Systematics of the Excitation Spectrum and Form Factors of Baryons in Holographic QCD: from Confinement to Quark Degrees of Freedom

Guy F. de Téramond

Universidad de Costa Rica

Nucleon Resonance Structure

in Exclusive Electroproduction

at High Photon Virtualities

USC, August 13 - 15, 2012

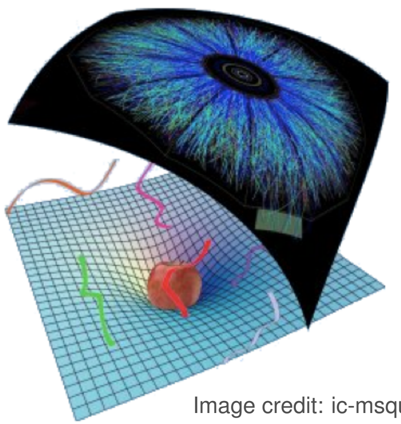
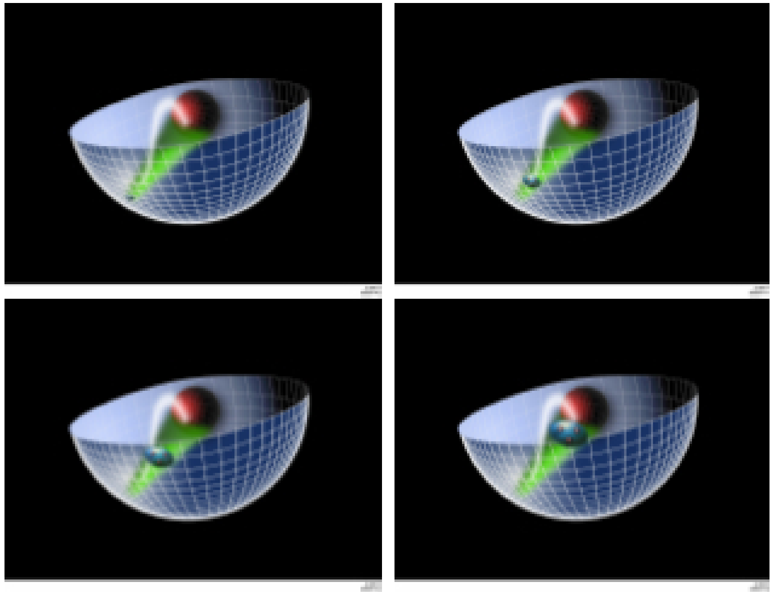


Image credit: ic-msquare



Recent Review: GdT and S.J. Brodsky, arXiv:1203.4025 [hep-ph]

Gauge/Gravity Correspondence and QCD

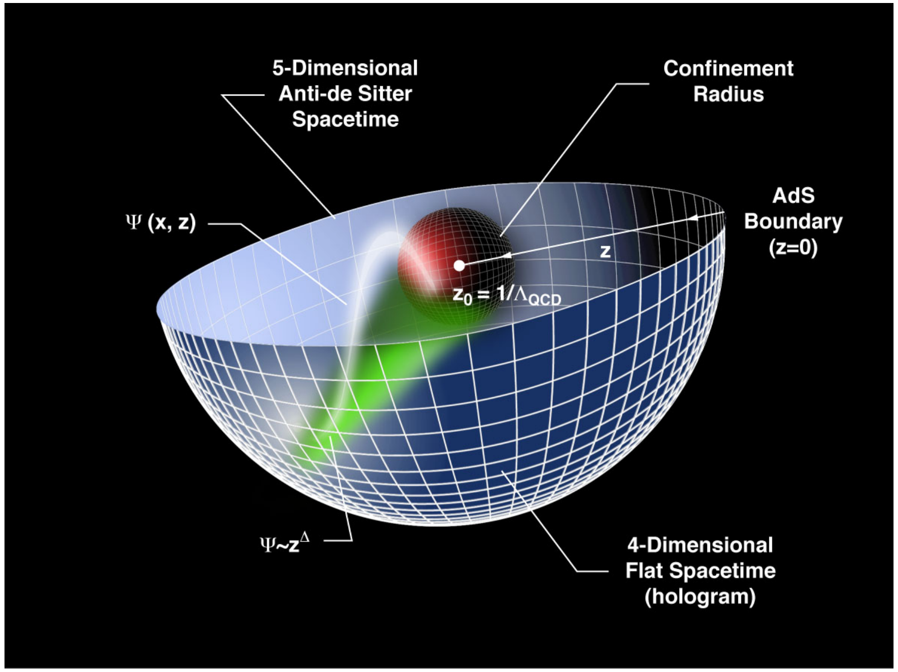
- Review recent analytical insights into the nonperturbative nature of light-hadron bound states using the gauge/gravity correspondence [Maldacena (1998)]
- Description of strongly coupled ultra relativistic system using a dual gravity description in a higher dimensional space (holographic)
- Why is AdS space important? AdS$_5$ is a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D,$ with the group of isometries of AdS$_5$ [a]
- Mapping of AdS gravity to QCD quantized at fixed light-front time gives a precise relation between wave functions in AdS space and the LF wavefunctions describing the internal structure of hadrons

---

[a] Isometry group: most general group of transformations which leave invariant the distance between two points: dimension of isometry group of AdS$_{d+1}$ is $\frac{(d+1)(d+2)}{2}$

- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$



8-2007
8685A14

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz=0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R}\, L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space

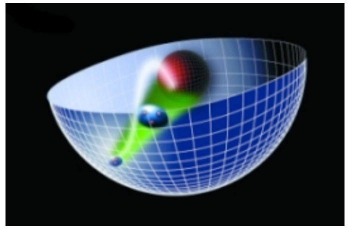
- Short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS, $z \sim 1/\Lambda_{\rm QCD}$

- Use isometries of AdS to map local interpolating operators at the UV boundary into modes propagating inside AdS

AdS Gravity Action

$$\mathcal{R}_{NKLM} = -\frac{1}{R^2}\left(g_{NL}g_{KM} - g_{NM}g_{KL}\right)$$



- AdS$_5$ metric $x^M = (x^\mu, z)$:

$$ds^2 = g_{MN}dx^M dx^N = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

- Action for gravity coupled to scalar field in AdS$_5$ $\left(\Lambda = -\frac{6}{R^2}\right)$:

$$S = \int d^4x dz \sqrt{g}\left(\frac{1}{\kappa^2}\left(\mathcal{R} - 2\Lambda\right) + \frac{1}{2}\left(g^{MN}\partial_M\Phi\partial_N\Phi - \mu^2\Phi^2\right)\right)$$

- Equations of motion $\left(\sqrt{g} = (R/z)^5\right)$

$$\mathcal{R}_{MN} - \frac{1}{2}g_{MN}\mathcal{R} - \Lambda g_{MN} = 0$$

$$z^3\partial_z\left(\frac{1}{z^3}\partial_z\Phi\right) - \partial_\nu\partial^\nu\Phi - \left(\frac{\mu R}{z}\right)^2\Phi = 0$$

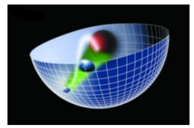
Light-Front Holographic Mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Physical modes are plane-waves along x^μ-coordinates with four-momentum P^μ and invariant mass $P_\mu P^\mu = M^2$: $\Phi_P(x,z) = e^{-iP\cdot x}\Phi(z)$

- Find AdS eom

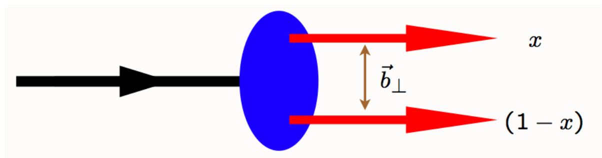
$$\left[-z^3\partial_z\left(\frac{1}{z^3}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi = M^2\Phi$$



- Upon substitution $z \rightarrow \zeta$ and $\phi(\zeta) \sim \zeta^{-3/2}\,\Phi(\zeta)$ in AdS eom we find

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

with $U(\zeta) = 0$ in the conformal AdS limit and $(\mu R)^2 = -4 + L^2$



- Identical with LFWE from Hamiltonian LF eom $P_\mu P^\mu|\phi\rangle = M^2|\phi\rangle$, where ζ is the invariant transverse distance between two partons $\zeta^2 = x(1-x)b_\perp^2$ and the effective interaction U acts only on the valence sector

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

Meson Spectrum in Hard Wall Model

[LF Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

- How to break conformality and compute the hadronic spectrum ?
- Conformal model up to the confinement scale $1/\Lambda_{\rm QCD}$ [Polchinski and Strassler (2002)]

$$U(\zeta) = \begin{cases} 0 & \text{if} \quad \zeta \leq \frac{1}{\Lambda_{\rm QCD}} \\ \infty & \text{if} \quad \zeta > \frac{1}{\Lambda_{\rm QCD}} \end{cases}$$

