Systematics of the Excitation Spectrum and Form Factors of Baryons in Holographic QCD: from Confinement to Quark Degrees of Freedom

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Nucleon Resonance Structure in Exclusive Electroproduction at High Photon Virtualities

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Gauge/Gravity Correspondence and QCD

- Review recent analytical insights into the nonperturbative nature of light-hadron bound states using the gauge/gravity correspondence [Maldacena (1998)]

- Description of strongly coupled ultra relativistic system using a dual gravity description in a higher dimensional space (holographic)

- Why is AdS space important? AdS$_5$ is a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu$, $M^{\mu\nu}$, $K^\mu$, $D$, with the group of isometries of AdS$_5$

- Mapping of AdS gravity to QCD quantized at fixed light-front time gives a precise relation between wave functions in AdS space and the LF wavefunctions describing the internal structure of hadrons

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Isometry group: most general group of transformations which leave invariant the distance between two points: dimension of isometry group of AdS$_{d+1}$ is \( \frac{(d+1)(d+2)}{2} \)
• AdS$_5$ metric:

\[
\frac{ds^2}{L_{\text{AdS}}} = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)
\]

\[L_{\text{Minkowski}}\]

• A distance $L_{\text{AdS}}$ shrinks by a warp factor $z/R$ as observed in Minkowski space ($dz = 0$):

\[
L_{\text{Minkowski}} \sim \frac{z}{R} L_{\text{AdS}}
\]

• Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable $z$ acts like a scaling variable in Minkowski space

• Short distances $x_\mu x^\mu \rightarrow 0$ maps to UV conformal AdS boundary $z \rightarrow 0$

• Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$ maps to large IR region of AdS, $z \sim 1/\Lambda_{\text{QCD}}$

• Use isometries of AdS to map local interpolating operators at the UV boundary into modes propagating inside AdS
AdS Gravity Action

- AdS$_5$ metric $x^M = (x^\mu, z)$:

\[ ds^2 = g_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \]

- Action for gravity coupled to scalar field in AdS$_5$ ($\Lambda = -\frac{6}{R^2}$):

\[ S = \int d^4 x dz \sqrt{g} \left( \frac{1}{\kappa^2} (\mathcal{R} - 2\Lambda) + \frac{1}{2} (g^{MN} \partial_M \Phi \partial_N \Phi - \mu^2 \Phi^2) \right) \]

- Equations of motion ($\sqrt{g} = (R/z)^5$)

\[ \mathcal{R}_{MN} - \frac{1}{2} g_{MN} \mathcal{R} - \Lambda g_{MN} = 0 \]

\[ z^3 \partial_z \left( \frac{1}{z^3} \partial_z \Phi \right) - \partial_\nu \partial^\nu \Phi - \left( \frac{\mu R}{z} \right)^2 \Phi = 0 \]
Light-Front Holographic Mapping

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Physical modes are plane-waves along $x^\mu$-coordinates with four-momentum $P^\mu$ and invariant mass $P_\mu P^\mu = M^2$: $\Phi_P(x, z) = e^{-i P \cdot x} \Phi(z)$

- Find AdS eom

$$\left[ -z^3 \partial_z \left( \frac{1}{z^3} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi = M^2 \Phi$$

- Upon substitution $z \to \zeta$ and $\phi(\zeta) \sim \zeta^{-3/2} \Phi(\zeta)$ in AdS eom we find

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with $U(\zeta) = 0$ in the conformal AdS limit and $(\mu R)^2 = -4 + L^2$

- Identical with LFWE from Hamiltonian LF eom $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$, where $\zeta$ is the invariant transverse distance between two partons $\zeta^2 = x(1 - x)b_\perp^2$ and the effective interaction $U$ acts only on the valence sector

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
Meson Spectrum in Hard Wall Model

[LF Hard wall model: GdT and S. J. Brodsky, PRL 94, 201601 (2005)]

- How to break conformality and compute the hadronic spectrum?

