Hadron Phenomenology and QCDs DSEs

Lecture 1: *An Introduction to Non-Perturbative QCD*

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USC Summer Academy on Non-Perturbative Physics, July–August 2012

Building Blocks of the Universe

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

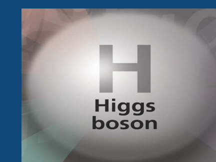
BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0



- Fundamental constituents of the Standard Model (SM) of particle physics
  - Quantum Chromodynamics (QCD) & Electroweak (EW) theories
- Local non-abelian gauge field theories
  - a special type of relativistic quantum field theory
- SM Lagrangian has gauge symmetries: $SU(3)_c \otimes SU(2)_L \otimes U_Y(1)$
  - SM has 19 parameters which need to be determined by experiment
  - however only 2 parameters are intrinsic to QCD: Λ_{QCD} & $\theta_{QCD} \leqslant 10^{-9}$

Motivation of Lectures

- Explore non-perturbative structure of QCD as it relates to hadron structure
- The tools available are:
  - lattice QCD
  - chiral perturbation theory
  - QCD inspired models
- We will investigate QCDs Dyson-Schwinger Equations (DSEs)
  - these are the equations of motion of the theory; represented by an infinite tower of coupled integral equations
  - a solution to these equations is a solution to QCD
  - in practice this tower must be truncated $\Longleftrightarrow$ modeling
- Some of the advantages of models over lattice and χPT are
  - can explore a wider array of physics topics
  - provide intuition
  - facilitate a dynamic interplay between experiment and theory

Plan of Lectures

- Part 1 – introduction to QCD and the non-perturbative frameworks of the Dyson–Schwinger equations (DSEs) and the Nambu–Jona-Lasinio (NJL) model

- Part 2 – pion and nucleon form factors within the DSE and NJL approaches to non-perturbative QCD

- Part 3 – parton distribution functions within the DSE and NJL approaches to non-perturbative QCD

- Part 4 – the study of quark degrees of freedom in nuclei and nuclear matter within the NJL approach to non-perturbative QCD

Recommended References

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Quantum Chromodynamics (QCD)

- QCD is the fundamental theory of the strong interaction, where the quarks and gluons are the basic degrees of freedom

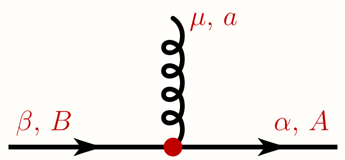
$$(q_\alpha)_f^A \quad \begin{cases} \text{colour} & A = 1,\, 2,\, 3 \\ \text{spin} & \alpha = \uparrow, \downarrow \\ \text{flavour} & f = u,\, d,\, s,\, c,\, b,\, t \end{cases} \qquad\qquad A_\mu^a \quad \begin{cases} \text{colour} & a = 1, \ldots, 8 \\ \text{spin} & \varepsilon_\mu^\pm \end{cases}$$

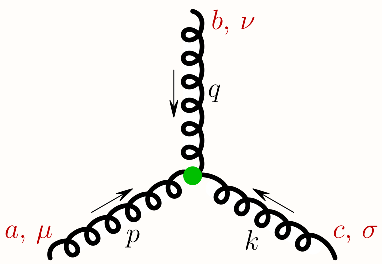
- QCD is a non-abelian gauge theory whose dynamics are governed by the Lagrangian

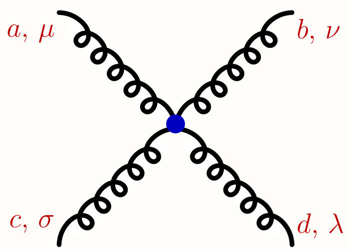
$$\mathcal{L} = \bar{q}_f \left(i\not{D} + m_f \right) q_f - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu};$$

$$i\not{D} = \gamma^\mu \left(i\partial_\mu + g_s\, A_\mu^a\, T^a \right)$$

$$F_{\mu\nu}^a = \partial_\mu\, A_\nu^a - \partial_\nu\, A_\mu^a + g_s\, f_{abc}\, A_\mu^b\, A_\nu^c$$