- Confinement scale $\frac{1}{\Lambda_{\rm QCD}} \sim$ 1 Fm, $\quad \Lambda_{\rm QCD} \sim 200$ MeV
- Covariant version of MIT bag model: quarks permanently confined inside a finite region of space
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int_0^{\Lambda_{\rm QCD}^{-1}} d\zeta \, \phi^2(z) = 1$

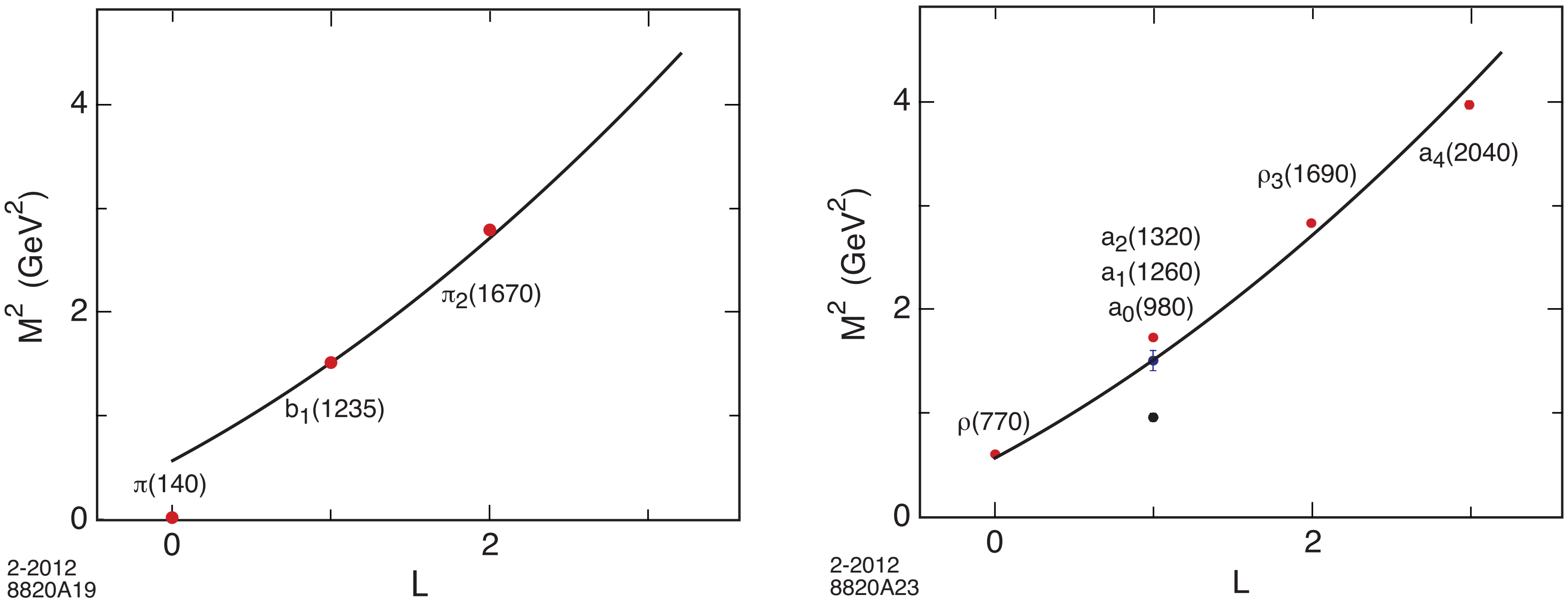
$$\phi_{L,k}(\zeta) = \frac{\sqrt{2}\Lambda_{QCD}}{J_{1+L}(\beta_{L,k})} \sqrt{\zeta} J_L(\zeta \beta_{L,k} \Lambda_{QCD})$$

- Eigenvalues

$$\mathcal{M}_{L,k} = \beta_{L,k} \Lambda_{\rm QCD}$$

Table 1: $I = 1$ mesons. For a $q\bar{q}$ state $P = (-1)^{L+1}$, $C = (-1)^{L+S}$

L	S	n	J^{PC}	$I = 1$ Meson
0	0	0	0^{-+}	$\pi(140)$
0	0	1	0^{-+}	$\pi(1300)$
0	0	2	0^{-+}	$\pi(1800)$
0	1	0	1^{--}	$\rho(770)$
0	1	1	1^{--}	$\rho(1450)$
0	1	2	1^{--}	$\rho(1700)$
1	0	0	1^{+-}	$b_1(1235)$
1	1	0	0^{++}	$a_0(980)$
1	1	1	0^{++}	$a_0(1450)$
1	1	0	1^{++}	$a_1(1260)$
1	1	0	2^{++}	$a_2(1320)$
2	0	0	2^{-+}	$\pi_2(1670)$
2	0	1	2^{-+}	$\pi_2(1880)$
2	1	0	3^{--}	$\rho_3(1690)$
3	1	0	4^{++}	$a_4(2040)$



Orbital and radial excitations for the π and the ρ I=1meson families ($\Lambda_{\rm QCD}$ = 0.32 GeV)

- Pion is not chiral
- $\mathcal{M} \sim 2n + L$ in contrast to usual Regge dependence $\mathcal{M}^2 \sim n + L$
- Important $J - L$ splitting (different J for same L) in mesons not described by hard-wall model
- Radial modes not well described in hard-wall model

Higher Spin Wave Equations in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Spin-J in AdS represented by totally symmetric rank J tensor field $\Phi_{M_1 \cdots M_J}$

- Action for spin-J field in AdS$_{d+1}$ $\left(x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{MN} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_M \Phi_{M_1 \cdots M_J} D_N \Phi_{M_1' \cdots M_J'} - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport (affine connection)

$$D_M \Phi_{M_1 \cdots M_J} = \partial_M \Phi_{M_1 \cdots M_J} - \Gamma^K_{M M_1} \Phi_{K \cdots M_J} - \cdots - \Gamma^K_{M M_J} \Phi_{M_1 \cdots K}$$

- Dilaton background $\varphi(z)$ breaks conformality of the theory (vanishes in the UV limit)

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

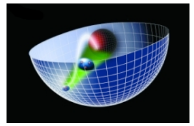
$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{-iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \quad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin-J modes Φ_J with only physical degrees of freedom
  [H. G. Dosch, S. J. Brodsky and GdT]

- Find AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1\cdots\mu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = \mathcal{M}^2\Phi_J(z)}$$



[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

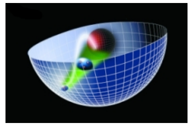
Dual QCD Light-Front Wave Equation

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

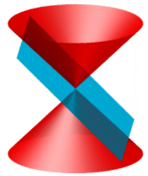
- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$



find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$



with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms of the partons inside a hadron
- Interaction terms in the QCD Lagrangian build the effective confining potential $U(\zeta)$ which acts on the valence sector and correspond to the truncation of AdS space in an effective dual gravity approximation

Meson Spectrum in Soft Wall Model

- Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]
- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LFWE

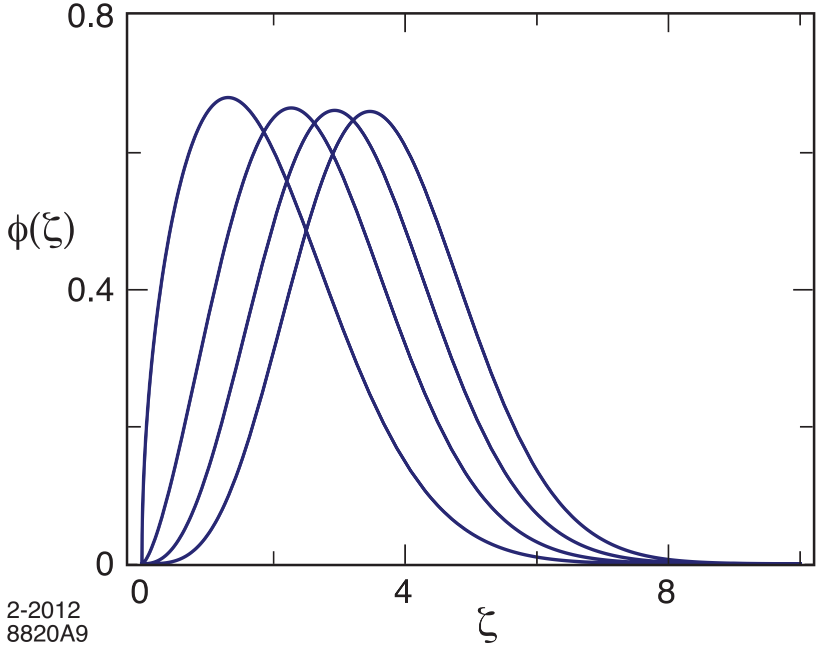
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