- Conformal model up to the confinement scale $1/\Lambda_{QCD}$ [Polchinski and Strassler (2002)]

$$U(\zeta) = \begin{cases} 0 & \text{if } \zeta \leq \frac{1}{\Lambda_{QCD}} \\ \infty & \text{if } \zeta > \frac{1}{\Lambda_{QCD}} \end{cases}$$

- Confinement scale $\frac{1}{\Lambda_{QCD}} \sim 1$ Fm, $\Lambda_{QCD} \sim 200$ MeV

- Covariant version of MIT bag model: quarks permanently confined inside a finite region of space

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int_0^{\Lambda_{QCD}^{-1}} d\zeta \phi^2(z) = 1$

$$\phi_{L,k}(\zeta) = \frac{\sqrt{2}\Lambda_{QCD}}{J_{1+L}(\beta_{L,k})} \sqrt{\zeta} J_L(\zeta \beta_{L,k} \Lambda_{QCD})$$

- Eigenvalues

$$\mathcal{M}_{L,k} = \beta_{L,k} \Lambda_{QCD}$$
Table 1: $I = 1$ mesons. For a $q\bar{q}$ state $P = (-1)^{L+1}$, $C = (-1)^{L+S}$

<table>
<thead>
<tr>
<th>$L$</th>
<th>$S$</th>
<th>$n$</th>
<th>$J^{PC}$</th>
<th>$I = 1$ Meson</th>
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<td>0</td>
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<td>1$^{--}$</td>
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<td>0$^{++}$</td>
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<td>4$^{++}$</td>
<td>$a_4(2040)$</td>
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</table>
Orbital and radial excitations for the $\pi$ and the $\rho$ I=1 meson families ($\Lambda_{QCD} = 0.32$ GeV)

- Pion is not chiral
- $M \sim 2n + L$ in contrast to usual Regge dependence $M^2 \sim n + L$
- Important $J - L$ splitting (different $J$ for same $L$) in mesons not described by hard-wall model
- Radial modes not well described in hard-wall model
Higher Spin Wave Equations in AdS Space

• Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

• Spin-$J$ in AdS represented by totally symmetric rank $J$ tensor field $\Phi_{M_1 \cdots M_J}$

• Action for spin-$J$ field in AdS$_{d+1}$ $(x^M = (x^\mu, z))$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left( g^{MN} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_M \Phi_{M_1 \cdots M_J} D_N \Phi_{M_1' \cdots M_J'} - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \right)$$

where $D_M$ is the covariant derivative which includes parallel transport (affine connection)

$$D_M \Phi_{M_1 \cdots M_J} = \partial_M \Phi_{M_1 \cdots M_J} - \Gamma^K_{MM_1} \Phi_{K \cdots M_J} - \cdots - \Gamma^K_{M_{M_J}} \Phi_{M_1 \cdots K}$$

• Dilaton background $\varphi(z)$ breaks conformality of the theory (vanishes in the UV limit)
• Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1...\mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1...\mu_J}, \quad \Phi_{z\mu_2...\mu_J} = \cdots = \Phi_{\mu_1\mu_2...z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu = M^2$

• Construct effective action in terms of spin-$J$ modes $\Phi_J$ with only physical degrees of freedom

[H. G. Dosch, S. J. Brodsky and GdT]

• Find AdS wave equation for spin-$J$ mode $\Phi_J = \Phi_{\mu_1...\mu_J}$

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = M^2 \Phi_J(z)$$

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Dual QCD Light-Front Wave Equation

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^d-1-2J}{e^{\varphi(z)}} \frac{\partial}{\partial z} \left(\frac{z^{d-1-2J} \partial_z}{e^{\varphi(z)}}\right) + \left(\frac{\mu_R}{z}\right)^2\right] \Phi_J(z) = M^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms of the partons inside a hadron

- Interaction terms in the QCD Lagrangian build the effective confining potential $U(\zeta)$ which acts on the valence sector and correspond to the truncation of AdS space in an effective dual gravity approximation
Meson Spectrum in Soft Wall Model

• Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

• Dilaton profile $\varphi(z) = +\kappa^2 z^2$

• Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$

• LFWE

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \right) \phi_J(\zeta) = M_J^2 \phi_J(\zeta)
\]

• Normalized eigenfunctions

$$\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

• Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J + L}{2} \right)$$
LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes
\[ J = L + S, \ I = 1 \] meson families \[ \mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2) \]