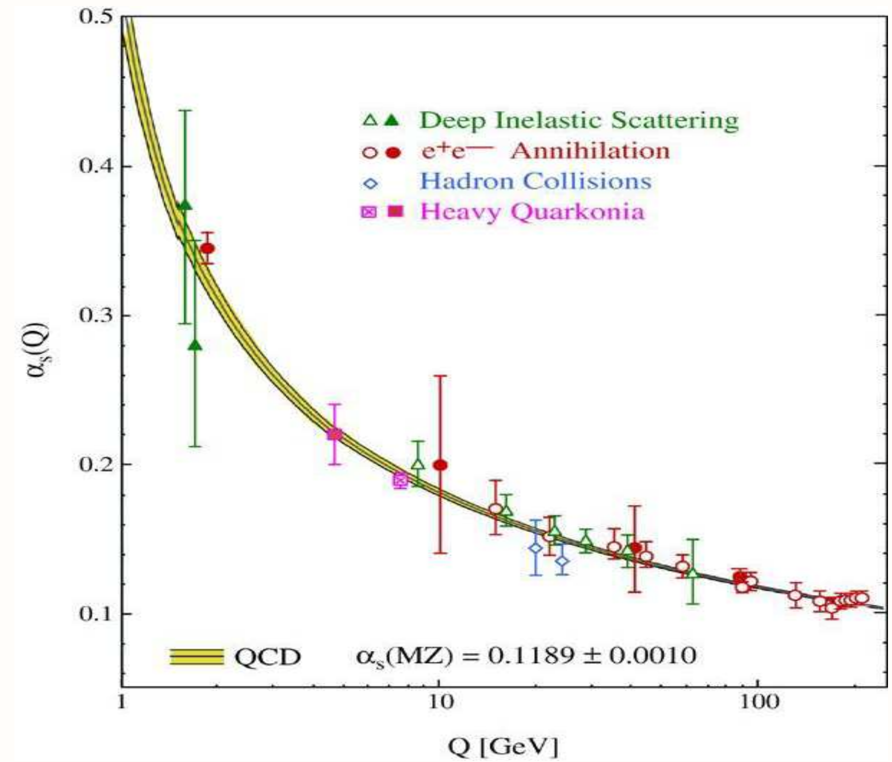


- Gluon self-interactions have many profound consequences

Asymptotic Freedom

- At large Q^2 or short distances interaction strength becomes logarithmically small
  - a striking features of QCD
  - QED has opposite behaviour: $\alpha_e \simeq \frac{1}{137}$

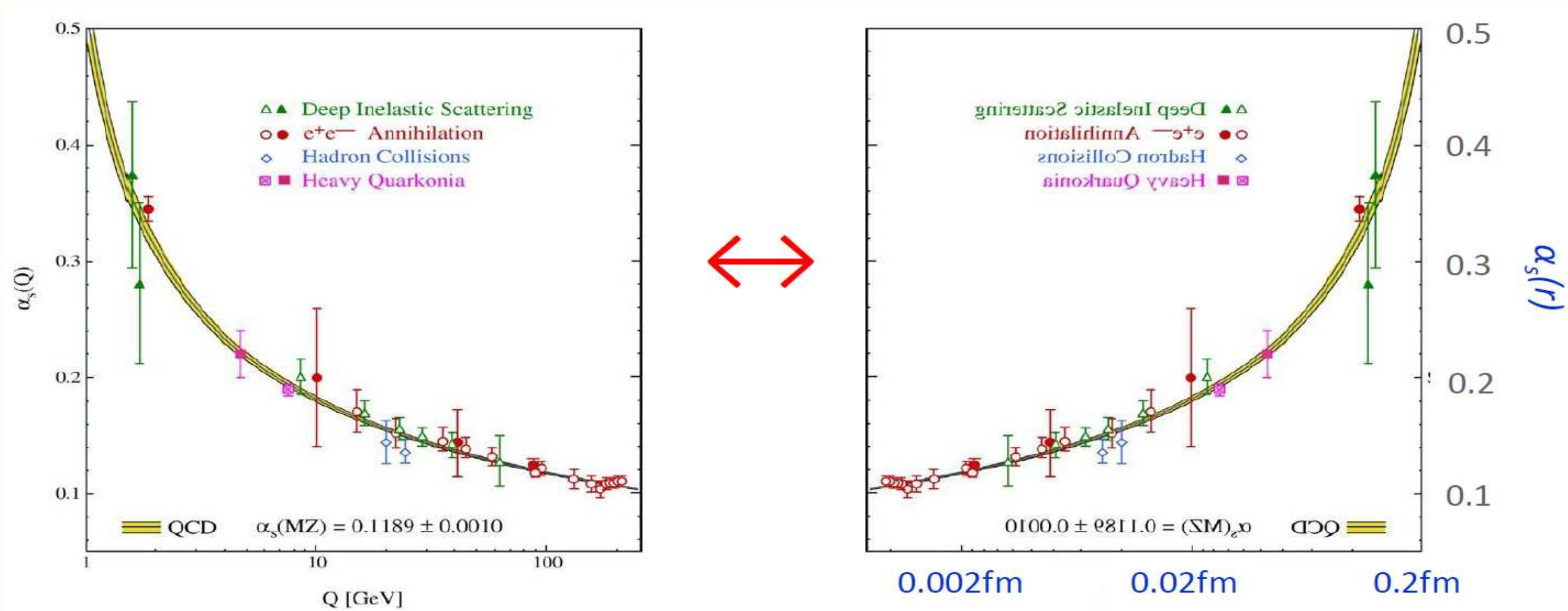
$$\alpha_s^{LO}(Q^2) = \frac{4\,\pi}{\left(11 - \frac{2}{3}\,N_f\right)\ln\left(Q^2/\Lambda^2_{QCD}\right)}$$



- Asymptotic Freedom – 2004 Nobel Prize – Gross, Politzer and Wilczek
- Λ_{QCD} is the most important parameter in QCD
  - $\Lambda_{QCD} \simeq 200$ MeV $\simeq 1$ fm^{-1} – sets scale, QCDs “standard kilogram”
- Momentum-dependent coupling $\Longleftrightarrow$ coupling depends on separation
  - interaction strength between quarks and gluons grows with separation

Asymptotic Freedom (2)

- Use $1\,\text{fm} = \frac{1}{197.3}\,\text{MeV}^{-1}$ to plot $\alpha_s(Q)$ as function of separation r
- For $r \simeq 0.2\,\text{fm} = \frac{1}{4}\,r_{\text{proton}}$ coupling is huge!
  - ✦ perturbation theory completely breaks down in this domain
- What happens to coupling as $r \to \infty$? Is $\alpha_s(r)$ unbounded?
  - ✦ could it require an infinite amount of energy to extract a quark or gluon from inside a hadron? *Confinement*
- QCD and hadron physics is inherently non-perturbative!



