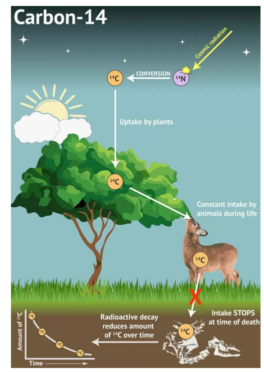
<u>511 Nuclear Physics</u> University of South Carolina **<u>Spring 25</u>**

Midterm Exam Instructor: Ralf W. Gothe **3/6/25**

Name: . . . . **Signature:** . . . .

1) Decay Rates



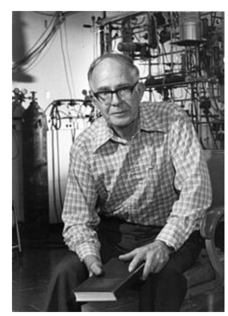
1.1) **(3)** The carbon isotope ${}^{14}_{6}C$ is produced in nuclear reactions of cosmic rays in the atmosphere. It is β-unstable.

$$ {}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e + 0.156\,MeV, $$

with a lifetime of 8270 years. It is found that $1\,g$ of carbon, newly extracted from the atmosphere, has on average 15.3 such radioactive decays per minute. What is the proportion of the ${}^{14}_{6}C$ isotope in carbon?

1.2) **(2)** You have extracted $1\,g$ of carbon from recently excavated human bones and measured 3.83 decays per minute. How old are the bones?

2) The Davis Experiment and the Solar Neutrino Problem

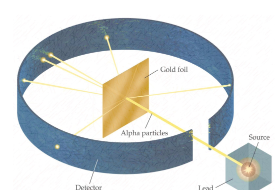


2.1) [**7 P**] Calculate the threshold energy for electron-neutrino ν_e absorption by ${}^{37}_{17}Cl$. Start by formulating the reaction. Fill in for the question marks and interpret your result! Assume the neutrino is massless as well as $m_n c^2 = 939.566\,MeV$, $m_H c^2 = 938.783\,MeV$ $a_V = 15.67\,MeV$, $a_S = 17.23\,MeV$, $a_C = 0.714\,MeV$, $a_A = 93.15\,MeV$, and

$$ \delta = \begin{cases} -11.2\,MeV & \text{for ?-? nuclei} \\ 0\,MeV & \text{for ?-? nuclei} \\ +11.2\,MeV & \text{for ?-? nuclei} \end{cases} $$

2.2) [**3 P**] Do recoil corrections influence the significance of your results? Show why or why not!

3) Cross Section



3.1) [**3 P**] Calculate the number of target nuclei per cm^2 for a $2.5\,\mu m$ thin gold (${}^{197}Au$) target! The density of gold is $\rho_{Au} = 19.3\,\frac{g}{cm^3}$.

3.2) [**2 P**] Calculate the number of beam particles per s for an α (${}^{4}He^{++}$) beam of $1\,nA$!

3.3) [**2 P**] Calculate from the luminosity and a given event rate $\dot{N} = 471963\,Hz$ the total cross section in $barn$ for $\alpha({}^{197}Au, {}^{197}Au)\alpha$!

4) **General Relativistic Kinematics**

4.1) **[2 P]** Derive the fully relativistic function $E(p, m_0)$ from $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $m = \gamma m_0$, and $E = mc^2$. Distinguish clearly between rest mass m_0 and total mass m.

4.2) **[2 P]** How are $\beta = \frac{v}{c}$ and $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ defined by the rest mass m_0, the momentum p, and the total energy E for a given particle?

4.3) **[2 P]** What are Lorentz scalars and why are they of particular interest? Calculate explicitely $P_\mu P^\mu$ of a particle with the rest mass m_0 and interpret the result.

4.4) **[+3 P]** How is the invariant mass of a two body system defined? How is it related to the sum of the energies in the center of mass frame. When is the invariant mass the sum of the two rest masses?

4.5) **[+2 P]** What happens if a particle with no rest mass moves through a central gravitational field? Explain why!

5) **Power Plants**

5.1) **[2 P]** The combustion of methane

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O,$$

releases $9\,eV$ per methane molecule. Estimate the relative energy release per unit mass for nuclear to chemical fuel! The fission of one ^{235}U atom releases $205\,MeV$.

5.2) **[3 P]** Show that a nuclear power plant, producing $1\,GW$ of heat, consumes $1\,kg$ of ^{235}U per day!

5.3) **[1 P]** Show that a power station burning natural gas, producing $1\,GW$ of heat, will discharge $4.38\,Gg$ of the greenhouse gas carbon dioxide per day into the atmosphere!

5.4) **[2 P]** How much anti-matter per day would Scotty need to produce $1\,GW$ of continuous heat.

6) **Probabilities**

6.1) **[2 P]** The probability of measuring no electron in a given time interval from a ^{90}Sr source is $P_0 = 0.05$. Calculate the mean value and the probability of measuring five electrons?

6.2) **[2 P]** Calculate the mean value of measured electrons when P_0 is equal to P_6.