- Triplet splitting for the \( L = 1, \ J = 0, 1, 2, \ I = 1 \) vector meson \( a \)-states
  \[
  \mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}
  \]

- \( J - L \) splitting in mesons and radial excitations are well described in soft-wall model

Orbital and radial excitations for the \( \pi \ (\kappa = 0.59 \text{ GeV}) \) and the \( \rho \ I=1 \) meson families \( (\kappa = 0.54 \text{ GeV}) \)
Fermionic Modes in AdS Space and Baryon Spectrum

[GdT and S. J. Brodsky, PRL 94, 201601 (2005)]

- Lattice calculations of the ground state hadron masses agree very well with experimental values.

- However, excitation spectrum of nucleon represents important challenge to LQCD due to enormous computational complexity beyond ground state configuration and multi-hadron thresholds.

- Large basis of interpolating operators required in LQCD since excited nucleon states are classified according to irreducible representations of the lattice, not the angular momentum.

- The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods.

- Analytical exploration of systematics of light-baryon resonances and nucleon form factors.

- Extension of holographic ideas to spin-\(\frac{1}{2}\) (and higher half-integral \(J\)) hadrons by considering propagation of RS spinor field \(\Psi_{\alpha M_1...M_{J-1/2}}\) in AdS space.
Higher Spin Wave Equations in AdS Space

- For fermion fields in AdS one cannot break conformality with introduction of dilaton background since it can be scaled away leaving the action conformally invariant [I. Kirsch (2006)]

- Introduce an effective confining potential $V(z)$ in the action for a Dirac field in $\text{AdS}_{d+1}$

$$S_F = \int d^d x dz \sqrt{\text{det} g} g^{M_1 M'_1} \cdots g^{M_T M'_T} \left( \overline{\Psi}_{M_1 \cdots M_T} \left( i e_A^M \Gamma_A^M D_M - \mu - V(z) \right) \Psi_{M'_1 \cdots M'_T} + \cdots \right)$$

where $D_M$ is the covariant derivative of the spinor field $\Psi_{\alpha M_1 \cdots M_T}$, $T = J - \frac{1}{2}$

$$D_M \Psi_{M_1 \cdots M_T} = \partial_M \Psi_{M_1 \cdots M_T} - \frac{i}{2} \omega^A_M \Sigma_{AB} \Psi_{M_1 \cdots M_T} - \Gamma^K_{M M_1} \Psi_{K M_1 \cdots M_T} - \cdots - \Gamma^K_{M M_T} \Psi_{M_1 \cdots K}$$

- $M, N = 1, \cdots, d + 1$ curved space indices, $A, B = 1, \cdots, d + 1$ tangent indices

- $e_A^M$ is the vielbein, $w^A_M$ spin connection, $\Sigma_{AB}$ generators of the Lorentz group, $\Sigma_{AB} = \frac{i}{4} [\Gamma_A, \Gamma_B]$

- $\Gamma^A$ tangent space Dirac matrices $\{\Gamma^A, \Gamma^B\} = \eta^{AB}$

- For $d$ even we choose $\Gamma^A = (\Gamma^\mu, \Gamma^z)$ with $\Gamma_z = -\Gamma^z = \Gamma_0 \Gamma_1 \cdots \Gamma_{d-1}$

- For $d = 4$: $\Gamma^A = (\gamma^\mu, i \gamma_5)$
• Physical hadron has plane-wave, spinors, and polarization along $3+1$ physical coordinates

$$\Psi_P(x, z)_{\mu_1\ldots\mu_T} = e^{-iP\cdot x} \Psi(z)_{\mu_1\ldots\mu_T}, \quad \Psi_{z\mu_2\ldots\mu_T} = \cdots = \Psi_{\mu_1\mu_2\ldots z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu = M^2$

• Construct effective action in terms of spin-$J$ modes $\Psi_J$ with only physical degrees of freedom

[H. G. Dosch, S. J. Brodsky and GdT]

• Find AdS wave equation for spin-$J$ mode $\Phi_J = \Phi_{\mu_1\ldots\mu_{J-1/2}}$

$$\left[ i \left( z \eta^{MN} \Gamma_M \partial_N + \frac{d}{2} \Gamma_z \right) - \mu R - RV(z) \right] \Psi_J = 0$$

upon $\mu$-rescaling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Light-Front Mapping and Cluster Decomposition