Confinement

- Hadron structure & QCD is characterized by two emergent phenomena
  - confinement and dynamical chiral symmetry breaking (DCSB)

- Both of these phenomena are not evident from the QCD Lagrangian
- All known hadrons are colour singlets, even though they are composed of coloured quarks and gluons: baryons (qqq) & mesons $(\bar{q}q)$
- Confinement conjecture: *particles that carry the colour charge cannot be isolated and can therefore not be directly observed*
- Related to \$1 million Millennium Prize:

Yang-Mills Existence And Mass Gap: *Prove that for any compact simple gauge group G, quantum Yang-Mills theory on $\mathbf{R}^4$ exists and has a mass gap $\Delta > 0$.*

  - for $SU(3)_c$ must prove that glueballs have a lower bound on their mass
  - partial explanation as to why strong force is short ranged

- Understanding confinement should be intimately related to the infra-red properties of $\alpha_s(Q^2)$ or QCDs β-function: $\beta(g_s) = \mu \frac{\partial g_s}{\partial \mu}$ & $\alpha_s = \frac{g_s^2}{4\pi}$

Chiral Symmetry

- Define left- and right-handed fields: $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\,\psi$
- The QCD Lagrangian then takes the form $[\mathbf{M} = \mathrm{diag}\,(m_u, m_d, m_s, \ldots)]$

$$\mathcal{L} = \bar{\psi}_L\, i\not{D}\,\psi_L + \bar{\psi}_R\, i\not{D}\,\psi_R - \bar{\psi}_R\,\mathbf{M}\,\psi_L - \bar{\psi}_L\,\mathbf{M}\,\psi_R - \frac{1}{4}\,F^a_{\mu\nu}F^{\mu\nu}_a$$

- Therefore for $\mathbf{M} = 0$ QCD Lagrangian is chirally symmetric

$$SU(N_f)_L \otimes SU(N_f)_R \implies \psi_{L,R} \to e^{-i\,\omega^a_{L,R}\,T^a}\,\psi_{L,R}$$

- $SU(N_f)_L \otimes SU(N_f)_R$ chiral symmetry is equivalent to

$$SU(N_f)_V \otimes SU(N_f)_A \implies \psi \to e^{-i\,\omega^a_V\,T^a}\psi, \quad \psi \to e^{-i\,\omega^a_A\,T^a\,\gamma_5}\psi$$

- Global symmetries: Wigner-Weyl or Nambu-Goldstone modes
  - ✦ Wigner-Weyl mode: vacuum is also invariant
  - ✦ Nambu-Goldstone mode: vacuum breaks symmetry

Dynamical Chiral Symmetry Breaking

- Recall for $\mathbf{M} = 0$ QCD Lagrangian is invariant under

$$SU(N_f)_L \otimes SU(N_f)_R \iff SU(N_f)_V \otimes SU(N_f)_A$$

- Transformations $SU(N_f)_V$ form generalized isospin subgroup
  - hadronic mass spectrum tells us nature respects isospin symmetry
  - $m_{\pi^-} \simeq m_{\pi^0} \simeq m_{\pi^+}, \quad m_p \simeq m_n, \quad m_{\Sigma^-} \simeq m_{\Sigma^0} \simeq m_{\Sigma^+}$
  - therefore $SU(N_f)_V$ is realized in the Wigner-Weyl mode
- $SU(N_f)_A$ transformations mix states of opposite parities
  - expect hadronic mass spectrum to exhibit parity degeneracy
  - $m_{a_1} - m_\rho \simeq 490$ MeV, $\quad m_{N(940)} - m_{N^*(1535)} \simeq 600$ MeV, etc
  - recall: $m_u \simeq m_d \simeq 5$ MeV $\Longrightarrow$ cannot produce large mass splittings
  - therefore $SU(N_f)_A$ must be realized in the Nambu-Goldstone mode
- Therefore chiral symmetry is dynamically broken

$$SU(N_f)_L \otimes SU(N_f)_R \xRightarrow{\text{vacuum/interactions}} SU(N_f)_V$$

Goldstone's Theorem

- Goldstone's theorem: *if a continuous global symmetry is broken dynamically, then for each broken group generator there must appear in the theory a massless spinless particle (Goldstone boson)*
- QCDs chiral symmetry is explicitly broken by small current quark masses

$$m_u = 1.5 - 3.3\,\text{MeV}, \quad m_d = 3.5 - 6.0\,\text{MeV}, \quad m_s = 70 - 130\,\text{MeV} \quad (\ll \Lambda_{QCD})$$