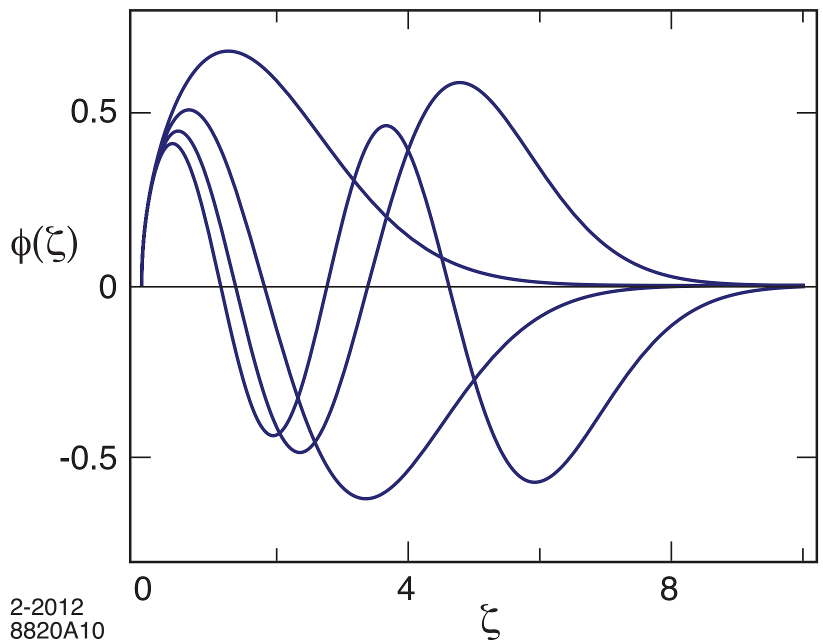
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

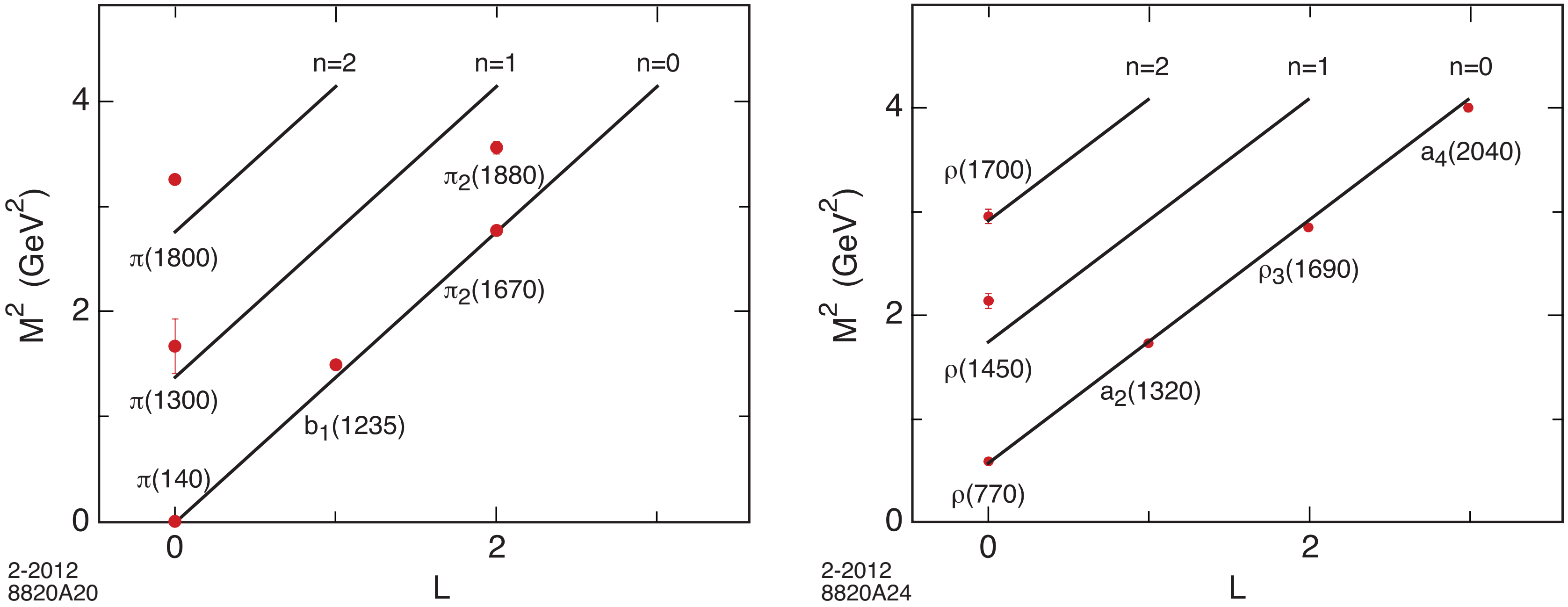




LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes

- $J = L + S,\ I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ I=1meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $L = 1,\, J = 0, 1, 2,\, I = 1$ vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

- $J - L$ splitting in mesons and radial excitations are well described in soft-wall model

Fermionic Modes in AdS Space and Baryon Spectrum

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

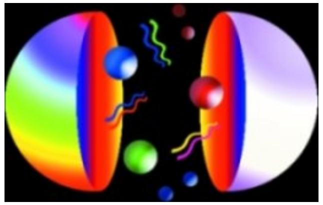


Image credit: N. Evans

- Lattice calculations of the ground state hadron masses agree very with experimental values
- However, excitation spectrum of nucleon represents important challenge to LQCD due to enormous computational complexity beyond ground state configuration and multi-hadron thresholds
- Large basis of interpolating operators required in LQCD since excited nucleon states are classified according to irreducible representations of the lattice, not the angular momentum
- The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods
- Analytical exploration of systematics of light-baryon resonances and nucleon form factors
- Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral J) hadrons by considering propagation of RS spinor field $\Psi_{\alpha M_1 \cdots M_{J-1/2}}$ in AdS space

Higher Spin Wave Equations in AdS Space

- For fermion fields in AdS one cannot break conformality with introduction of dilaton background since it can be scaled away leaving the action conformally invariant [I. Kirsch (2006)]

- Introduce an effective confining potential $V(z)$ in the action for a Dirac field in AdS$_{d+1}$

$$S_F = \int d^dx\, dz \sqrt{g} g^{M_1 M_1'} \cdots g^{M_T M_T'} \left(\overline{\Psi}_{M_1 \cdots M_T} \left(i e^M_A \Gamma^A D_M - \mu - V(z) \right) \Psi_{M_1' \cdots M_T'} + \cdots \right)$$

where D_M is the covariant derivative of the spinor field $\Psi_{\alpha M_1 \cdots M_T}$, $\quad T = J - \frac{1}{2}$

$$D_M \Psi_{M_1 \cdots M_T} = \partial_M \Psi_{M_1 \cdots M_T} - \frac{i}{2} \omega^{AB}_M \Sigma_{AB} \Psi_{M_1 \cdots M_T} - \Gamma^K_{M M_1} \Psi_{K \cdots M_T} - \cdots - \Gamma^K_{M M_T} \Psi_{M_1 \cdots K}$$

- $M, N = 1, \cdots, d+1$ curved space indices, $\quad A, B = 1, \cdots, d+1$ tangent indices
- e^M_A is the vielbein, w^{AB}_M spin connection, Σ_{AB} generators of the Lorentz group, $\Sigma_{AB} = \frac{i}{4} [\Gamma_A, \Gamma_B]$
- Γ^A tangent space Dirac matrices $\quad \{\Gamma^A, \Gamma^B\} = \eta^{AB}$
- For d even we choose $\Gamma^A = (\Gamma^\mu, \Gamma^z)$ with $\Gamma_z = -\Gamma^z = \Gamma_0 \Gamma_1 \cdots \Gamma_{d-1}$
- For $d = 4$: $\quad \Gamma^A = (\gamma^\mu, i\gamma_5)$

- Physical hadron has plane-wave, spinors, and polarization along $3+1$ physical coordinates

$$\Psi_P(x,z)_{\mu_1\cdots\mu_T} = e^{-iP\cdot x}\Psi(z)_{\mu_1\cdots\mu_T}, \quad \Psi_{z\mu_2\cdots\mu_T} = \cdots = \Psi_{\mu_1\mu_2\cdots z} = 0$$

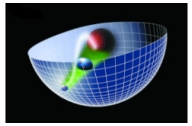
with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin-J modes Ψ_J with only physical degrees of freedom

[H. G. Dosch, S. J. Brodsky and GdT]

- Find AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1\cdots\mu_{J-1/2}}$

$$\left[i\left(z\eta^{MN}\Gamma_M\partial_N + \frac{d}{2}\Gamma_z\right) - \mu R - RV(z)\right]\Psi_J = 0$$



upon μ-rescaling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

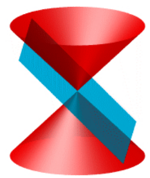
Light-Front Mapping and Cluster Decomposition

- Upon substitution $z \to \zeta$ and

$$\Psi(x,z) = e^{-iP\cdot x} z^2 \psi(z) u(P),$$

find LFWE for $d = 4$

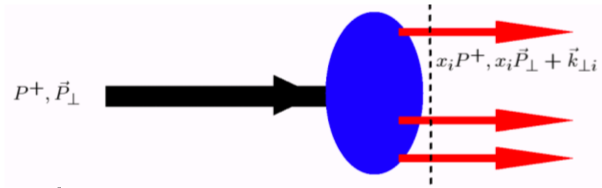
$$\begin{aligned} \frac{d}{d\zeta}\psi_+ + \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ + U(\zeta)\psi_+ &= \mathcal{M}\psi_-, \\ -\frac{d}{d\zeta}\psi_- + \frac{\nu+\frac{1}{2}}{\zeta}\psi_- + U(\zeta)\psi_- &= \mathcal{M}\psi_+, \end{aligned}$$



where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- ζ is the x-weighted definition of the transverse impact variable of the $n-1$ spectator system [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$



where $x = x_n$ is the longitudinal momentum fraction of the active quark

- Same multiplicity of states for mesons and baryons !

