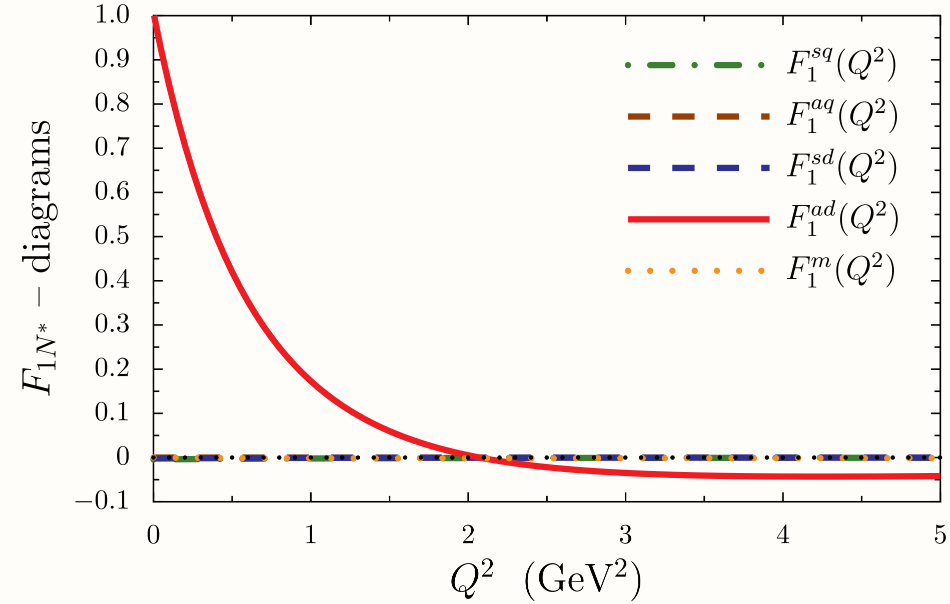




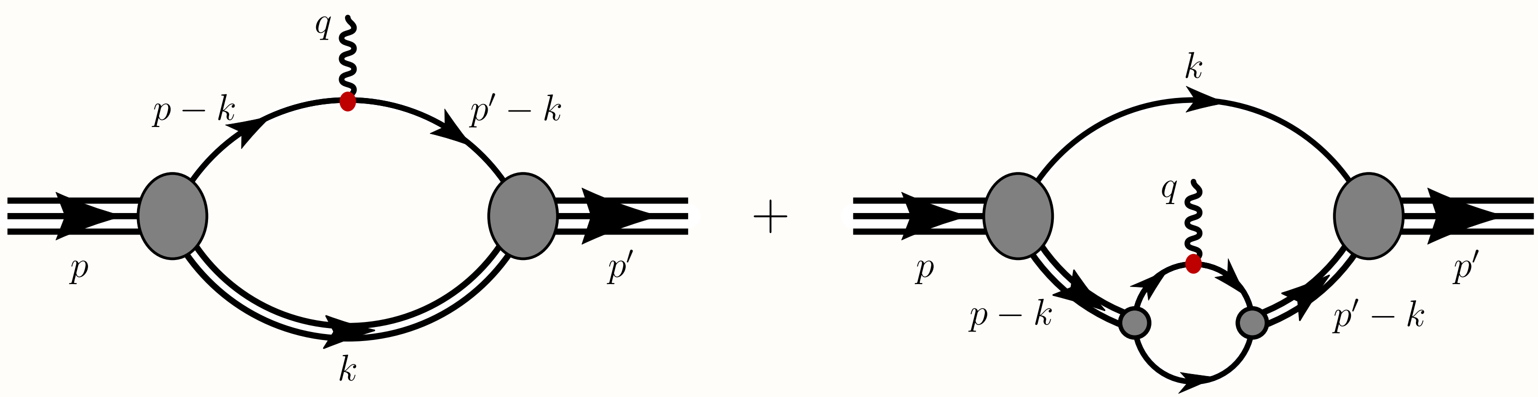
- Note these results are obtained within the constant mass function framework
  - therefore moderate to large Q^2 behaviour is poor
- Pion cloud effects have been ignored
  - expect magnetic moments and radii to be too small
- However we find N^* radii are 10% larger than the protons
- Find a zero in both F_1 and F_2 for Roper