- For $N_f = 3$ expect $N_f^2 - 1 = 8$ Goldstone bosons
  - $\pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ \eta$
  - physical particle masses are not zero – $m_\pi \sim 140\,$MeV, $m_K \sim 495\,$MeV etc – because of explicit chiral symmetry breaking: $m_{u,d,s} \neq 0$
- Chiral symmetry and its dynamical breaking has profound consequences for the QCD mass spectrum and hadron structure
  - this is not apparent from the QCD Lagrangian and is an innately non-perturbative (emergent) phenomena
- Need non-perturbative methods to fully understand consequences of QCD

Chiral Condensate; GT & GMOR Relations

- If a symmetry is dynamically broken some operator must acquire a vacuum expectation value, that is, $\langle 0\,|\Theta|\,0\rangle \neq 0$ ❁
  - ✦ operator must be Lorentz scalar: QCD $\Longrightarrow$ composite operator
  - ✦ colour singlet
- Simplest candidate for the DSCB order parameter is $\langle\bar\psi\psi\rangle = \langle\bar u u + \bar d d\rangle$

$$\langle 0\,|\bar\psi\psi|\,0\rangle^{\mu=2\,\mathrm{GeV}}_{\overline{\mathrm{MS}}} \simeq -\,(230\,\mathrm{MeV})^3$$

- Some important non-trivial consequences of DCSB ($M = 0$)
  - ✦ $f_\pi\, g_{\pi NN} = M_N\, g_A$ Goldberger–Treiman (GT) relation
    [we will study this relation in detail later]
  - ✦ $f_\pi^2\, m_\pi^2 = \frac{1}{2}\,(m_u + m_d)\,\langle\bar u u + \bar d d\rangle$ Gell-Mann–Oakes–Renner (GMOR)

❁ *This is the standard interpretation, Craig will discuss his idea that in fact the vacuum condensate equals zero and the order parameters for DCSB are the in-hadron condensates, for example,* $\langle\pi\,|\bar q q|\,\pi\rangle$

Proof of Gell-Mann–Oakes–Renner

- The axial-vector and pseudoscalar currents are

$$A_a^\mu(x) = \overline{\psi}(x)\,\gamma^\mu\gamma_5\,t_a\,\psi(x) \quad \& \quad P_a(x) = \overline{\psi}(x)\,i\gamma_5\,t_a\,\psi(x).$$

- Pion to vacuum matrix elements of these operators are

$$\langle 0\,|A_a^\mu(0)|\,\pi_b(p)\rangle = \delta_{ab}\,i\,f_\pi\,p^\mu \quad \& \quad \langle 0\,|P_a(0)|\,\pi_b(p)\rangle = \delta_{ab}\,g_\pi.$$

- PCAC: $\partial_\mu A_a^\mu = \overline{\psi}\,i\gamma_5\,\{m, t_a\}\,\psi \quad \Longrightarrow \quad \partial_\mu A_a^\mu = (m_u + m_d)\,P_a,$

- $\langle 0\,|\partial_\mu A_a^\mu|\,\pi_b(p)\rangle = \delta_{ab}\,f_\pi\,p^2 = (m_u+m_d)\,\langle 0\,|P_a|\,\pi_b(p)\rangle = (m_u+m_d)\,\delta_{ab}\,g_\pi$

