High $Q^2$ Helicity Amplitudes in the hypercentral Constituent Quark Model

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Genova

Nucleon Resonances: From Photoproduction to High Photon Virtualities
Trento, 12-16 October 2015
Outline of the talk

The model

Some remarks on the spectrum

The helicity amplitudes
  Photocouplings
  $Q^2$ dependence

Relativity
  Elastic form factors
  N-$\Delta$ transition

Asymptotic behaviour
The model
Hypercental Constituent Quark Model

\[ H_{3q} = T + V_{3q}(\rho, \lambda) + H_{hyp} \]

- SU(6) symmetry
- SU(6) violation

SU(6) configurations \( L^P_t \) \( t=A,M,S \) (symmetry type)

The quark interaction contains:
- A long range spin-independent confinement
- A short range spin dependent term

A separation typical of any CQM

Hyperspherical coordinates: \( \rho, \lambda \rightarrow (x, t, \Omega_\rho, \Omega_\lambda) \)

\[
x = (\rho^2 + \lambda^2)^{1/2} \\
t = \arctg \frac{\rho}{\lambda}
\]

hyperradius
hyperangle
(size)
(form)

In the Schrödinger equation

\[
L^2(\theta, \phi) \rightarrow L^2( t, \Omega_\rho, \Omega_\lambda) \\
C_2(O(3)) \quad C_2(O(6))
\]

\[
L^2( t, \Omega_\rho, \Omega_\lambda) Y_{[\gamma]} ( t, \Omega_\rho, \Omega_\lambda) = -\gamma (\gamma+4) Y_{[\gamma]} ( t, \Omega_\rho, \Omega_\lambda)
\]

\[
Y_{[\gamma]} ( t, \Omega_\rho, \Omega_\lambda) \quad \text{hyperspherical harmonics}
\]

\[
\gamma = 2n + l_\rho + l_\lambda \quad \text{grand angular quantum number}
\]
Hypercentral hypothesis

\[ V = V(x) \]

Factorization of the wf

\[ \Psi(x, t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) Y_{[\gamma]}(\Omega) \]

\( \nu \) number of nodes
\( \gamma \) grand angular quantum number

Only one “hypercentral” equation for \( \psi_{\nu\gamma}(x) \)
Hypercentral Model

\[ H_{3q} = T + -\tau/\chi + \alpha \chi + H_{hyp} \]

three free parameters fitted to the spectrum

Having fixed the parameters \( \Rightarrow \) predictions

Some remarks on the spectrum
Two analytical solutions of the hypercentral equation

\[ \sum_{i<j} \frac{1}{2} k (r_i - r_j)^2 = \frac{3}{2} k x^2 \]

- \[ \tau/x \]

<table>
<thead>
<tr>
<th>Harmonic Oscillator</th>
<th>HyperCoulomb potential</th>
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</thead>
<tbody>
<tr>
<td>[ \nu=1 ] ( 0^+_S )</td>
<td>[ \nu=2 ] ( 0^+_S )</td>
</tr>
<tr>
<td>[ \nu=0 ] ( 1^-_M )</td>
<td>[ \nu=1 ] ( 1^-_M )</td>
</tr>
<tr>
<td>[ \gamma=0 ] ( 0^+_S )</td>
<td>[ \gamma=0 ] ( 0^+_S )</td>
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</table>
\[ V(x) = -\frac{\tau}{x} + \alpha x \]
Negative parity states

**SU(6) configuration** $1^-_M$

Possible 3-quark states

obtained combining the orbital angular momentum $L=1$ with the spin values

Notation $^{2s+1}\text{SU}(3)$

<table>
<thead>
<tr>
<th>$^2$8</th>
<th>$^4$8</th>
<th>$^2$10</th>
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<tbody>
<tr>
<td>S=1/2</td>
<td>S=3/2</td>
<td>S=1/2</td>
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<tr>
<td>N 1/2$^-$</td>
<td>N 1/2$^-$</td>
<td>$\Delta$ 1/2$^-$</td>
</tr>
<tr>
<td>N 3/2$^-$</td>
<td>N 3/2$^-$</td>
<td>$\Delta$ 3/2$^-$</td>
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<tr>
<td>N 5/2$^-$</td>
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The SU(6) configuration $1^{-M}$