Nucleon and N^* – F_1 Form Factors Results



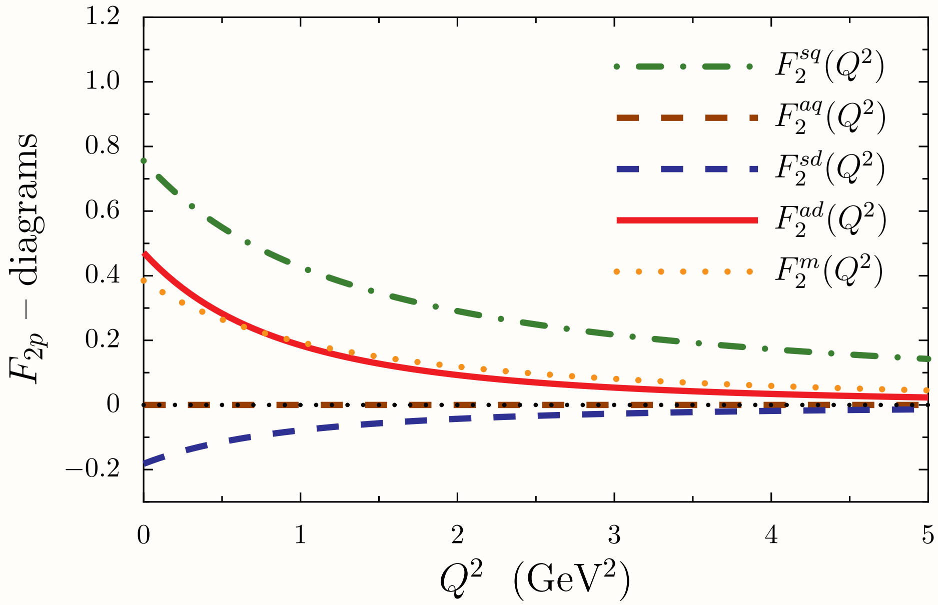


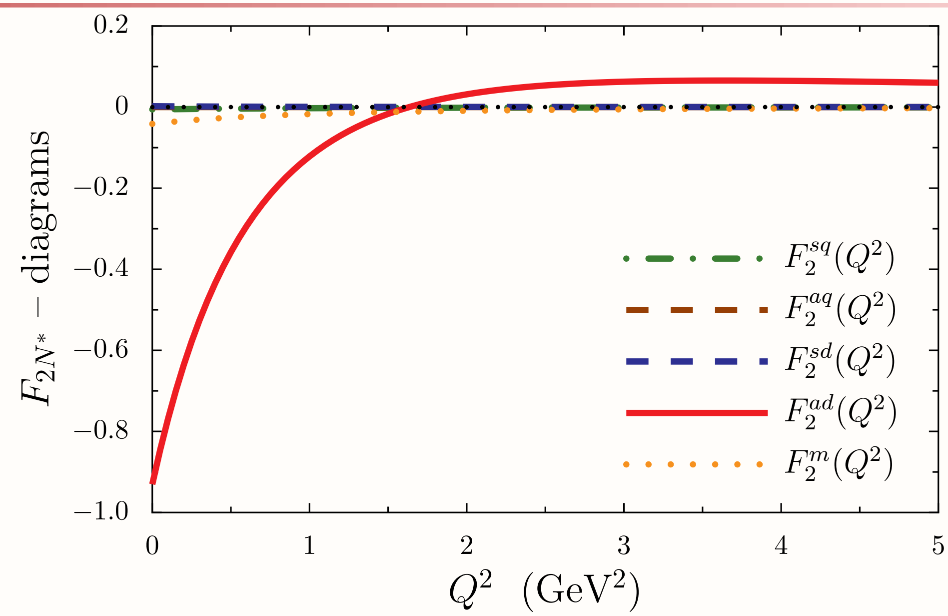
- Contributions originate from the following diagrams