- *This gives the exact relation in QCD:* $f_\pi\,m_\pi^2 = (m_u+m_d)\,g_\pi$

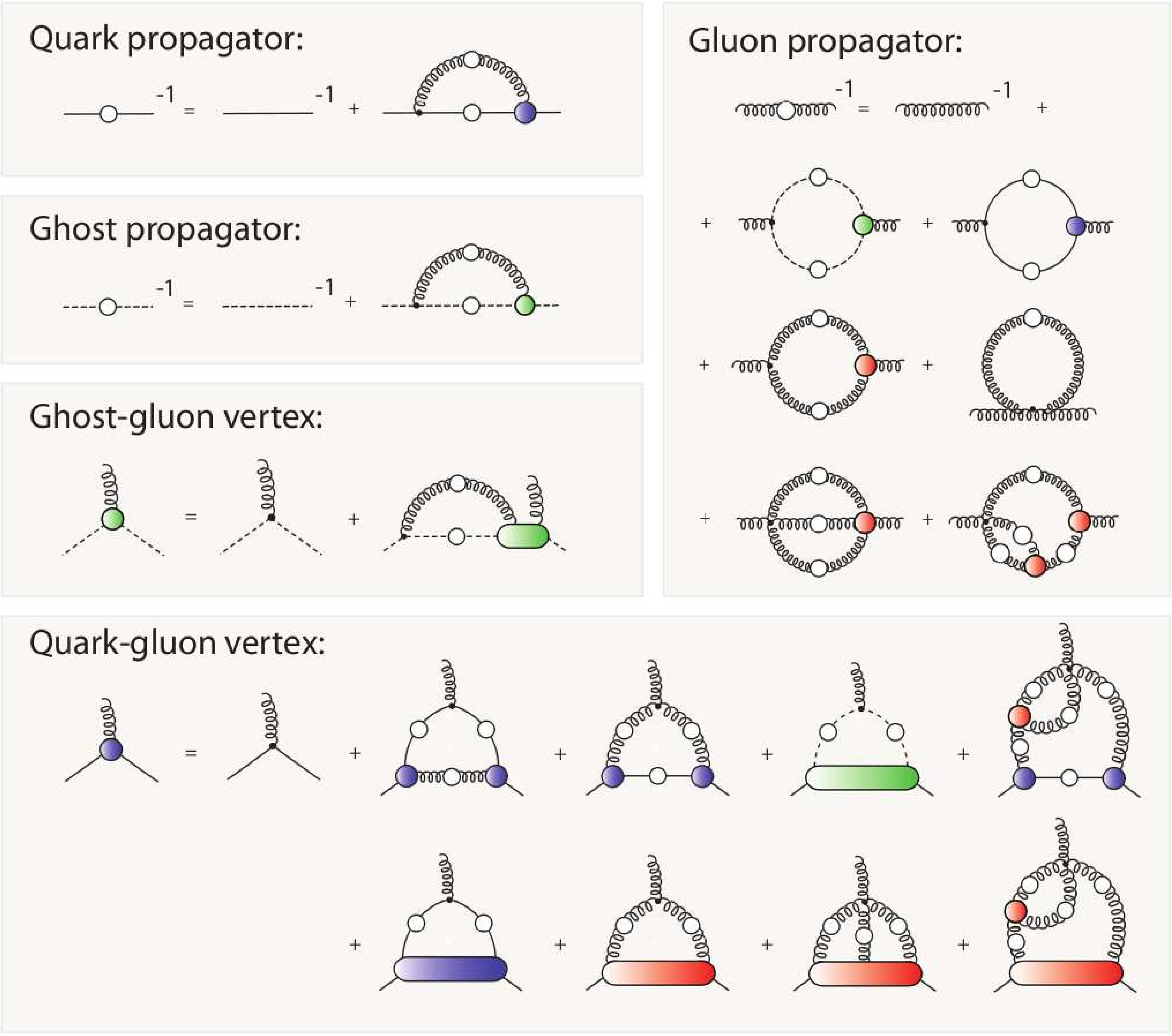
- In chiral limit – $f_\pi\,m_\pi^2 = 0$ – important consequences
  - ground state: $m_\pi = 0 \implies f_\pi \neq 0$; excited states: $m_\pi \neq 0 \implies f_\pi = 0$
  - decay constants for pseudoscalar excited states are zero

- To complete the proof: $[Q_A^a,\,P_b] = -\delta_{ab}\,\frac{i}{2}\,\overline{\psi}\psi;\quad \int \frac{d^3p}{2\,p^0(2\,\pi^3)^3}\ |\pi_a\rangle\,\langle\pi_a| = 1$

Dyson–Schwinger Equations

- DSEs are the equations of motion for a quantum field theory
  - ✦ infinite tower of coupled integral equations
  - ✦ usually a solution is only possible after a truncation
- Some important aspects of the Dyson–Schwinger Equations approach:
  - ✦ study hadrons as bound states of quarks AND gluons
  - ✦ Poincaré covariance
  - ✦ renormalizable
  - ✦ exhibits dynamical chiral symmetry breaking
    - → generation of fermion mass from nothing
  - ✦ usually formulated in Euclidean space
  - ✦ yields Schwinger functions $\Longleftrightarrow$ Euclidean space Green functions
- Very useful tool for building/guiding models of QCD

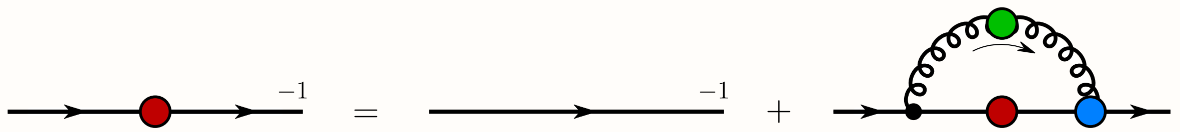
QCDs Dyson–Schwinger Equations



ETC!

QCDs Gap Equation: simplest DSE

- QCDs gap equation $\implies$ quark propagator – most important DSE

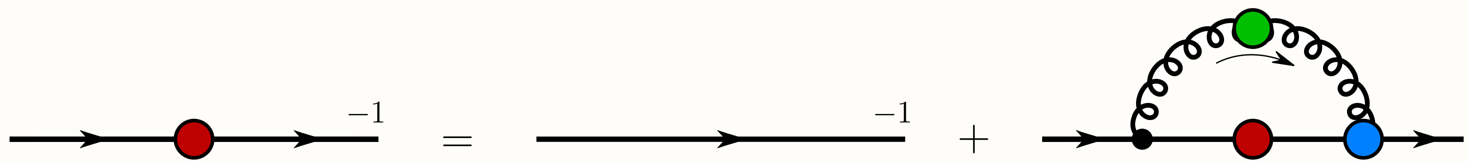


  - ✦ ingredients – dressed gluon propagator & quark-gluon vertex

$$S(p)^{-1} = Z_2\,(i\,\not{p} + m_0) + Z_1 \int \frac{d^4k}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-k)\,\frac{\lambda^a}{2}\,\gamma^\mu\, S(q)\,\Gamma^{a,\nu}(p,k)$$

- Ingredients
  - ✦ $S(p)$ dressed quark propagator
  - ✦ $D_{\mu\nu}(p-k)$ dressed gluon propagator
  - ✦ $\Gamma^{a,\nu}(p,k)$ dressed quark-gluon vertex
  - ✦ m_0 bare current quark mass
  - ✦ Z_1, Z_2 vertex and quark wave function renormalization constants
- Recall $D_{\mu\nu}$ and $\Gamma_{\nu,a}$ satisfy their own DSE