Contains all the $4^*$ & $3^*$ resonances known prior up to 2010

<table>
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<tr>
<th>28</th>
<th>48</th>
<th>210</th>
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<tbody>
<tr>
<td>$^2$N(1535)1/2$^-$</td>
<td>$^4$N(1650)1/2$^-$</td>
<td>$^2$Δ(1620)1/2$^-$</td>
</tr>
<tr>
<td>$^2$N(1520)3/2$^-$</td>
<td>$^4$N(1700)3/2$^-$</td>
<td>$^2$Δ(1700)3/2$^-$</td>
</tr>
<tr>
<td></td>
<td>$^4$N(1675)5/2$^-$</td>
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BUT
in the PDG 2102-2014 there are new entries

$3^*$ $^3$N(1875) 3/2$^-$  where should it be placed?

(there are also 5 new 2* states!)
$N(1875) \frac{3}{2}^-$

Harmonic Oscillator

HyperCoulomb potential

\begin{align*}
\nu &= 0 \\
\nu &= 1 \\
\gamma &= 0 \\
\gamma &= 1 \\
\gamma &= 2
\end{align*}

M.G., E. Santopinto, nucl-th 1510.00582
The helicity amplitudes
HELICITY AMPLITUDES

Extracted from electroproduction of mesons
Definition

\[ A_{1/2} = \langle N^* J_z = 1/2 \mid H_{em}^T \mid N J_z = -1/2 \rangle \]
\[ A_{3/2} = \langle N^* J_z = 3/2 \mid H_{em}^T \mid N J_z = 1/2 \rangle \]
\[ S_{1/2} = \langle N^* J_z = 1/2 \mid H_{em}^L \mid N J_z = 1/2 \rangle \]

\[ N, N^* \] nucleon and resonance as 3q states
\[ H_{em}^T, H_{em}^L \] model transition operator

\[ \text{calculated in the Breit System} \]

\[ \text{results for the negative parity resonances:} \]

Systematic predictions for transverse and longitudinal amplitudes

Proton and neutron electro-excitation to 14 resonances
The photocouplings
$10^{-3} \text{ GeV}^{-1/2}$

Δ excitation

$10^{-3}$ GeV$^{-1/2}$

Helicity amplitudes

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<tbody>
<tr>
<td><strong>Q^2 = 0 values</strong></td>
<td></td>
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<tr>
<td><strong>Q^2 = 0</strong></td>
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</tr>
<tr>
<td>D13 (1520)</td>
<td>-65.7</td>
<td>66.8</td>
<td>78.2</td>
<td>-1.4</td>
<td>-61.1</td>
<td>-79.6</td>
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<tr>
<td>D13 (1700)</td>
<td>8.0</td>
<td>-10.9</td>
<td>-7.9</td>
<td>12</td>
<td>70.1</td>
<td>8.1</td>
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<tr>
<td>D15 (1675)</td>
<td>1.4</td>
<td>1.9</td>
<td>0</td>
<td>-36.6</td>
<td>-51.1</td>
<td>-0.2</td>
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<tr>
<td>D33(1700)</td>
<td>80.9</td>
<td>70.2</td>
<td>78.2</td>
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<tr>
<td>F15 (1680)</td>
<td>-35.4</td>
<td>24.1</td>
<td>27.4</td>
<td>37.7</td>
<td>14.8</td>
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<tr>
<td>F35(1905)</td>
<td>-16.6</td>
<td>-50.5</td>
<td>-4.6</td>
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<tr>
<td>F37(1950)</td>
<td>-28.0</td>
<td>-35.1</td>
<td>-0.4</td>
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<tr>
<td>P11(1440)</td>
<td>87.7</td>
<td>65.4</td>
<td>57.9</td>
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<tr>
<td>P11(1710)</td>
<td>42.5</td>
<td>-22.6</td>
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<td>18.4</td>
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<tr>
<td>P13(1720)</td>
<td>94.1</td>
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<td>-35.8</td>
<td>-47.6</td>
<td>3.9</td>
<td>13.5</td>
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<tr>
<td>P33(1232)</td>
<td>-96.9</td>
<td>-169</td>
<td>-0.6</td>
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<tr>
<td>S11(1535)</td>
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<tr>
<td>S11(1650)</td>
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<tr>
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<td>29.7</td>
<td>-55.3</td>
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**10^{-3} GeV^{-1/2}**