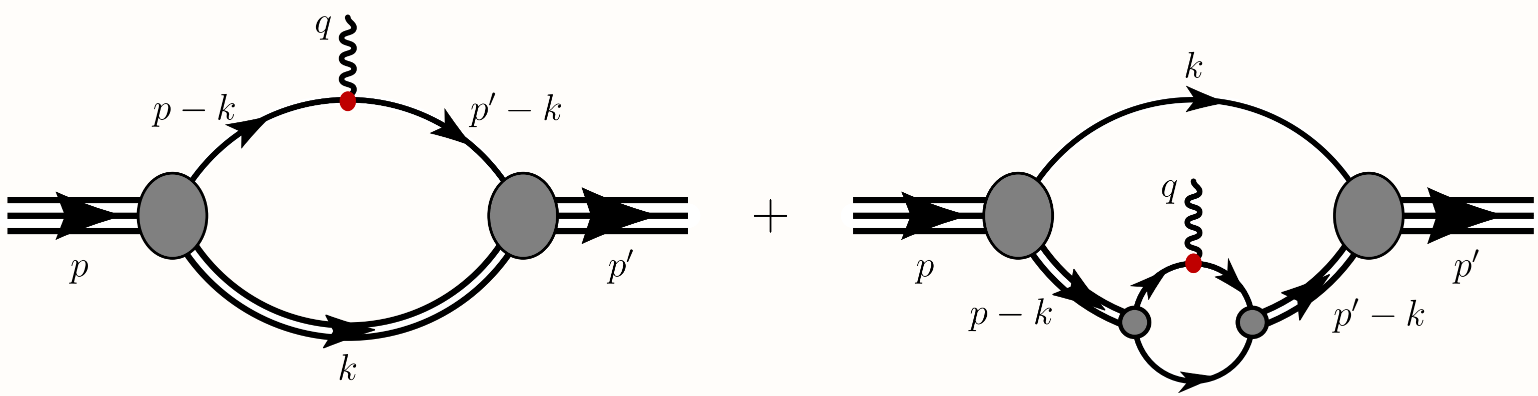
- Find that N^* form factors are axial–vector diquark dominated

