PHYSICS 713

ADVANCED QUANTUM THEORY I

FALL 2012

Meeting time: Tuesday and Thursday 9:30 am - 10:45 am

Place: SUM 333

Office hours: Mondays 1:00 - 3:00 pm,
and on Tuesdays and Thursdays after class

Instructor: Professor Pawel O. Mazur

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The purpose of the course is to introduce to graduate students of physics the basic results in Relativistic Quantum Mechanics and in Quantum Field Theory (QFT). Relativistic wave equations for massive particles of spin-zero (the Klein-Gordon equation) and of spin-$\frac{1}{2}$ (the Dirac equation), and for a massless particle of spin-1 (a photon) will be discussed in detail. The unifying concept of a quantum field operator will be introduced in two different ways. First the concept of a quantum field treated as a book-keeping device in the framework of the method of second quantization will be introduced in the contexts of non-relativistic quantum mechanics of a fixed number of particles, and relativistic quantum mechanics of an arbitrary number of particles. The second way by which the concept of a quantum field will be introduced to graduate students is by considering canonical quantization of 'classical' fields. Interacting quantum fields and the formula for the S-matrix will be introduced. Graduate students will learn how to compute the simplest matrix elements of the scattering operator S on the example of Quantum Electrodynamics. They will be taught the modern methods of computations in QFT including the Feynman diagram techniques and the methods of regularization of divergent loop integrals.

The prerequisites to this course are PHYS 701, 703, 704, 711 and 712

Texts:

Quantum Field Theory, by C. Itzykson and J.-B. Zuber
Dover Publications, 2005, ISBN - 13: 978-0-486-44568-7;
ISBN - 10: 0-486-44568-2

Quantum Field Theory : A Modern Perspective,
by V. Parameswaran Nair
Springer: Graduate Texts in Contemporary Physics, 2005, ISBN 0-387-21386-4

I suggest that students taking PHYS 713 do acquaint themselves with the contents of several selected chapters of Professor Nair's book as an additional reading assigment: Chapters 1-7, pp. 1-101

Instructional delivery strategy: In this course I will lecture on the material which can be found in the selected chapters of books which are listed above. I did all the necessary derivations of formulae and computations and I have included them in my lecture notes. The material which is covered in my lectures is not so easy therefore I will be performing all the necessary computations on a blackboard, showing you at the same time how to do computations in QFT effectively. Some parts of these computations will be left for you to complete as a homework assignment. 80 percent of the material covered in this course will be in the form of lectures and 20 percent in the reading assignments.

Reading assignments: The relevant chapters of the books listed above should be read prior to and after the class. The reading assignments for the next class will be given at the end of the earlier class. The text of the book by V. Parameswaran Nair is supplementary to the book by Itzykson and Zuber.

Homeworks: There will be no tests given during the class. Up to 8 'long' homeworks will be assigned during the course. This amounts to an approximately one homework per two weeks, but if the number of 'long' homeworks is smaller than 8, then the average time for a 'long' homework could be longer. Solving homework problems will involve sometimes lengthy computations and this is why you will have two or more weeks to complete those computations. You are welcome to visit my office, preferably during my regular office hours on Mondays, 1:00 - 3:00 pm, if you will need help with the course material and with your homeworks. I am also available on Tuesdays and Thursdays after the morning PHYS 713 class. Your homework grade will count toward 80 percent of your course score.

Final exam: The final exam is comprehensive and will cover all material covered in class. The final exam counts toward 20 percent of your course score. All students must take the final exam. The exam will last 2.5 hours.

Grading: Each homework carries a weight of 20 points. The final exam counts for 40 points. The maximal number of points is therefore 200 if there are 8 'long' homeworks assigned. The translation of the course score to letter grade will occur only when the complete scores have been calculated after the final exam. The ranges of grade of A, B, C, D, and F are traditional for this course. A (score ≥ 170, or ≥ 85 percent), B (score ≥ 150, or ≥ 75 percent), C (score ≥ 130, or ≥ 65 percent), D (score ≥ 110, or ≥ 55 percent), F (score ≤ 100, or ≤ 50 percent). If the number of homeworks is smaller than 8 the rule for computing the final grade is 20 percent for the final exam and each 'long' homework counts for 80 percent divided by the number of 'long' homeworks.