- Upon substitution $z \rightarrow \zeta$ and

$$\Psi(x, z) = e^{-iP \cdot x z^2} \psi(z) u(P),$$

find LFWE for $d = 4$

$$\frac{d}{d\zeta} \psi_+ + \frac{\nu + \frac{1}{2}}{\zeta} \psi_+ + U(\zeta) \psi_+ = \mathcal{M} \psi_-, \quad -\frac{d}{d\zeta} \psi_- + \frac{\nu + \frac{1}{2}}{\zeta} \psi_- + U(\zeta) \psi_- = \mathcal{M} \psi_+,$$

where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- $\zeta$ is the $x$-weighted definition of the transverse impact variable of the $n-1$ spectator system

[S. J. Brodsky and GdT, PRL 96, 201601 (2006)]

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|$$

where $x = x_n$ is the longitudinal momentum fraction of the active quark

- Same multiplicity of states for mesons and baryons!
Baryon Spectrum in Soft-Wall Model

• Choose linear potential $U = \kappa^2 \zeta$

• LF nucleon eigenfunctions $\nu = L + 1 \quad (\tau = 3)$

\[
\psi_+ (\zeta) = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)
\]

\[
\psi_- (\zeta) = \kappa^{3+L} \sqrt{\frac{1}{n+L+2}} \zeta^{5+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)
\]

• Normalization

\[
\int d\zeta \psi_+^2 (\zeta) = \int d\zeta \psi_-^2 (\zeta) = 1
\]

• Eigenvalues

\[
\mathcal{M}_{n,L}^2 = 4\kappa^2 (n + L + 2) + C
\]

• Full $J - L$ degeneracy (different $J$ for same $L$) for baryons along given trajectory!
<table>
<thead>
<tr>
<th>SU(6)</th>
<th>K</th>
<th>S</th>
<th>L</th>
<th>n</th>
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<tr>
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<tr>
<td></td>
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<td>3/2</td>
<td>4</td>
<td>0</td>
<td>(\Delta_{5/2}^+) (\Delta_{7/2}^+) (\Delta_{9/2}^+) (\Delta_{11/2}^+ (2420))</td>
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<tr>
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</table>
• Gap scale $4\kappa^2$ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families!

• No $J - L$ splitting!

Plus-minus nucleon spectrum gap for $\kappa = 0.49$ GeV

• Fix the energy scale to the proton mass for the lowest state $n = 0, L = 0$: $C = -4\kappa^2$
- Phenomenological rules for increase in mass $M^2$ to construct full baryon spectrum from proton state
  
  $4\kappa^2$ for $\Delta n = 1$
  $4\kappa^2$ for $\Delta L = 1$
  $2\kappa^2$ for $\Delta S = 1$
  $2\kappa^2$ for $\Delta P = \pm$

- Eigenvalues
  
  $M^2_{n,L,S}^{(\pm)} = 4\kappa^2 \left( n + L + S/2 + 3/4 \right)$
  $M^2_{n,L,S}^{(-)} = 4\kappa^2 \left( n + L + S/2 + 5/4 \right)$
Orbital and radial excitations for positive parity $N$ and $\Delta$ baryon families ($\kappa = 0.49 - 0.51$ GeV)
Since \( M_{n,L,S=\frac{3}{2}}^2(+) = M_{n,L,S=\frac{1}{2}}^2(-) \) positive and negative-parity \( \Delta \) states are in the same trajectory.

[See also: H. Forkel, M. Beyer and T. Frederico, JHEP 0707, 077 (2007)]

\[ \Delta \text{ orbital trajectories for } n = 0 \text{ and } \kappa = 0.51 \text{ GeV} \]

- \( \Delta(1930) \) quantum number assignment (E. Klempt and J. M. Richard (2010): \( S = 3/2, L = 1, n = 1 \))

- Find \( M_{\Delta(1930)} = 4\kappa \approx 2 \text{ GeV} \) compared with experimental value 1.96 GeV