Nucleon and N^* – F_2 Form Factors Results



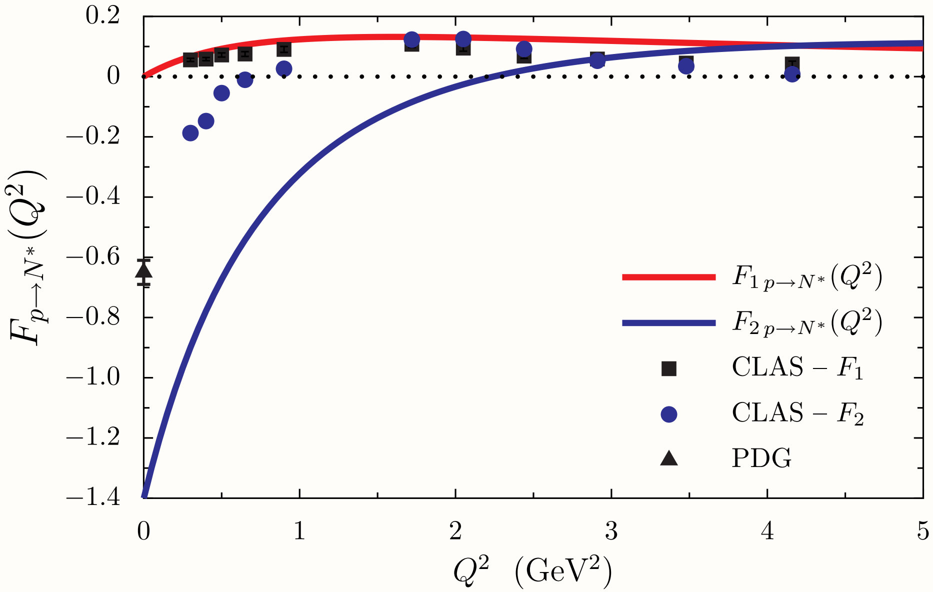


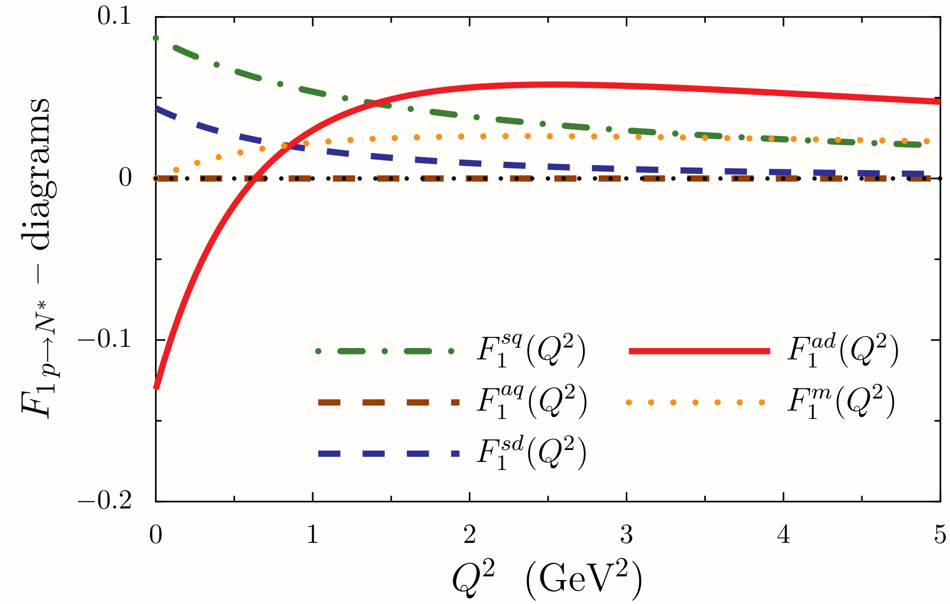
- Contributions originate from the following diagrams

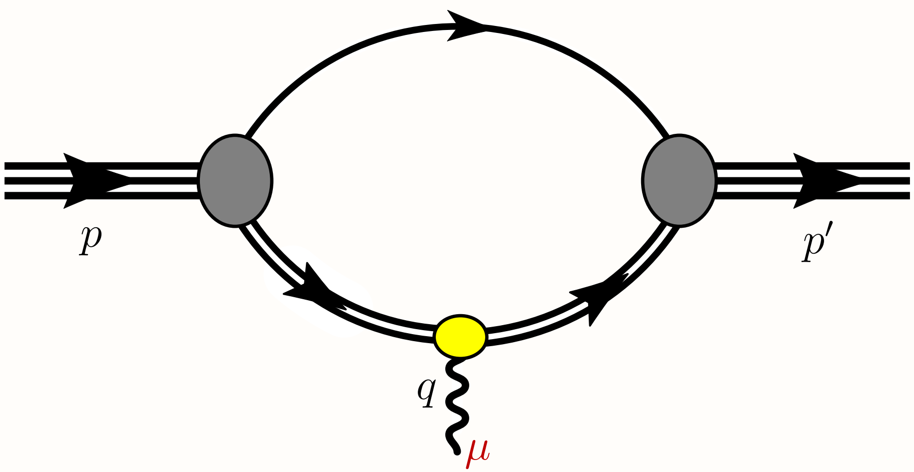


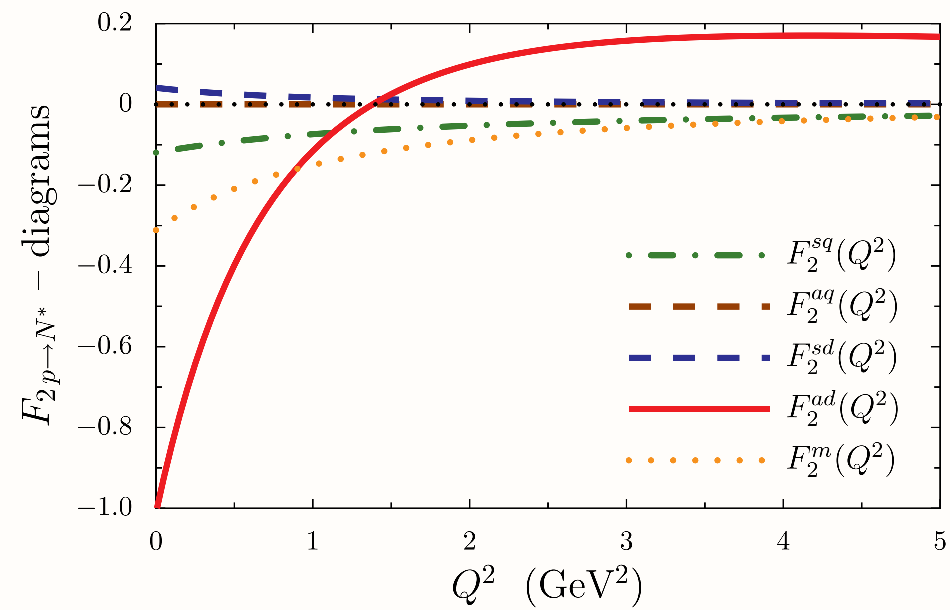
- Find that N^* form factors are axial–vector diquark dominated