$Q^2$ dependence
N(1520) $3/2^-$ transition amplitudes

M. Aiello et al.,
Phys. Lett. B387,
215 (1996)
E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)
N(1440) $\frac{1}{2}^+$ (Roper) transition amplitudes

E. Santopinto, M.G.
$\Delta(1232)\, 3/2^+$ transition amplitudes

There is missing strength at low $Q^2$

The reason is attributed to the lack of
Quark-antiquark pair mechanisms
not present in a three-quark model

E. Santopinto, Ph. D. Thesis (Genova 1996).
solid: MAID
dotted: dynamical model
dashed: hCQM predictions

Relativity

Various levels

- Lorentz boosts
- Relativistic dynamics

- quark-antiquark pair effects (meson cloud)
- [relativistic equations (BS, DS)]
Relativistic corrections to form factors

• Lorentz boosts applied to the initial and final state
• Expansion of current matrix elements up to first order in quark momentum

• Results

\[ A_{\text{rel}} (Q^2) = F \ A_{\text{n.rel}} (Q^2_{\text{eff}}) \]

\[ F = \text{kin factor} \quad Q^2_{\text{eff}} = Q^2 \left( \frac{M_N}{E_N} \right)^2 \]
My De Sanctis et al.: A relativistic study of the nucleon helicity amplitudes

Fig. 1. Comparison between the experimental data for the helicity amplitudes $A_{1/2}$, $A_{3/2}$ for the $D_{13}$ resonance and the calculations with the relativistic corrections (full curve). The data are from the compilation of [3]. In the figure we report also the non-relativistic calculations in the EVF (dashed curve) and in the Breit frame (dotted curve).

The coefficients $S$ are the generalization for the inelastic transitions of the corresponding quantities introduced for the elastic case [tx].

Within our approximation, we note that the relativistic corrections introduce two kinds of modifications with respect to the non-relativistic treatment: a multiplicative factor coming from the expansion of the quark spinors and the substitution of the momentum transfer $q$ with the effective momentum $q_{\text{eff}}$ in the non-relativistic matrix elements [uli].

The matrix elements $A_{s}$, $S_{s}$, $I_{s}$ can be calculated using as input the wave functions obtained in a non-relativistic quark model. We present the results for the three-body force hypercentral potential [tu] introduced in Sec3.s which has been already used for the description of the spectrum [tusx], the photocouplings [S+sx] and the elastic form factors with relativistic corrections [tx].

We perform the calculations for the negative parity resonances, choosing those for which there are some experimental data available, namely the $D_{13}$, $S_{11}$, $D_{33}$ states. The results are given in Figs3.ts Ss ls = s 4 and x3. We report the relativistic form factors of $A_{x}$ in the EVF compared with the results without relativistic correction in the same frame. For comparison, we give also the non-relativistic transition form factors in the Breit system [u].

We can observe from the various figures that the relativistic corrections modify slightly the high $Q^2$ behaviours which remain in agreement with data. On the contrary, the relativistic corrections give a significant contribution at low $Q^2$ as already observed by [4]. It is interesting to observe that even if one takes into account the relativistic kinematics, there still remains a strong discrepancy with the experimental data at low $Q^2$. This fact is in our opinion an indication that the present description of the $e^{+}e^{-}$ excitation of baryons has some deficiency. The problem is not that of finding a better quark wave function as proved by the similar results obtained with different constituent quark models [ulisst = su].

Actually, some fundamental mechanism is lacking, for instance the production of $q\overline{q}$ pairs and/or sea-quark effects.

In the figures, the non-relativistic calculations in the Breit frame are not drastically different from the non-relativistic ones in the EVF. For the electromagnetic excitations, one can calculate the transition radius. With the constituent quark model.
In the case of the **helicity amplitudes** the application of Lorentz boosts does not alter the results

**BUT**

for the **elastic form factors** the situation is different
With Lorentz boosts:

improvement of the elastic f.f.
depletion of the ratio $G_E^P/G_M^P$
A fully relativistic treatment is necessary

But

The relativistic effects are expected to be more important for the elastic form factors

(the ground state)
Calculated values

Point Form

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current
With quark form factors

Δ(1232)

Structure similar to the nucleon

Spin-isospin splitting of the ground state

Relativistic effects important also for the excitation?
Dash non relativistic
Full point form

Relativity is an important issue for the description of elastic and inelastic form factors but it is not the only important issue.