Baryon Spectrum in Soft-Wall Model

- Choose linear potential $U = \kappa^2 \zeta$
- LF nucleon eigenfunctions $\nu = L + 1 \quad (\tau = 3)$

$$\psi_+(\zeta) = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{\frac{3}{2}+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}(\kappa^2\zeta^2)$$

$$\psi_-(\zeta) = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{\frac{5}{2}+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}(\kappa^2\zeta^2)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

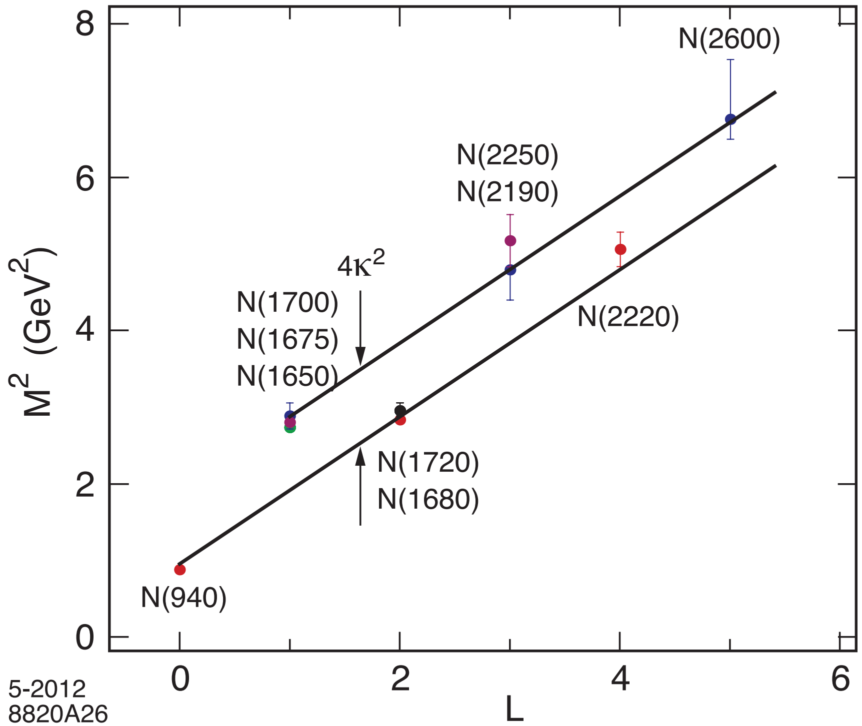
$$\mathcal{M}_{n,L}^2 = 4\kappa^2(n+L+2) + C$$

- Full $J - L$ degeneracy (different J for same L) for baryons along given trajectory !

$SU(6)$	K	S	L	n	Baryon State
56	0	$\frac{1}{2}$	0	0	$N\,\frac{1}{2}^+(940)$
	0	$\frac{3}{2}$	0	0	$\Delta\,\frac{3}{2}^+(1232)$
56	1	$\frac{1}{2}$	0	1	$N\,\frac{1}{2}^+(1440)$
	1	$\frac{3}{2}$	0	1	$\Delta\,\frac{3}{2}^+(1600)$
70	1	$\frac{1}{2}$	1	0	$N\,\frac{1}{2}^-(1535)$ $N\,\frac{3}{2}^-(1520)$
	1	$\frac{3}{2}$	1	0	$N\,\frac{1}{2}^-(1650)$ $N\,\frac{3}{2}^-(1700)$ $N\,\frac{5}{2}^-(1675)$
	1	$\frac{1}{2}$	1	0	$\Delta\,\frac{1}{2}^-(1620)$ $\Delta\,\frac{3}{2}^-(1700)$
56	2	$\frac{1}{2}$	0	2	$N\,\frac{1}{2}^+(1710)$
	2	$\frac{1}{2}$	2	0	$N\,\frac{3}{2}^+(1720)$ $N\,\frac{5}{2}^+(1680)$
	2	$\frac{3}{2}$	2	0	$\Delta\,\frac{1}{2}^+(1910)$ $\Delta\,\frac{3}{2}^+(1920)$ $\Delta\,\frac{5}{2}^+(1905)$ $\Delta\,\frac{7}{2}^+(1950)$
70	2	$\frac{3}{2}$	1	1	$N\,\frac{1}{2}^-$ $N\,\frac{3}{2}^-(1875)$ $N\,\frac{5}{2}^-$
	2	$\frac{3}{2}$	1	1	$\Delta\,\frac{5}{2}^-(1930)$
56	3	$\frac{1}{2}$	2	1	$N\,\frac{3}{2}^+(1900)$ $N\,\frac{5}{2}^+$
70	3	$\frac{1}{2}$	3	0	$N\,\frac{5}{2}^-$ $N\,\frac{7}{2}^-$
	3	$\frac{3}{2}$	3	0	$N\,\frac{3}{2}^-$ $N\,\frac{5}{2}^-$ $N\,\frac{7}{2}^-(2190)$ $N\,\frac{9}{2}^-(2250)$
	3	$\frac{1}{2}$	3	0	$\Delta\,\frac{5}{2}^-$ $\Delta\,\frac{7}{2}^-$
56	4	$\frac{1}{2}$	4	0	$N\,\frac{7}{2}^+$ $N\,\frac{9}{2}^+(2220)$
	4	$\frac{3}{2}$	4	0	$\Delta\,\frac{5}{2}^+$ $\Delta\,\frac{7}{2}^+$ $\Delta\,\frac{9}{2}^+$ $\Delta\,\frac{11}{2}^+(2420)$
70	5	$\frac{1}{2}$	5	0	$N\,\frac{9}{2}^-$ $N\,\frac{11}{2}^-$
	5	$\frac{3}{2}$	5	0	$N\,\frac{7}{2}^-$ $N\,\frac{9}{2}^-$ $N\,\frac{11}{2}^-(2600)$ $N\,\frac{13}{2}^-$

- Gap scale $4\kappa^2$ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families !

- No $J - L$ splitting !



Plus-minus nucleon spectrum gap for $\kappa = 0.49$ GeV

- Fix the energy scale to the proton mass for the lowest state $n = 0, L = 0$: $C = -4\kappa^2$

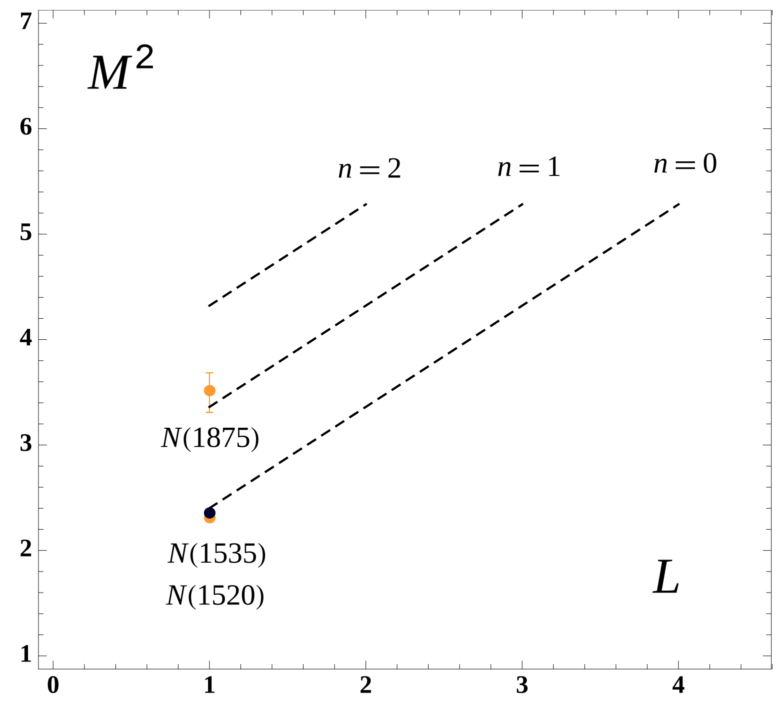
- Phenomenological rules for increase in mass $\mathcal{M}^2$ to construct full baryon spectrum from proton state

