PHY 721 - Problem Set 7

- 1. SU(N) is a continuous group of all N×N matrices which are unitary and have determinant +1. If we define an element of SU(N) as $U(\vec{\theta})$ where $\vec{\theta}$ is the n-component vector of continuously variable parameters, we may also define the generators of the group by

$$G_i \equiv i \lim_{\delta\theta_i \to 0} \left(\frac{U(0, \ldots, \delta\theta_i, \ldots, 0) - I}{\delta\theta_i} \right) \equiv i \frac{\partial U}{\partial \theta_i}\bigg|_{\vec{\theta}=0}$$

where the identity element I is the element for which $\vec{\theta} = 0$. Thus, $U(\vec{\theta}) = \exp(-i\vec{\theta}\cdot\vec{G})$.

1. How many parameters n does SU(N) have?
2. Denote the elements of the unitary matrices by U_{ij}. Elements of the matrices infinitesimally different from the identity matrix may be written as

$$U_{ij} = \delta_{ij} + du_{ij}$$

From the properties of unitarity and +1 determinant derive conditions on the du_{ij}. Show that the matrices composed of the du_{ij} must be traceless and anti-Hermitian. Thus show that the generators of SU(N) are Hermitian and traceless.