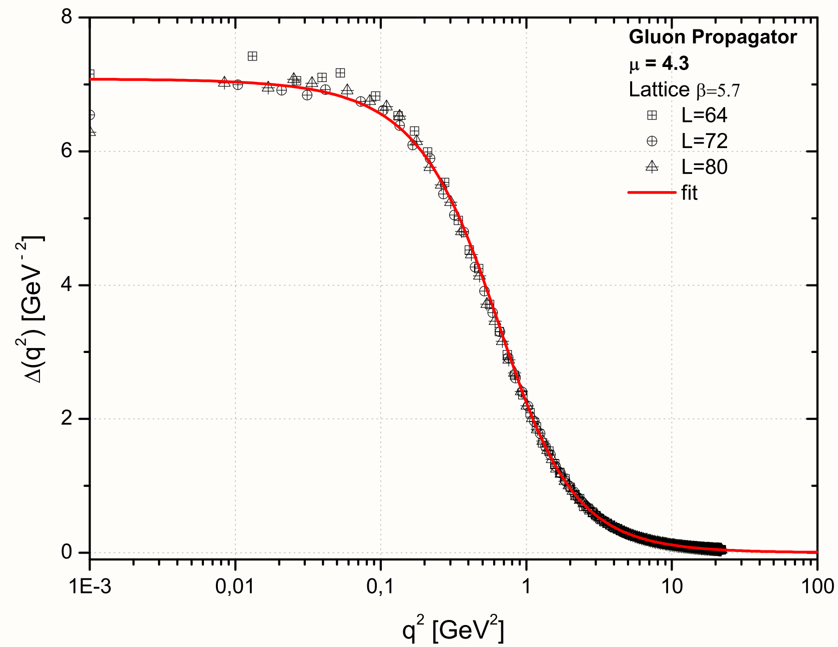
Gap Equation: Key Ingredients



- *Dressed gluon propagator*

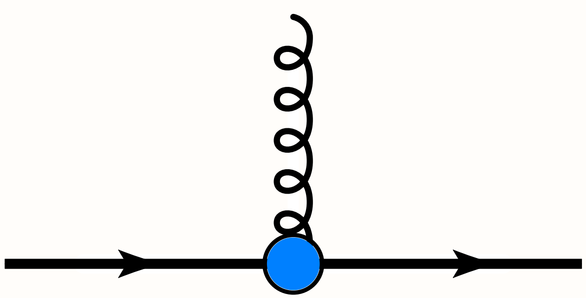
$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\Delta(q^2) + \xi\,\frac{q^\mu q^\nu}{q^4}$$

- Characterized by one dressing function $\Delta(p^2)$ & a gauge parameter ξ
- Choose Landau gauge $\xi = 0$ (fixed point of RG)



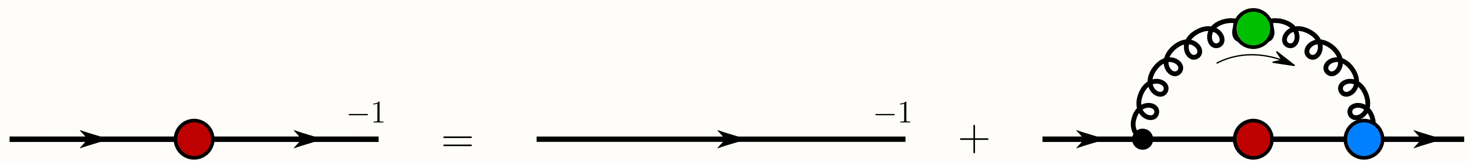
A. C. Aguilar *et al*, Phys. Rev. **D81**, 034003 (2010).

- *Quark–gluon vertex*



$$\Gamma^{a,\mu}_{gqq}(p',p) = \frac{\lambda^a}{2}\sum_{i=1}^{12}\,\lambda_i^\mu\; f_i(p'^2,p^2,q^2) = \Gamma^\mu_L + \Gamma^\mu_T$$

Challenge: Symmetry Preserving Trucations



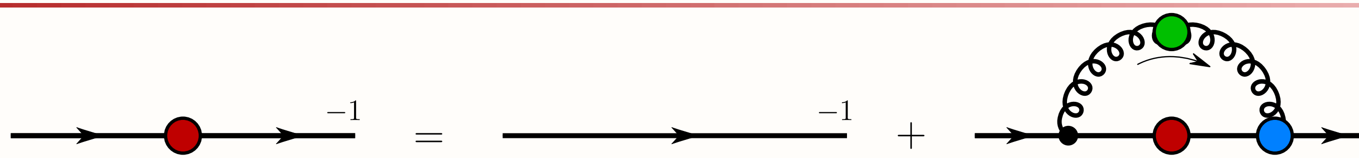
$$S(p)^{-1} = Z_2\,(i\,\not{p} + m_0) + Z_1 \int \frac{d^4k}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-k)\, \frac{\lambda^a}{2}\, \gamma^\mu\, S(k)\, \frac{\lambda^a}{2}\, \Gamma^\nu(p,k)$$