  $4\kappa^2$ for $\Delta n = 1$

  $4\kappa^2$ for $\Delta L = 1$

  $2\kappa^2$ for $\Delta S = 1$

  $2\kappa^2$ for $\Delta P = \pm$

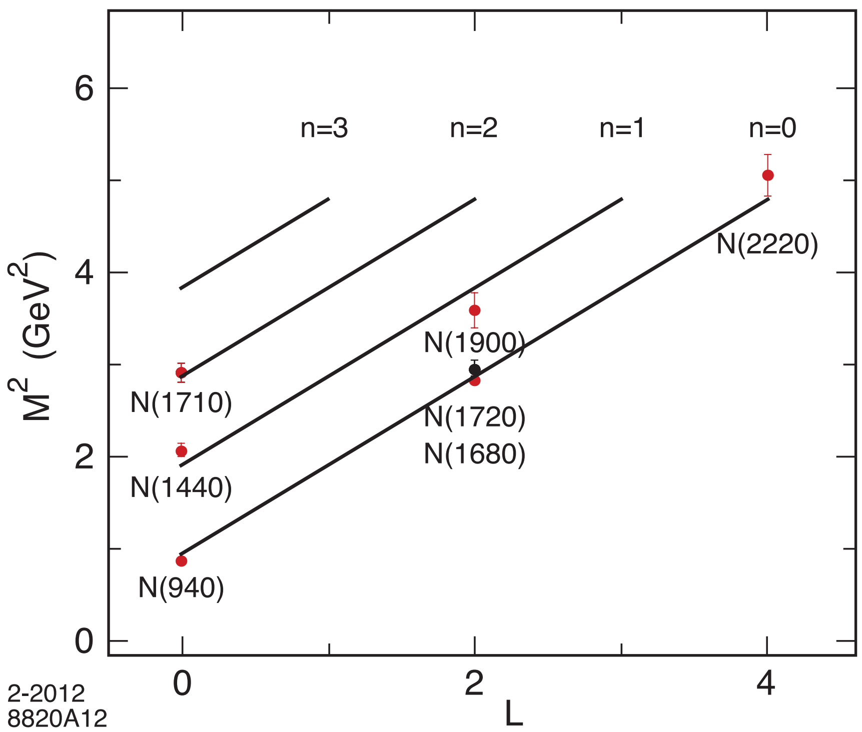


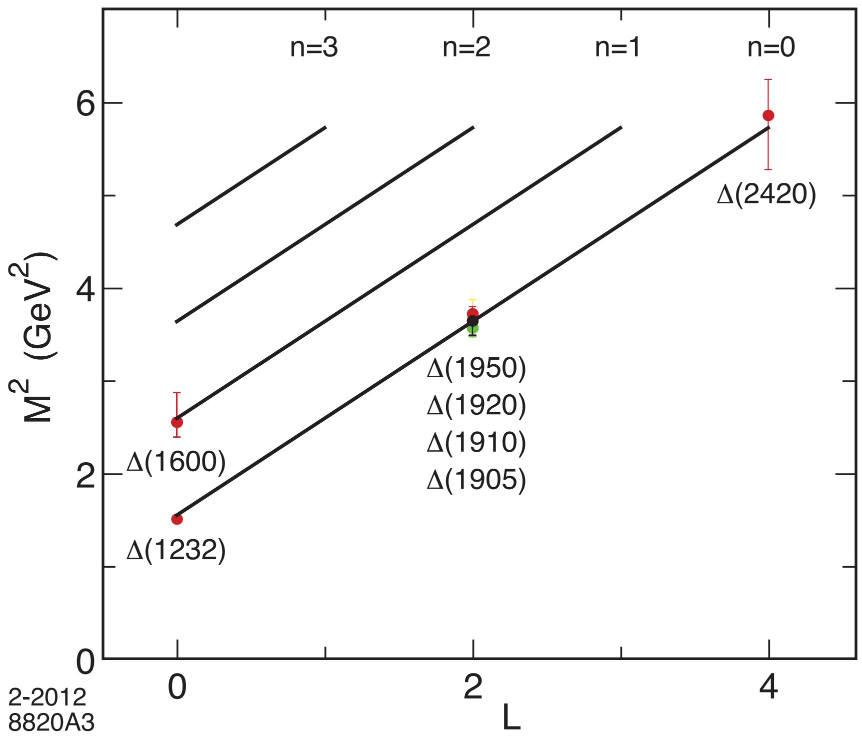
New state $N(1875)$ for $\kappa = 0.49$ GeV

- Eigenvalues

$$\mathcal{M}^{2(+)}_{n,L,S} = 4\kappa^2\left(n + L + S/2 + 3/4\right)$$

$$\mathcal{M}^{2(-)}_{n,L,S} = 4\kappa^2\left(n + L + S/2 + 5/4\right)$$

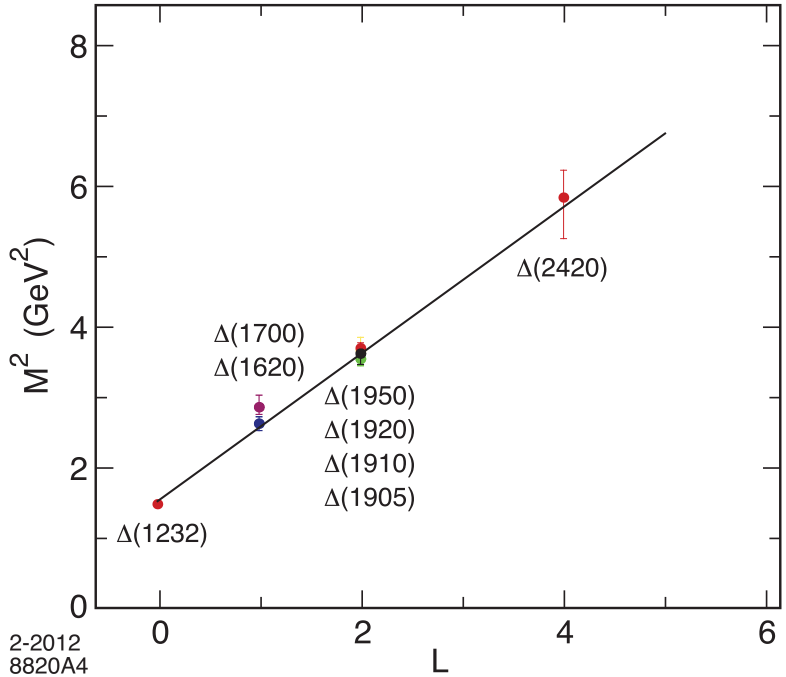




Orbital and radial excitations for positive parity N and Δ baryon families ($\kappa = 0.49 - 0.51$ GeV)

- Since $\mathcal{M}^{2\,(+)}_{n,L,S=\frac{3}{2}} = \mathcal{M}^{2\,(-)}_{n,L,S=\frac{1}{2}}$ positive and negative-parity Δ states are in the same trajectory

  [See also: H. Forkel, M. Beyer and T. Frederico, JHEP **0707**, 077 (2007)]



Δ orbital trajectories for $n = 0$ and $\kappa = 0.51$ GeV

- $\Delta(1930)$ quantum number assignment (E. Klempt and J. M. Richard (2010): $S = 3/2, L = 1, n = 1$
- Find $\mathcal{M}_{\Delta(1930)} = 4\kappa \simeq 2$ GeV compared with experimental value 1.96 GeV
- All known baryons well described by holographic formulas for $\mathcal{M}^{2\,(+)}_{n,L,S}$ and $\mathcal{M}^{2\,(-)}_{n,L,S}$

Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)   Mapping of EM currents

[S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]   Mapping of energy-momentum tensor

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle P'|J^\mu|P\rangle = (P + P')^\mu F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \overline{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, A^M(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_M \Phi_P(x,z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \left(P + P'\right)^\mu F(Q^2)$$

- Expressions for the transition amplitudes look very different but a precise mapping of the matrix elements can be carried out at fixed light-front time : $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

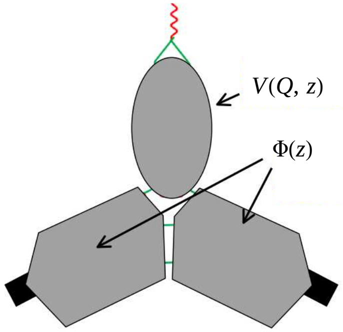


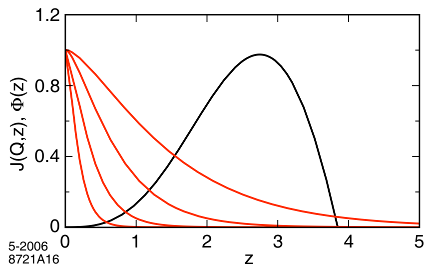
Image credit: F. Gross

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \quad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external EM source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, V(Q,z)\, \Phi^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$

$$V(Q,z) \to zQK_1(zQ)$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = (\zeta/R)^{-3/2}\,\Phi(\zeta), \quad z \to \zeta$$

- Dressed current for soft-wall model

$$V(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$

expanded as a sum of poles [Grigoryan and Radyushkin, Phys. Lett. B **650**, 421 (2007)]

$$V(Q,z) = 4\kappa^4 z^2 \sum_{n=0}^{\infty} \frac{L_n^1(\kappa^2 z^2)}{Q^2 + M_n^2}$$

