

# THE DARK SIDE OF THE COSMOLOGICAL CONSTANT

CAMILO POSADA AGUIRRE

University of South Carolina  
Department of Physics and Astronomy

09/23/11

# Outline

- 1 Einstein's Greatest Blunder
- 2 The FLRW Universe
- 3 A Dynamical Universe
- 4 Resurrection of  $\Lambda$
- 5 Conclusions
- 6 References

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# Einstein's Greatest Blunder

- (1916) A. Einstein published the Field Equations of the General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

- Einstein equations predicts a **dynamical universe**
- The cosmological observations around (1917), showed a very low relative velocity of the stars
- A man with no faith. Einstein introduces  $\Lambda$ :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Far away, Einstein demanded:

$$\Lambda = 8\pi G\rho = a^{-2}$$

- This is absurdly ad-hoc and Bad Physics too because:  $\rho \sim a^{-3}!!$

- (1917) W. de Sitter finds a solution to the Einstein equations with  $\Lambda \neq 0$  and  $T_{\mu\nu} = 0$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$ds^2 = - \left(1 - \frac{\Lambda r^2}{3}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{\Lambda r^2}{3}\right)} + r^2 d\Omega^2$$

- Dynamics of the Universe dominated by  $\Lambda$
- (1924) Friedmann finds the evolutive homogeneous solution (Death of  $\Lambda$ ?)
- (1927) Lemaitre finds a solution which describes an expanding universe  $\Rightarrow$  Big Bang!

- (1929) Hubble observations  $\Rightarrow$  Expanding Universe!
- Einstein: *"If there is no quasi-static world, then away with the cosmological constant"*
- After open, Pandora's box is not easily closed again.  $\Lambda$  is a legitimate addition to the Field Equations
- Eddington keeps  $\Lambda$ . May solve the problem of the age of the Universe:

$$t_{uni} \sim 10^{12} s < \text{Edad de la Tierra}$$

- The Hubble parameter is checked: age problem solved  $\Rightarrow \Lambda$  is unnecessary
- (1967)  $\Lambda$  reborns. It may explain the strong redshift of some quasars ( $z \approx 2$ )

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# The Friedmann-Lemaitre-Robertson-Walker metric

- In a good approximation, the universe at the large-scale ( $10^{13} Mpc$ ), can be described by the Robertson-Walker metric:

$$ds^2 = -dt^2 + R(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

- $k$ : parameter which defines the space-time curvature
- $R(t)$ : scale factor which equals 1 at  $t_0$
- $r^2 d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$  metric on a two-sphere
- Redshift:

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{R_o}{R(t)}$$

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# Dynamics of a FLRW Universe

- Standard cosmology  $\rightarrow$  Universe is modelled as an ideal fluid, determined by an energy density  $\rho$  and a pressure  $p$ :

$$T_{\mu\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- For this energy-momentum tensor, the Einstein equations gives:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{R^2} = H^2$$

- H is the **Hubble parameter**. This equation describes the dynamics of an expanding Universe

- The second Friedmann equation reads:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P)$$

- Exists a critical value for  $\rho$  such that ( $k = 0$ ):

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

- $\ddot{R} \neq 0 \rightarrow$  General Relativity predicts an expanding universe!!  
Einstein modifies his equations to keep a static universe

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

# The Einstein's biggest blunder?

- Observations by Hubble pointed out an expanding universe!
- Einstein attempted to put the genie back in the bottle but he failed.
- Eddington: “ $\Lambda$  is a legitimate addition to the Einstein equations”

$$\nabla_{\mu}(G_{\mu\nu} + \Lambda g_{\mu\nu}) = 0$$

- $\Lambda$  only can removed if is less than  $G_{\mu\nu}$ !
- $\Lambda$  remains a focal point of cosmology (accelerated cosmological expansion!)
- $\Lambda$  in QED is associated to the energy density of the vacuum  
 $\rho_{\Lambda} \neq 0$

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# Dark Energy

$$\frac{8}{3}\pi G\rho + \frac{\Lambda}{3} - \frac{k}{R^2} = H^2$$

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

- $\Omega_m = \frac{8\pi G\rho}{3H^2}$ : matter (baryonic and non-barionic)
- $\Omega_\Lambda$ : Dark energy density
- $\Omega_k$ : effect of the space-time curvature
- CMB observations appears to point  $\Omega_k \approx 0$
- According to the  $\Lambda$ CDM model:  $\Omega_m = \Omega_b + \Omega_{\text{darkmatter}}$
- Current observations gives:

$$\Omega_b \sim 0.0227 \pm 0.0006$$

$$\Omega_\Lambda \sim 0.74 \pm 0.03$$

- The Universe is filled of unknown dark energy!!  $\Lambda$  is here to stay!

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# Conclusions

- The cosmological constant is a completely natural term in the Einstein equations
- Current observations suggest that  $\Lambda$  is very important in the cosmological scenario
- The nature of  $\Lambda$ , and therefore of the 70% of the universe, is still a mystery

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# References

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