- All known baryons well described by holographic formulas for \( M_{n,L,S}^2(+) \) and \( M_{n,L,S}^2(-) \)
Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL 96, 201601 (2006)] Mapping of EM currents
[S. J. Brodsky and GdT, PRD 78, 025032 (2008)] Mapping of energy-momentum tensor

• EM transition matrix element in QCD: local coupling to pointlike constituents

\[ \langle P' | J^\mu | P \rangle = (P + P')^\mu F(Q^2) \]

where \( Q = P' - P \) and \( J^\mu = e_q \overline{q} \gamma^\mu q \)

• EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode \( \Phi(x, z) \)

\[
\int d^4x \; dz \sqrt{g} A^M(x, z) \Phi_P^*(x, z) \hat{\partial}_M \Phi_P(x, z) \\
\sim (2\pi)^4 \delta^4(P' - P) \epsilon_\mu (P + P')^\mu F(Q^2)
\]

• Expressions for the transition amplitudes look very different but a precise mapping of the matrix elements can be carried out at fixed light-front time: \( \Phi_P(z) \Leftrightarrow \psi(P) \)
• Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$
\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \to z^\tau, \quad z \to 0
$$

• Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons $\Phi_P$ and $\Phi_P'$, with the non-normalizable mode $V(Q, z)$ dual to external EM source [Polchinski and Strassler (2002)].

$$
F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi^2(z) \to \left( \frac{1}{Q^2} \right)^{\tau-1} V(Q, z) \to zQ K_1(zQ)
$$

At large $Q$ important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

• Compare with electromagnetic FF in LF QCD for arbitrary $Q$. Expressions can be matched only if LFWF is factorized

$$
\psi(x, \zeta, \varphi) = e^{iM \varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}
$$

• Find

$$
X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = (\zeta/R)^{-3/2} \Phi(\zeta), \quad z \to \zeta
$$
• Dressed current for soft-wall model

\[ V(Q, z) = \Gamma \left(1 + \frac{Q^2}{4\kappa^2}\right) U \left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) \]


\[ V(Q, z) = 4\kappa^4 z^2 \sum_{n=0}^{\infty} \frac{L_n^{1}(\kappa^2 z^2)}{Q^2 + M_n^2} \]

• Form factor in soft-wall model expressed as \( \tau - 1 \) product of poles along vector radial trajectory (twist \( \tau = N + L \)) [Brodsky and GdT, Phys. Rev. D77 (2008) 056007]

\[ F_{\tau}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{M_{\rho^{\tau-2}}^2}\right)} \]

• Analytical form \( F(Q^2) \) incorporates correct scaling from constituents and mass gap from confinement

• \( M_{\rho_{n}}^2 \rightarrow 4\kappa^2(n + 1/2) \) since VM is twist-2 \( q\bar{q} \) and not twist 3 squark-squark with \( L = 1 \)
- Finite charge radius and nonperturbative pole structure generated with "dressed" EM current in AdS

Continuous line: confined current, dashed line free current.

- Effective LF wave function

\[ \psi(x, b_\perp) = \kappa \frac{(1 - x)}{\sqrt{\pi \ln \left( \frac{1}{x} \right)}} e^{-\frac{1}{2} \kappa^2 b_\perp^2 (1-x)^2 / \ln(\frac{1}{x})} \]

• Nucleon EM form factor

\[
\langle P' | J^\mu(0) | P \rangle = u(P') \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2M} F_2(q^2) \right] u(P)
\]

• EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode \( \Psi_P(x, z) \)

\[
\int d^4x \, dz \, \sqrt{g} \, \bar{\Psi}_{P'}(x, z) \, e^A_M \Gamma_A A^M(x, z) \Psi_P(x, z)
\sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu u(P') \gamma^\mu F_1(q^2) u(P)
\]

• Effective AdS/QCD model: additional 'anomalous' term in the 5-dim action


\[
\int d^4x \, dz \, \sqrt{g} \, \bar{\Psi} e^A_M e^B_N \left[ \Gamma_A, \Gamma_B \right] F^{MN} \Psi
\sim (2\pi)^4 \delta^4(P' - P - q) \epsilon_\mu u(P') \frac{i\sigma^{\mu\nu}q^\nu}{2M} F_2(q^2) u(P)
\]