PHYS 713 TIME TABLE

Week 1. Relativistic Wave Equations: the case of a massive spin-zero particle. The Klein-Gordon equation for a scalar field.

Week 2. Relativistic Wave Equations: the case of a massive spin-$\frac{1}{2}$ particle. The Dirac equation for a bi-spinor field. Relativistic transformation properties and the spin.

Week 3. The Dirac equation and the Clifford algebra generated by Dirac's γ-matrices. Charge conjugation. Antiparticles. The basis of solutions for particle and antiparticle wave functions. Preparing the ground for the introduction of a quantum field operator for fermions.

Week 4. Relativistic Wave Equations: the case of a massless spin-one particle, a photon. E. Majorana's re-interpretation of Maxwell's vacuum equations as the Schrödinger equation for a single photon. Relativistic transformation properties and the spin.

Week 5. The action principle and the canonical formalism for 'classical' fields. The cases of scalar, bi-spinor, and vector fields. Preparing the ground for canonical quantization formalism.

Week 6. Quantum field operator as a book-keeping device. The construction of quantum fields from wave functions satisfying relativistic wave equations and from the Fock algebra of creation and annihilation operators.

The separate cases of bosons (spin-zero and spin-one particles) and fermions (spin-$\frac{1}{2}$). The spin-statistics connection for free fields finding its expression in the choices of the commutation and the anti-commutation relations for the generators of the Fock algebra for the integer and the half-integer spins, respectively.

Week 7. The method of second quantization applied to non-relativistic quantum mechanics (NRQM) of many particles. Construction of the quantum field. The Quantum Many-Body Problem (NRQM) on the examples of bosons and fermions. A rather brief presentation of these two particular applications of the method of second quantization in NRQM of many particles follows the book by E. M. Lifshitz and L. Pitaevski (Statistical Physics Part 2, The Landau Course in Theoretical Physics). This topic will also be a subject of a reading/essay assignment.

Week 8. The construction of Quantum Field Operator by canonical quantization of 'classical' fields. The method of second quantization applied to the cases of bosons (spin-zero, spin-one) and fermions (spin-$\frac{1}{2}$) via canonical quantization for which Relativistic Wave Equations were introduced in the first several weeks of this course.

No classes on Thursday, October 18th. The Fall Break.

Week 9. Interactions and the S-matrix. A general formula for the S-matrix. Wick's theorem. The chronological product and the Schwinger-Dyson series.

Week 10. The Electromagnetic Field. Quantization and photons. Interaction with charged particles. Quantum Electrodynamics (QED).

Week 11. Commutators, Propagators (Green Functions). Scalar field propagator. Propagator for Dirac fermions. Propagator for photons and the issue of gauge-fixing. The Feynman diagram rules for Quantum Electrodynamics (QED) of charged fermions.

No classes on Tuesday, November 6th. The General Election Day.

Week 12. Propagators, Vertices Continued. Examples of Scattering Processes. S-matrix elements. Decay rates and cross-sections.

Weeks 13-15. Elementary processes in QED. Vacuum polarization and charge renormalization. Electron's self-energy and mass renormalization. The anomalous magnetic moment of an electron. Methods of regularizations of divergent single loop integrals.

No classes on Thursday, November 22nd. Thanksgiving recess.

The **final exam** will take place on **Monday, December 10, 2012, 9:00-11:30 am.** The exam problems will be selected from the material covered in my lectures. The exam will last 150 minutes. Therefore, relatively easy three problems will be selected so you may expect to solve each of them during the allotted 50 minutes per problem if you have mastered the material covered during the classes (see **Attendance policy**).

Attendance policy: Attendance highly suggested. No more than three classes (at most 10 percent of classes) may be missed. The material covered during the classes which were missed must be mastered by a student.

Office of Student Disability Services policy statement: Any student with a documented disability should contact the Office of Student Disability Services at 803-777-6142 to make arrangements for appropriate accommodations.