THE DARK SIDE OF THE COSMOLOGICAL CONSTANT

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Outline

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Newton's apple

- Newtonian gravity $\Rightarrow$ field equation for the gravitational potential:

$$\nabla^2\Phi = 4\pi G\rho$$

- The gravitational force between two masses, is given by:

$$\vec{F} = -G\frac{mM}{r^2}\hat{e}_r$$

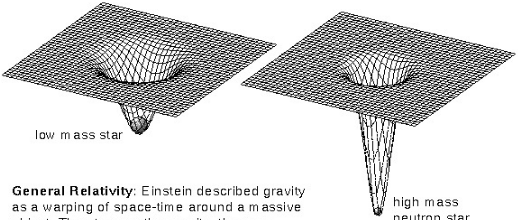
- This model (although useful), shows many problems:
  - Instantaneous action at distance $\rightarrow$ Newton: *"Hypotheses non fingo"* (I feign no hypotheses)
  - Fail to explain the perihelion precession of Mercury's orbit

Gravity is Geometry

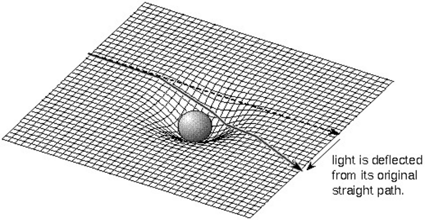
- (1916) Einstein published the General Relativity led by the following arguments:
  - Generalize Newtonian Gravity
  - No preferred coordinate system
  - Local conservation of energy-momentum for any space-time

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

- Geometrized units: $c = G = 1$.
- $1 = \frac{G}{c^2} = 7.425 \times 10^{-28} mkg^{-1}$. Mass measured in meters!.
- $T_{\mu\nu}$ =: energy momentum tensor
- $R_{\mu\nu}$: Ricci tensor
- $g_{\mu\nu}$: metric tensor
- Wheeler: *"Space acts on matter telling it how to move. In turn, matter reacts back on space, telling it how to curve"*



General Relativity: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

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Einstein's Greatest Blunder

- Einstein equations predicts a **dynamical universe**
- The cosmological observations around (1917), showed a very low relative velocity of the stars
- A man with no faith. Einstein introduces Λ:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Far away, Einstein demanded:

$$\Lambda = 8\pi G\rho = a^{-2}$$

- This is absurdly ad-hoc and Bad Physics too because: $\rho \sim a^{-3}$!!

- (1917) W. de Sitter finds a solution to the Einstein equations with $\Lambda \neq 0$ and $T_{\mu\nu} = 0$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$ds^2 = -\left(1 - \frac{\Lambda r^2}{3}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{\Lambda r^2}{3}\right)} + r^2 d\Omega^2$$

- Dynamics of the Universe dominated by Λ
- (1924) Friedmann finds the evolutive homogeneous solution (Death of Λ?)
- (1927) Lemaitre finds a solution which describes an expanding universe $\Rightarrow$ Big Bang!

- (1929) Hubble observations $\Rightarrow$ Expanding Universe!
- Einstein: *"If there is no quasi-static world, then away with the cosmological constant"*
- After open, Pandora's box is not easily closed again. Λ is a legitimate additon to the Field Equations
- Eddington keeps Λ. May solve the problem of the age of the Universe:

$$t_{uni} \sim 10^{12} s < \quad \text{Age of the Earth}$$

- The Hubble parameter is checked: age problem solved $\Rightarrow$ Λ is unnecessary
- (1967) Λ reborns. It may explain the strong redshift of some quasars ($z \approx 2$)

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The Friedmann-Lemaitre-Robertson-Walker metric

- In a good approximation, the universe at the large-scale ($10^{13} Mpc$), can be described by the Robertson-Walker metric:

$$ds^2 = -dt^2 + R(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

- k: parameter which defines the space-time curvature
- $R(t)$: scale factor which equals 1 at t_0
- $r^2 d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ metric on a two-sphere
- Redshift:

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{R_o}{R(t)}$$

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Dynamics of a FLRW Universe

- Standard cosmology $\rightarrow$ Universe is modelled as an ideal fluid, determined by an energy density ρ and a pressure p:

$$T_{\mu\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- For this energy-momentum tensor, the Einstein equations gives:

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{R^2} = H^2}$$

- H is the **Hubble parameter**. This equation describes the dynamics of an expanding Universe

- The second Friedmann equation reads:

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P)}$$

- Exists a critical value for ρ such that ($k = 0$):

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

- $\ddot{R} \neq 0 \rightarrow$ General Relativity predicts an expanding universe!!. Einstein modifies his equations to keep a static universe

$$\boxed{H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3}}$$

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}}$$

The Einstein's biggest blunder?

- Obervations by Hubble pointed out an expanding universe!
- Einstein attempted to put the genie back in the bottle but he failed.
- Eddington: "Λ is a legitimate addition to the Einstein equations"

$$\nabla_\mu (G_{\mu\nu} + \Lambda g_{\mu\nu}) = 0$$

- Λ only can removed if is less than $G_{\mu\nu}$!
- Λ remains a focal point of cosmology (accelerated cosmological expansion!)
- Λ in QED is associated to the energy density of the vacuum $\rho_\Lambda \neq 0$

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Dark Energy

$$\frac{8}{3}\pi G\rho + \frac{\Lambda}{3} - \frac{k}{R^2} = H^2$$

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

- $\Omega_m = \frac{8\pi G\rho}{3H^2}$: matter (baryonic and non-barionic)
- Ω_Λ: Dark energy density
- Ω_k: effect of the space-time curvature
- CMB observations appears to point $\Omega_k \approx 0$
- According to the ΛCDM model: $\Omega_m = \Omega_b + \Omega_{darkmatter}$
- Current observations gives:

$$\Omega_b \sim 0.0227 \pm 0.0006$$

$$\Omega_\Lambda \sim 0.74 \pm 0.03$$

- The Universe is filled of unknown dark energy!! Λ is here to stay!

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Conclusions

- The cosmological constant is a completely natural term in the Einstein equations
- Current observations suggest that Λ could be very important in the cosmological scenario
- The nature of Λ, and therefore of the 70% of the universe, is still a mystery to be solved

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References

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