- Need a sensible truncation scheme that must maintain symmetries of theory
- Conservation of vector and axial-vector currents is critical to a robust description of hadron structure. *Breaking the*
  - ✦ vector current $\Longrightarrow$ will not conserve charge
  - ✦ axial current incorrectly $\Longrightarrow$ will not respect chiral symmetry ($M = 0$)
- Axial–Vector Ward–Takahashi identity - encapsulates structure of DCSB

$$q_\mu\, \Gamma_5^{\mu,i}(p',p) = S^{-1}(p')\,\gamma_5\, t_i + t_i\,\gamma_5\, S^{-1}(p) + 2\,m\,\Gamma_\pi^i(p',p)$$

  - ✦ relates inhomogeneous axial-vector & pseudoscalar vertices with quark propagator

Rainbow Ladder Truncation



- Rainbow-ladder – a symmetry preserving truncation to QCDs DSEs

$$\frac{1}{4\pi}\, g^2\, D_{\mu\nu}(p-k)\,\Gamma_\nu(p,k) \quad\longrightarrow\quad \alpha_{\text{eff}}(p-k)\, D^{\text{free}}_{\mu\nu}(p-k)\,\gamma_\nu$$

- Need model for $\alpha_{\text{eff}}(k^2)$ – must agree with perturbative QCD as $k^2 \to \infty$
  - the "*Maris–Tandy model*" is historically the most successful example
    [P. Maris and P.C. Tandy, Phys. Rev. C 60, 055214 (1999).]

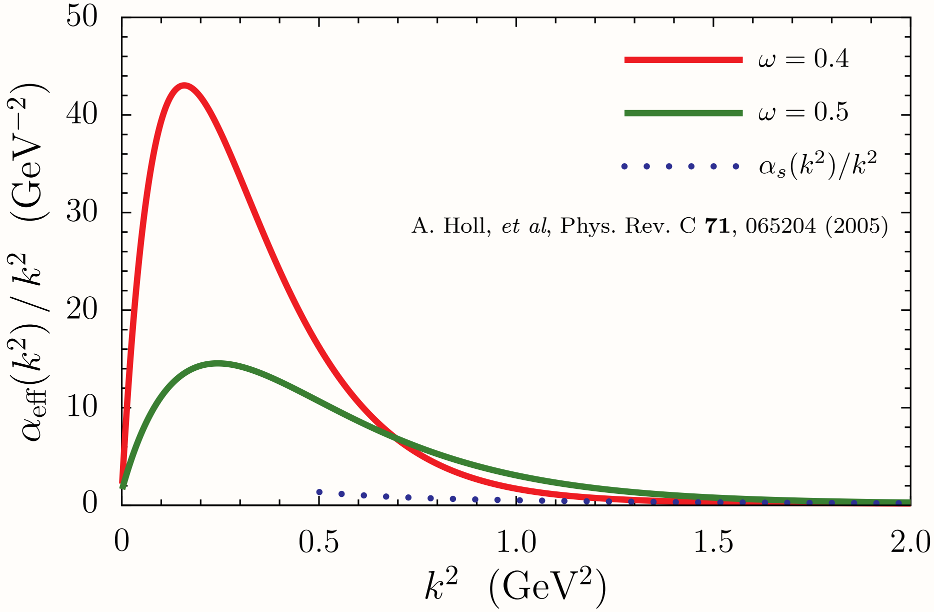
- Maris–Tandy effective running coupling is given by

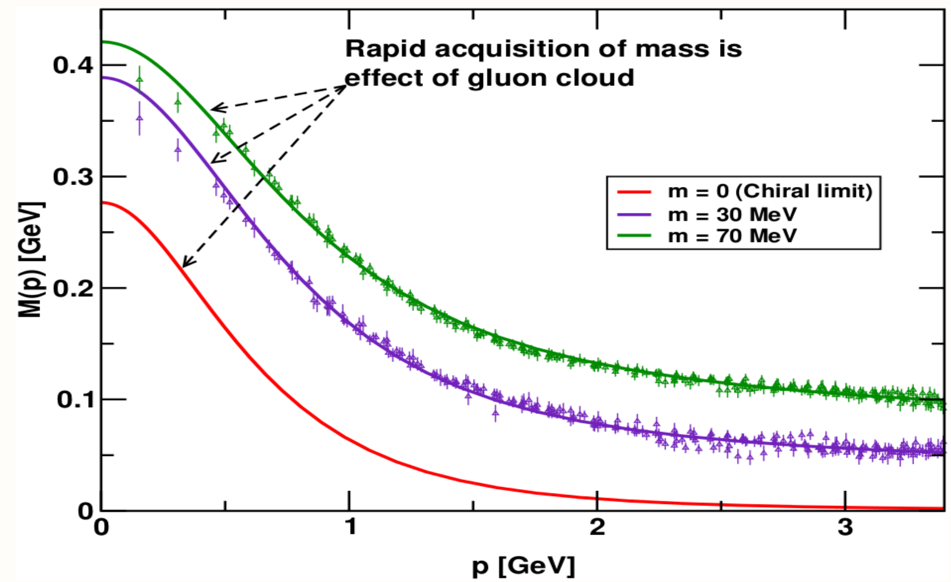
$$\frac{\alpha_{\text{eff}}(k^2)}{k^2} = \frac{\pi\, D}{\omega^6}\, k^2\, e^{-k^2/\omega^2} + \frac{24\,\pi}{25}\,\frac{1-e^{-k^2/\mu^2}}{k^2}\left[\ln\left[e^2 - 1 + \left(1 + \frac{k^2}{\Lambda^2_{QCD}}\right)^2\right]\right]^{-1}$$