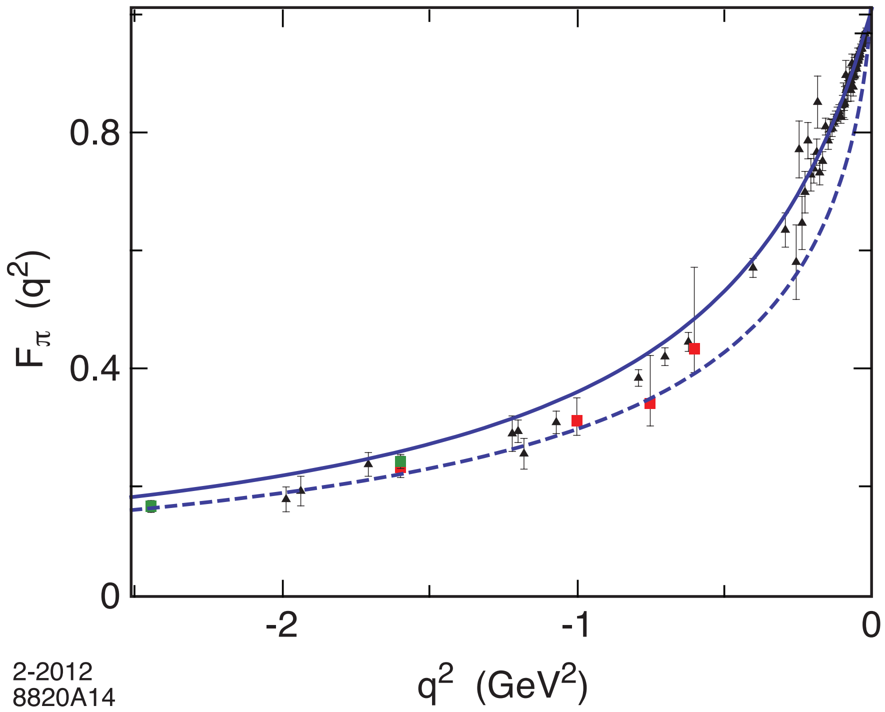
- Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory (twist $\tau = N + L$) [Brodsky and GdT, Phys. Rev. D77 (2008) 056007]

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{\tau-2}}^2}\right)}$$

- Analytical form $F(Q^2)$ incorporates correct scaling from constituents and mass gap from confinement

- $\mathcal{M}_{\rho_n}^{\,2} \to 4\kappa^2(n + 1/2)$ since VM is twist-2 $q\overline{q}$ and not twist 3 *squark-squark* with $L = 1$

- Finite charge radius and nonperturbative pole structure generated with ”dressed” EM current in AdS



Continuous line: confined current, dashed line free current.

- Effective LF wave function

$$\psi(x, \mathbf{b}_\perp) = \kappa \frac{(1-x)}{\sqrt{\pi \ln(\frac{1}{x})}}\, e^{-\frac{1}{2}\kappa^2 \mathbf{b}_\perp^2 (1-x)^2/\ln(\frac{1}{x})}$$

Nucleon Elastic Form Factors [Brodsky and GdT, arXiv:1203.4025 [hep-ph]]

- Nucleon EM form factor

$$\langle P'|J^\mu(0)|P\rangle = u(P')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2\mathcal{M}}F_2(q^2)\right]u(P)$$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode $\Psi_P(x,z)$

$$\int d^4x\, dz\, \sqrt{g}\; \overline{\Psi}_{P'}(x,z)\, e^A_M \Gamma_A A^M(x,z)\Psi_P(x,z)$$
$$\sim (2\pi)^4\delta^4\left(P'-P-q\right)\epsilon_\mu u(P')\gamma^\mu F_1(q^2)u(P)$$

- Effective AdS/QCD model: additional 'anomalous' term in the 5-dim action

[Abidin and Carlson, Phys. Rev. D79, 115003 (2009)]

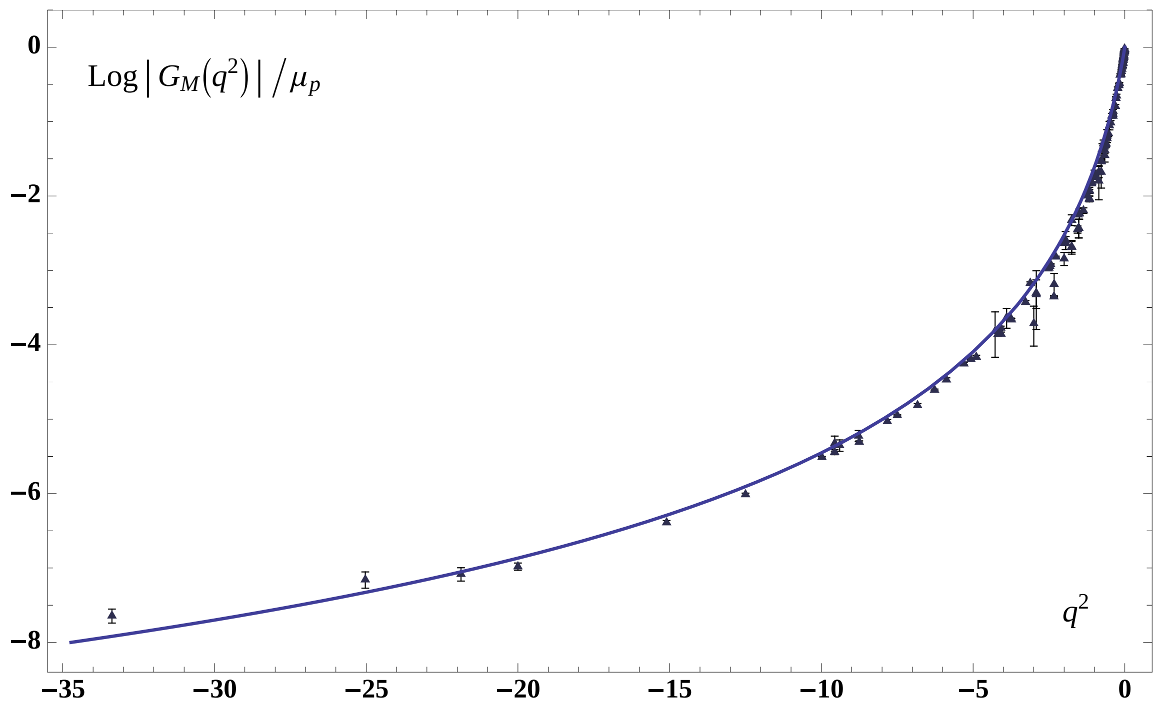
$$\int d^4x\, dz\, \sqrt{g}\; \overline{\Psi}\, e^A_M e^B_N \left[\Gamma_A, \Gamma_B\right] F^{MN}\Psi$$
$$\sim (2\pi)^4\delta^4\left(P'-P-q\right)\epsilon_\mu u(P')\frac{i\sigma^{\mu\nu}q^\nu}{2\mathcal{M}}F_2(q^2)u(P)$$

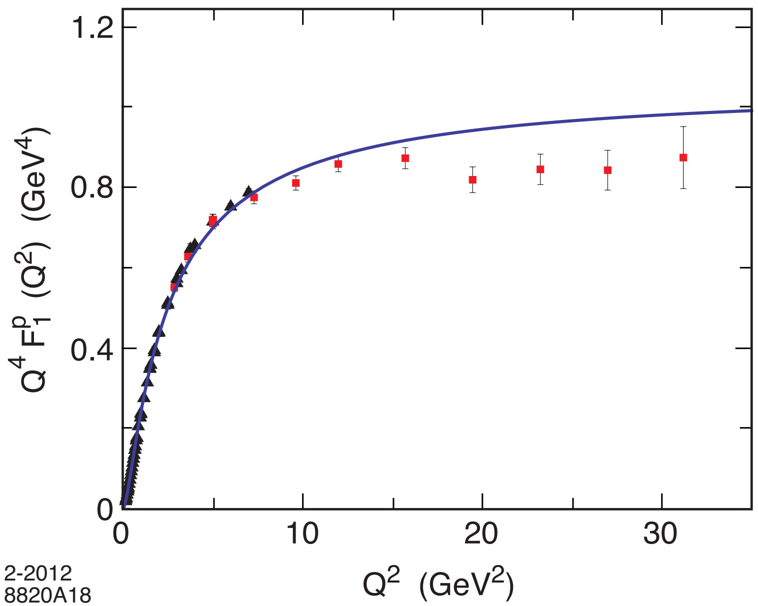
- Generalized Parton Distributions in AdS/QCD

