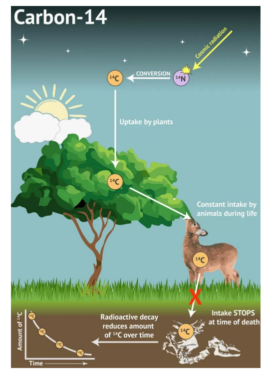
511 Nuclear Physics University of South Carolina **Spring 24**

Midterm Exam Instructor: Ralf W. Gothe **2/29/24**

Name: . . . . **Signature:** . . . .

1) **Decay Rates**



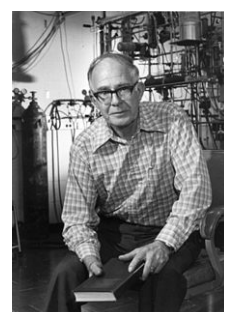
1.1) **(3)** The carbon isotope ${}^{14}_{6}C$ is produced in nuclear reactions of cosmic rays in the atmosphere. It is β-unstable.

$$ {}^{14}_{6}C \rightarrow {}^{14}_{7} N + e^- + \bar{\nu}_e + 0.156\, MeV, $$

with a lifetime of 8270 years. It is found that $1\, g$ of carbon, newly extracted from the atmosphere, has on average 15.3 such radioactive decays per minute. What is the proportion of the ${}^{14}_{6}C$ isotope in carbon?

1.2) **(2)** You have extracted $1\, g$ of carbon from recently excavated human bones and measured 3.83 decays per minute. How old are the bones?

2) **The Davis Experiment and the Solar Neutrino Problem**

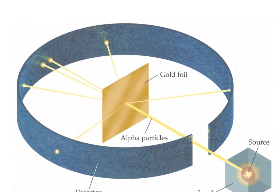


2.1) [**7 P**] Calculate the threshold energy for electron-neutrino ν_e absorption by ${}^{37}_{17}Cl$. Start by formulating the reaction. Fill in for the question marks and interpret your result! Assume the neutrino is massless as well as $m_n c^2 = 939.566\, MeV$, $m_H c^2 = 938.783\, MeV$ $a_V = 15.67\, MeV$, $a_S = 17.23\, MeV$, $a_C = 0.714\, MeV$, $a_A = 93.15\, MeV$, and

$$ \delta = \begin{cases} -11.2\, MeV & \text{for ?-? nuclei} \\ 0\, MeV & \text{for ?-? nuclei} \\ +11.2\, MeV & \text{for ?-? nuclei} \end{cases} $$

2.2) [**3 P**] Do recoil corrections influence the significance of your results? Show why or why not!

3) **Cross Section**



3.1) [**3 P**] Calculate the number of target nuclei per cm^2 for a $0.25\, \mu m$ thin gold (${}^{197}Au$) target! The density of gold is $\rho_{Au} = 19.3\, \frac{g}{cm^3}$.

3.2) [**2 P**] Calculate the number of beam particles per s for an α (${}^4He^{++}$) beam of $1\, nA$!

3.3) [**2 P**] Calculate from the luminosity and a given event rate $\dot{N} = 471963\, Hz$ the total cross section in *barn* for $\alpha({}^{197}Au, {}^{197}Au)\alpha$!

4) General Relativistic Kinematics

4.1) **[2 P]** Derive the fully relativistic function $E(p, m_0)$ from $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $m = \gamma m_0$, and $E = mc^2$. Distinguish clearly between rest mass m_0 and total mass m.

4.2) **[2 P]** How are $\beta = \frac{v}{c}$ and $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ defined by the rest mass m_0, the momentum p and the total energy E for a given particle?

4.3) **[3 P]** How is the invariant mass of a two body system defined? How is it related to m^*_Σ, the total relativistic mass in the CMS? When is the invariant mass the sum of the two rest masses?

4.4) **[+2 P]** What happens if a particle with no rest mass moves through a central gravitational field? Explain why!

5) Electron Scattering Kinematics

5.1) **[1 P]** Sketch the t channel Feynman diagram of elastic electron scattering off a nucleus and name your variables.

5.2) **[6 P]** Use the corresponding Mandelstam variable t to derive the dependence of the scattered electron energy k_{20} on the electron scattering angle ϑ_{12}, the beam energy k_{10}, and the mass of the nucleus M.

5.3) **[3 P]** Calculate the recoil energy and the mass square of the virtual photon t at maximum energy transfer for $k_{10} = 5.28\,GeV$ and two different target nuclei, $^{100}_{44}Ru$ and $^{1}_{1}H$, respectively! Assume $m_u c^2 = 931.5\,MeV$.

6) Probabilities

6.1) **[2 P]** The probability of measuring no electron from a ^{90}Sr source is $P_0 = 0.05$. Calculate the mean value and the probability of measuring five electrons?

6.2) **[2 P]** Calculate the mean value of measured electrons when P_0 is equal to P_6.