  - $\mu = 1\,\text{GeV}, \;\; \Lambda_{QCD} = \Lambda^{(4)}_{\overline{\text{MS}}} = 0.234\,\text{GeV}, \;\; \omega\, D = (0.72\,\text{GeV})^3$

- Correct LO perturbative limit is build in: $\alpha_{\text{eff}}(k^2) \overset{k^2\to\infty}{\longrightarrow} \frac{12}{25}\,\frac{\pi}{\ln\left[k^2/\Lambda^2_{\text{QCD}}\right]}$
  - one parameter model for QCDs infra-red behaviour

QCDs Quark Propagator





- Quark propagator: $S(p) = \dfrac{Z(p^2)}{i\not{p} + M(p^2)} = \dfrac{1}{i\not{p}\,A(p^2) + B(p^2)}$
- Dynamical mass generation, $M \propto \langle \bar{q}q \rangle \iff \langle \bar{q}q \rangle \neq 0 \iff$ DCSB
  - ✦ Higgs mechanism is almost irrelevant for light quarks
- DCSB generates 98% of the mass in the visible universe
- In perturbative QCD: $B(p^2) = m\left[1 - \frac{\alpha}{\pi}\ln\left(\frac{p^2}{m^2}\right) + \ldots\right] \stackrel{m\to 0}{\to} 0$
- QCD is an innately non-perturbative theory! The only example in nature

Solving the QCDs Gap Equation

$$S(p,\mu^2)^{-1} = Z_2(\mu^2,\Lambda^2)\, S_0(p) + \tfrac{4}{3}\, Z_1(\mu^2,\Lambda^2)\int_\Lambda g^2 D_{\mu\nu}(p-k)\gamma^\mu S(k,\mu^2)\Gamma^\nu(p,k)$$

- Use quark propagator: $S^{-1}(p,\mu^2) = i\not p\, A(p^2,\mu^2) + B(p^2,\mu^2)$
- Rainbow ladder truncation:

$$\frac{g^2}{4\pi}\,\Gamma^\nu(p,k) \to \alpha_{\text{eff}}(k^2)\,\gamma^\mu, \qquad D_{\mu\nu}(k) \to D^{\text{free}}_{\mu\nu}(k)$$

- Use *off-shell subtraction scheme* for renormalization:

$$S(p)^{-1}\Big|_{p^2=\mu^2} = i\not p + m(\mu^2)$$

  - ✦ $m(\mu^2)$ is the renormalized current quark mass: $m(\mu^2) = \frac{m_0(\Lambda^2)}{Z_m(\mu^2,\Lambda^2)}$

- Gap equation becomes set of coupled integral eqs. for $A(p^2)$ & $B(p^2)$:

$$A(p^2,\mu^2) = Z_2(\mu^2,\Lambda^2)\, A_0(p^2,\Lambda^2) \quad \& \quad B(p^2,\mu^2) = Z_2(\mu^2,\Lambda^2)\, B_0(p^2,\Lambda^2)$$

- Then solve the two coupled equations by iteration

Charting Interaction between light quarks

- Formally, hadronic observables are related to QCDs Schwinger functions
- For example, the quark propagator is a Schwinger Function and the gap equation relates this to:
  - the gluon propagator: $D^{\mu\nu}(k)$
  - the quark-gluon vertex $\Gamma^{a,\mu}_{\gamma qq}(p,p')$
  - the quark propagator is the building block of hadrons in the DSEs
- The DSEs are therefore a tool that can relate QCDs Schwinger Functions to hadronic observables
- Measurements of, for example, the hadron mass spectrum, elastic and transition form factors, PDFs, etc must provide information on the long-range interaction between light quarks and gluons
- Interplay between DSEs & experiment provides a sufficient framework to extract infra-red behaviour of QCDs Schwinger functions
- Within DSE framework can map out infra-red properties of QCDs running coupling $\alpha_s(Q^2) \iff$ confinement

Charting a Path Forward

- The full machinery of the DSE with a sophisticated quark-gluon vertex gives a solid connection between QCD and experiment
  - ✦ remains much to be explored, notably baryon PDFs, TMDs & GPDs
  - ✦ however DSEs calculations are time & resource intensive – useful to have some physics intuition before embarking upon DSE studies
  - ✦ very good reason to explore hadron and nuclear structure with a simplified quark-gluon interaction
- Replace gluon propagator with a δ-function in configuration space

$$g^2 D_{\mu\nu}(p-k)\Gamma^{\nu}(p,k) \;\to\; \frac{1}{m_G^2}\, g_{\mu\nu}\, \gamma^{\nu}$$

- This “contact interaction” framework is basically equivalent to the Nambu–Jona Lasinio (NJL) model
- The NJL model is powerful tool and can guide experiment
  - ✦ use as exploratory tool for subsequent DSE investigation

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