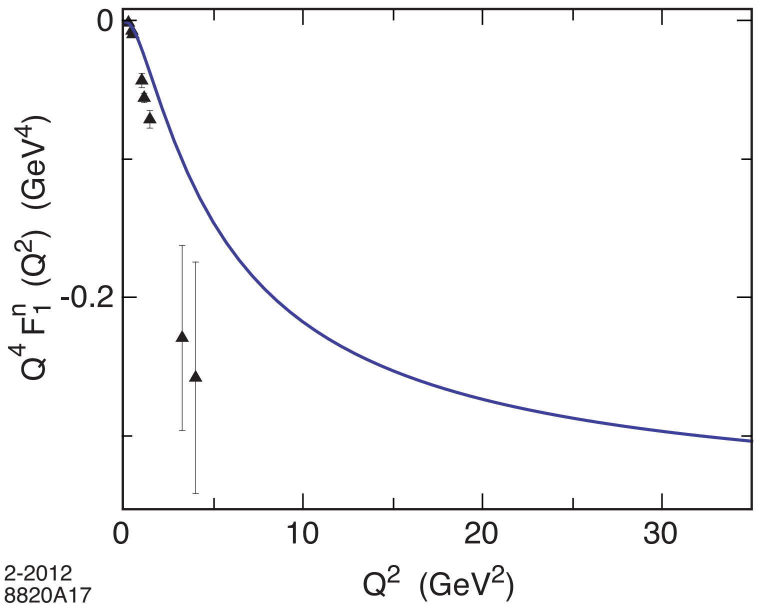
[Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

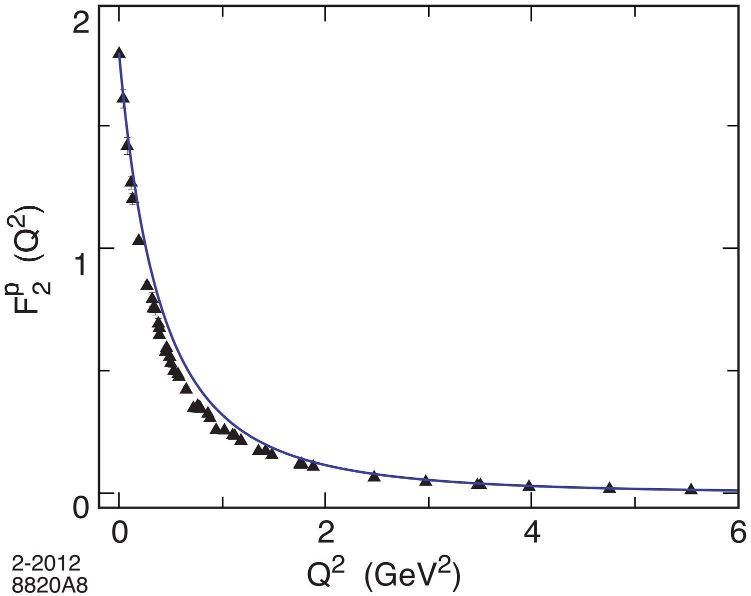
- Use $SU(6)$ flavor symmetry and normalization to static quantities $G_M(q^2) = F_1(q^2) + F_2(q^2)$

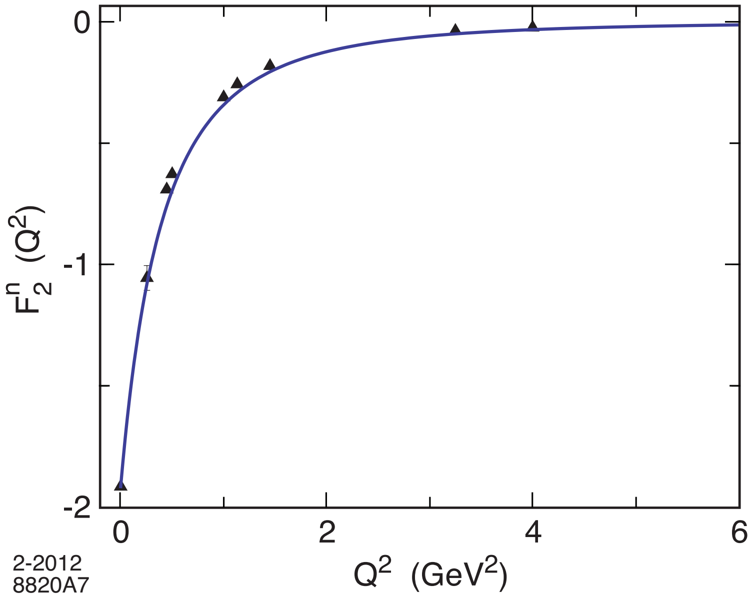
$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \qquad F_2^p(Q^2) = \frac{\chi_p}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$




$Q^4 F_1^p (Q^2)$ (GeV^4)
Q^2 (GeV^2)
2-2012
8820A18


$Q^4 F_1^n (Q^2)$ (GeV^4)
Q^2 (GeV^2)
2-2012
8820A17

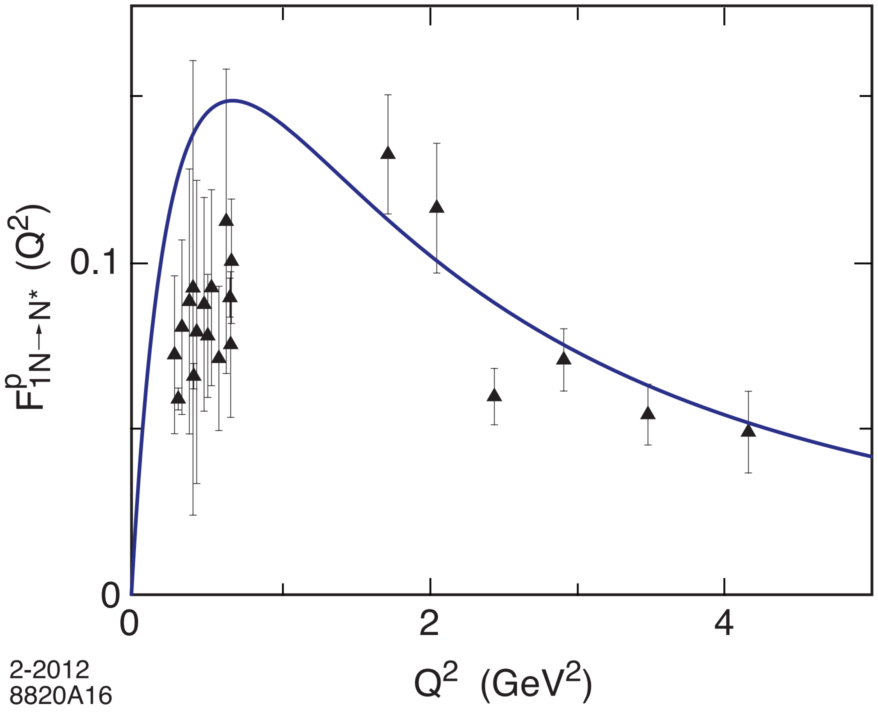

$F_2^p (Q^2)$
Q^2 (GeV^2)
2-2012
8820A8


$F_2^n (Q^2)$
Q^2 (GeV^2)
2-2012
8820A7

Nucleon Transition Form Factors

- Orthonormality of Laguerre functions $F_1{}^p_{N \to N^*}(0) = 0$

$$F_1{}^p_{N \to N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$



Proton transition form factor to the first radial excited state. Data from JLab

Flavor Decomposition of Elastic Nucleon Form Factors

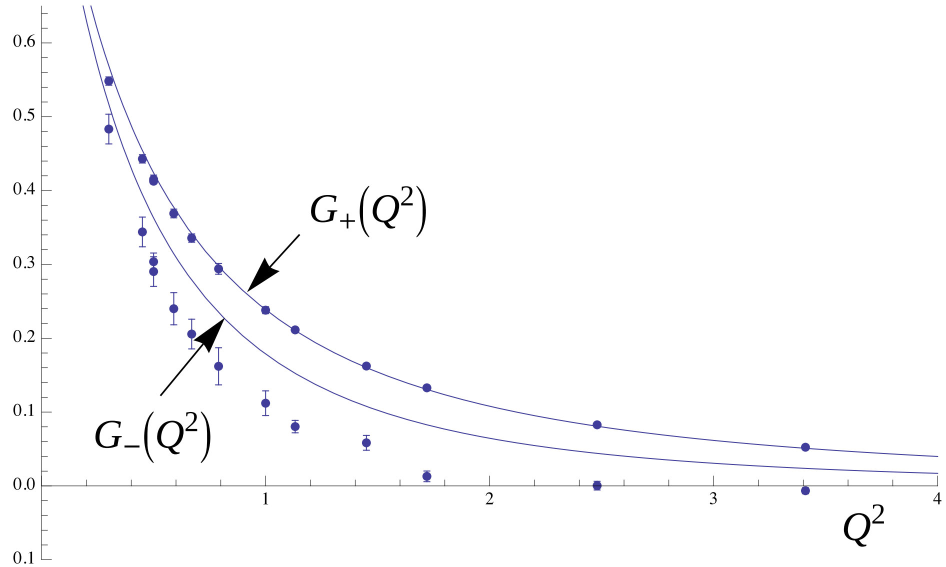
G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-,\qquad F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-,\qquad F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$

$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)}$$

and

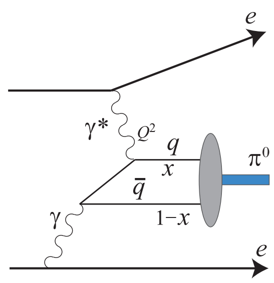
$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}$$