• Generalized Parton Distributions in AdS/QCD

[Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]
• Use $SU(6)$ flavor symmetry and normalization to static quantities

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right)}$$

$$F_2^p(Q^2) = \frac{\chi_p}{\left(1 + \frac{Q^2}{M_\rho^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right)\left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}$$
Nucleon Transition Form Factors

- Orthonormality of Laguerre functions
  \[ F_{1 N \rightarrow N^*}(0) = 0 \]

\[
F_{1 N \rightarrow N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{Q^2}{M^2_\rho} \left( 1 + \frac{Q^2}{M^2_\rho} \right) \left( 1 + \frac{Q^2}{M^2_{\rho'}} \right) \left( 1 + \frac{Q^2}{M^2_{\rho''}} \right)
\]

Proton transition form factor to the first radial excited state. Data from JLab
Flavor Decomposition of Elastic Nucleon Form Factors


- Proton SU(6) WF: \[ F_{u,1}^p = \frac{5}{3} G_+ + \frac{1}{3} G_- , \quad F_{d,1}^p = \frac{1}{3} G_+ + \frac{2}{3} G_- \]
- Neutron SU(6) WF: \[ F_{u,1}^n = \frac{1}{3} G_+ + \frac{2}{3} G_- , \quad F_{d,1}^n = \frac{5}{3} G_+ + \frac{1}{3} G_- \]

\[ G_+(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right)} \]

and

\[ G_-(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)} \]
Pion Transition Form-Factor

• Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \rightarrow \gamma$ vertex in the amplitude $e\pi \rightarrow e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2)\epsilon_{\mu\nu\rho\sigma}(p_\pi)\epsilon_\rho(k)q_\sigma, \quad k^2 = 0$$

• Asymptotic value of pion TFF is determined by first principles in QCD:

$$Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi \quad [\text{Lepage and Brodsky (1980)}]$$

• Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta(4)(p_\pi + q - k) F_{\pi\gamma}(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon_\mu(q)(p_\pi)\epsilon_\rho(k)q_\sigma$$

• Find $\phi(x) = \sqrt{3}f_\pi x(1 - x), \quad f_\pi = \kappa/\sqrt{2\pi}$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1 - x} \left[ 1 - e^{Q^2(1-x)/4\pi^2 f_\pi^2 x} \right]$$

normalized to the asymptotic DA [Musatov and Radyushkin (1997)]

• The CS form is local in AdS space and projects out only the asymptotic form of the pion DA
Pion-gamma transition form factor

\[ Q^2 F_{\pi\gamma}(Q^2) \text{(GeV)} \]

\[ Q^2 (\text{GeV}^2) \]

\[ Q^2 \left( \frac{\text{d}\sigma}{\text{d}Q^2} \right) \]

- Belle
- BaBar
- CLEO
- CELLO
- Free current; Twist 2
- Dressed current; Twist 2
- Dressed current; Twist 2+4

EmNN*2012, USC, August 15, 2012
Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ but also to $q \rightarrow qq\bar{q}$ and $\bar{q} \rightarrow \bar{q}q\bar{q}$

- Higher Fock states can have any number of extra $q\bar{q}$ pairs, but surprisingly no dynamical gluons

- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

$$|\pi\rangle = \psi_{qq/\pi}|qq\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \rightarrow q^2 + \sqrt{2i\mathcal{M}}\Gamma$ ($P_{q\bar{q}q\bar{q}} = 13\%$)
Conclusions

• The gauge/gravity duality leads to a simple analytical frame-independent nonperturbative semiclassical approximation to the light-front Hamiltonian problem for QCD: “Light-Front Holography”

• Unlike usual instant-time quantization the Hamiltonian equation in the light-front is frame independent and has a structure similar to eigenmode equations in AdS

• AdS transition matrix elements (overlap of AdS wave functions) map to current matrix elements in LF QCD (convolution of frame-independent light-front wave functions)

• Mapping of AdS gravity to boundary QFT quantized at fixed light-front time gives a precise relation between holographic wave functions in AdS and LFWFs describing the internal structure of hadrons

• No constituent gluons

• Improve the semiclassical approximation: introduce nonzero quark masses and short-range Coulomb-like gluonic corrections (heavy and heavy-light quark systems)