The medium $Q^2$ behaviour is fairly well reproduced ($1/x$ potential). There is lack of strength at low $Q^2$ (outer region) in the e.m. transitions specially for the A $3/2$ amplitudes.

3-quark core (about 0.5 fm) + quark-antiquark pairs (Meson cloud)

0.5 fm is the value predicted by hCQM

How to introduce it?
Two main approaches

- **the physical nucleon** $N$ is made of a bare nucleon dressed by a surrounding meson cloud

\[ |\tilde{N}\rangle = \Psi^N_{(3q)} |N(qqq)\rangle + \sum_{B,M} \Psi^{(BM)}_{(3q)(q\bar{q})} |B(qqq)M(q\bar{q})\rangle + \cdots \]

Problems of inconsistency

- **Introducing higher Fock components**

\[ |\Psi\rangle = \Psi_{3q} |qqq\rangle + \Psi_{3q\bar{q}} |3q\bar{q}\rangle \]

Consistency ok

But: how many components?

**Necessity of unquenching the quark model**
baryons

Unquenching the quark model

The $qq$-pair creation mechanism is introduced at the microscopical level → string-like $qq$ pair creation mechanism

(a) [Diagram]
(b) [Diagram]

Pair creation vertex

Σ over all the big towers of intermediate states

Construction of the formalism (group theory)
Problems that have been solved
sum over all intermediate states
permutational symmetry for all identical quarks
determination of the pair creation vertex

High $Q^2$ behaviour
High $Q^2$ behaviour

- Helicity ratio

$$Z = \frac{|A_{1/2}|^2 - |A_{3/2}|^2}{|A_{1/2}|^2 + |A_{3/2}|^2}$$

goes to 1 for increasing $Q^2$

(helicity conservation, Carlson 1986)
<table>
<thead>
<tr>
<th></th>
<th>proton</th>
<th>neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>P33</td>
<td>$\approx -0.5$</td>
<td></td>
</tr>
<tr>
<td>D13</td>
<td>ok</td>
<td>ok</td>
</tr>
<tr>
<td>F15</td>
<td>ok</td>
<td>$\approx 0.7$</td>
</tr>
<tr>
<td>D13*</td>
<td>ok</td>
<td>0.96</td>
</tr>
<tr>
<td>D33</td>
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</tr>
<tr>
<td>D15</td>
<td>$1/3$</td>
<td>$\approx 0.32$</td>
</tr>
<tr>
<td>F35</td>
<td>-0.82</td>
<td></td>
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<tr>
<td>F37</td>
<td>-0.32</td>
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</tr>
<tr>
<td>P13</td>
<td>ok</td>
<td>ok</td>
</tr>
</tbody>
</table>
Asymptotic behaviour of $\Delta$ excitation

$$A_{1/2} \approx G_M - 3 G_E$$
$$A_{3/2} \approx 3^{1/2} (G_M + G_E)$$

$$Z \rightarrow 1 \quad \text{if} \quad G_E \rightarrow -G_M$$

Simplified h.o. model for $N$ and $\Delta$ states

$$|N> = a_s |0^+_S > + a_m |0^+_M >$$
$$|\Delta> = b_s |0^+_S > + b_d |2^+_M >$$

D-wave

$$Z = 1 \quad \text{if} \quad b_d \approx 98\% !$$

Not possible in models with three quarks higher $L$ components?
Conclusions

• hCQM provides a simple and systematic approach to baryon properties
  (spectrum, helicity amplitudes, elastic ff)

• the hCQM structure of levels allows to describe all the new negative parity resonances without invoking higher shells

• relativity is important for the elastic ff and the Δ-excitation

• The missing strength at low Q^2 is due to the lack of quark-antiquark pair mechanisms

• Such mechanisms may be important also for the high Q^2 behaviour of elastic ff and resonance excitation, but also for the spectrum and the strong decays