PRELIMINARY

Pion Transition Form-Factor

[S. J. Brodsky, F.-G. Cao and GdT, arXiv:1005.39XX]



- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \to \gamma$ vertex in the amplitude $e\pi \to e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2) \epsilon_{\mu\nu\rho\sigma}(p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad k^2 = 0$$

- Asymptotic value of pion TFF is determined by first principles in QCD:
  $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$ [Lepage and Brodsky (1980)]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} (p_\pi + q - k) \, F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

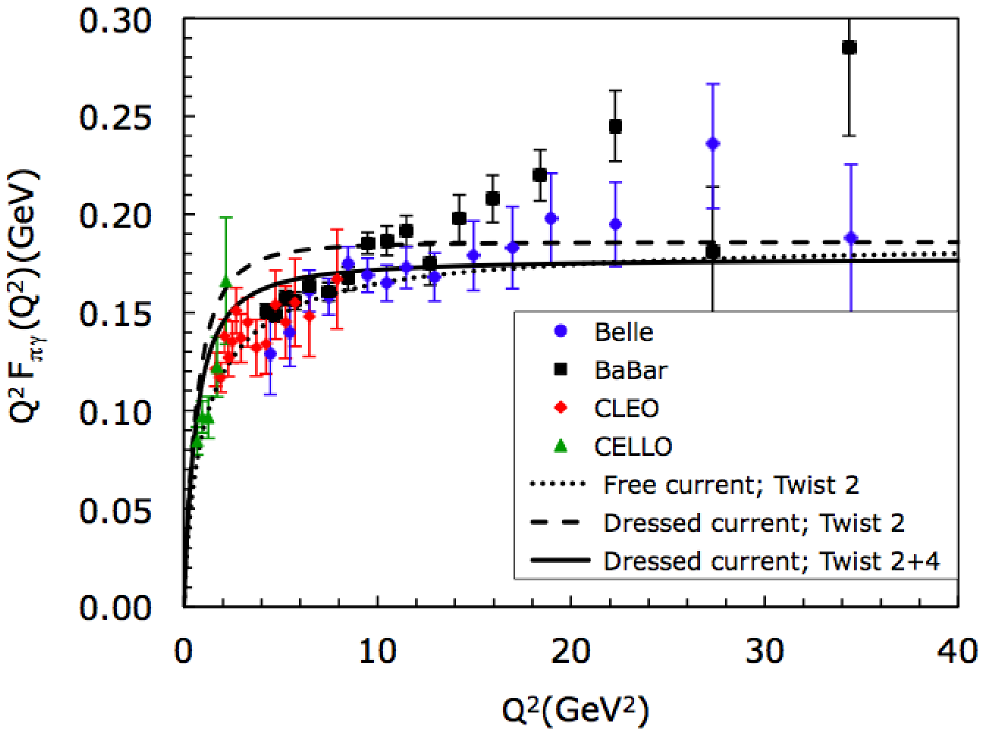
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \kappa/\sqrt{2}\pi\right)$

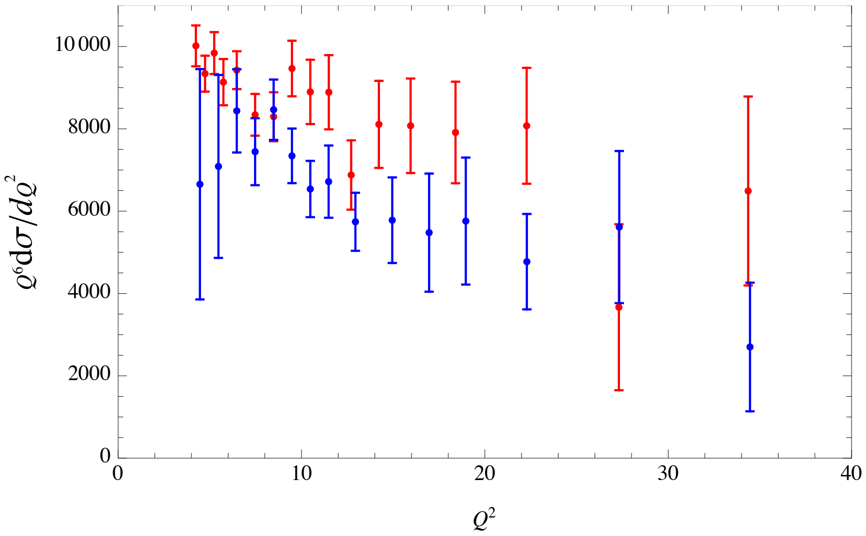
$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{Q^2(1-x)/4\pi^2 f_\pi^2 \, x}\right]$$

  normalized to the asymptotic DA [Musatov and Radyushkin (1997)]

- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

Pion-gamma transition form factor



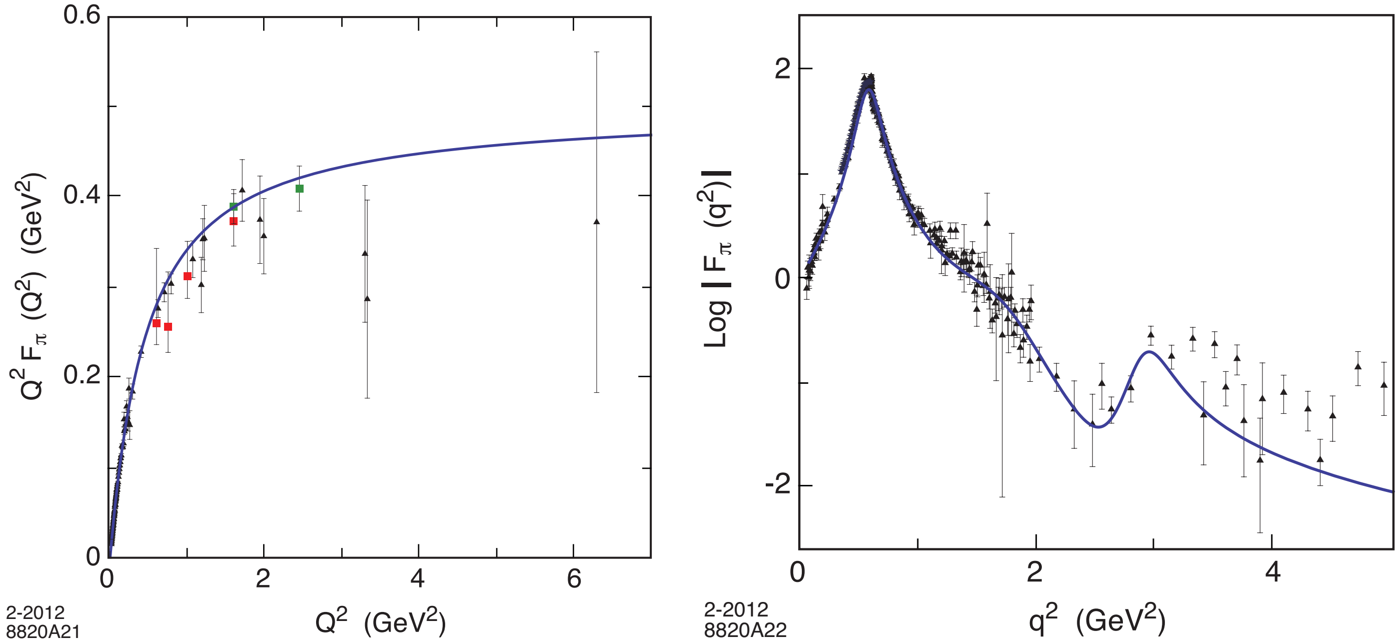


Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \to qq$, $q\overline{q} \to q\overline{q}$ but also to $q \to qq\overline{q}$ and $\overline{q} \to \overline{q}q\overline{q}$
- Higher Fock states can have any number of extra $q\overline{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

$$|\pi\rangle = \psi_{q\overline{q}/\pi}|q\overline{q}\rangle_{\tau=2} + \psi_{q\overline{q}q\overline{q}}|q\overline{q}q\overline{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \to q^2 + \sqrt{2}i\mathcal{M}\Gamma$ $\quad$ ($P_{q\overline{q}q\overline{q}}$ = 13 %)



Conclusions

- The gauge/gravity duality leads to a simple analytical frame-independent nonperturbative semiclassical approximation to the light-front Hamiltonian problem for QCD: “Light-Front Holography”
- Unlike usual instant-time quantization the Hamiltonian equation in the light-front is frame independent and has a structure similar to eigenmode equations in AdS
- AdS transition matrix elements (overlap of AdS wave functions) map to current matrix elements in LF QCD (convolution of frame-independent light-front wave functions)
- Mapping of AdS gravity to boundary QFT quantized at fixed light-front time gives a precise relation between holographic wave functions in AdS and LFWFs describing the internal structure of hadrons
- No constituent gluons
- Improve the semiclassical approximation: introduce nonzero quark masses and short-range Coulomb-like gluonic corrections (heavy and heavy-light quark systems)