$N \to N^*$ Transition Form Factors Results

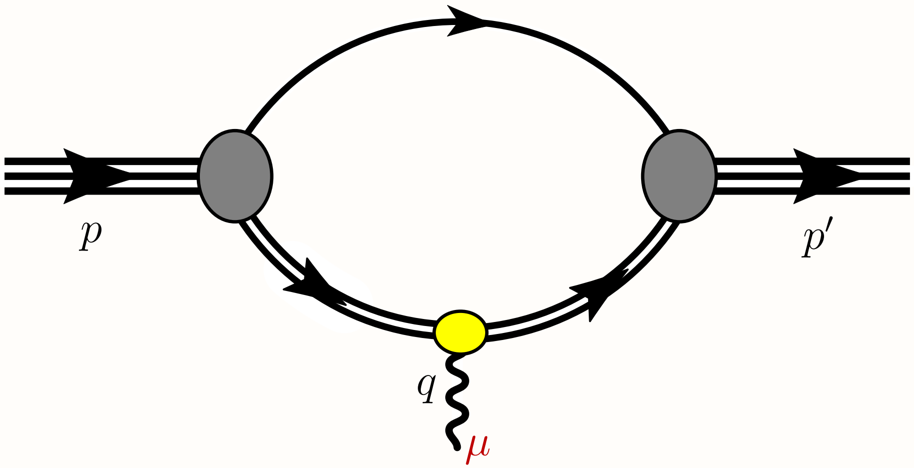


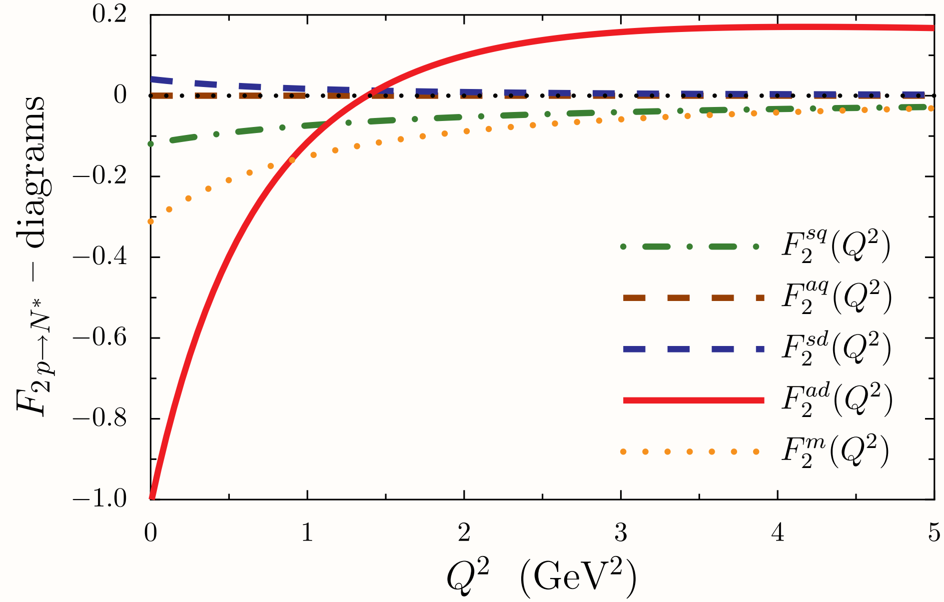






Why the Zero





- The photon–axial-vector diquark vertex has the form

$$\Lambda_{ax}^{\mu,\alpha\beta} = \left[g^{\alpha\beta} F_1(Q^2) - \frac{q^\alpha q^\beta}{2M_a^2} F_2(Q^2) \right] (p+p')^\mu - \left(q^\alpha g^{\mu\beta} - q^\beta g^{\mu\alpha} \right) F_3(Q^2)$$

- The three axial-vector diquark form factors are positive definite
- Cancellations between pieces of diagram give zero in $F_{2\,p\to N^*}$
- This zero is directly related to the zeros in the N^* form factors

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