7.39) We are given $u(\nu)\,d\nu = \dfrac{8\pi h}{c^3} \dfrac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$

From this $u(\nu)\,d\nu = \dfrac{8\pi \nu^2}{c^3} \cdot \dfrac{h\nu \cdot d\nu}{e^{h\nu/kT} - 1}$

we change variables : $\nu \to \lambda$ where $c = \lambda\nu$.
Since $u(\lambda)\,d\lambda = u(\nu)\,d\nu$ we have

$$u(\lambda) = u(\nu)\frac{d\nu}{d\lambda} = \frac{8\pi h}{\lambda^3} \cdot \frac{c\,d\lambda/\lambda^2}{e^{hc/kT\lambda} - 1} \qquad -(1)$$

(where we dropped the minus sign because the integration limits are reversed as well).

Let $b = \dfrac{hc}{kT}$ then $u(\lambda) = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{e^{b/\lambda} - 1}$.

The peak occurs at λ_{max} where $\left.\dfrac{du}{d\lambda}\right|_{\lambda=\lambda_{max}} = 0$ i.e.,

$$-\frac{5}{\lambda}u(\lambda) - \frac{u(\lambda)\,e^{b/\lambda}(-b/\lambda^2)}{(e^{b/\lambda} - 1)} = 0$$

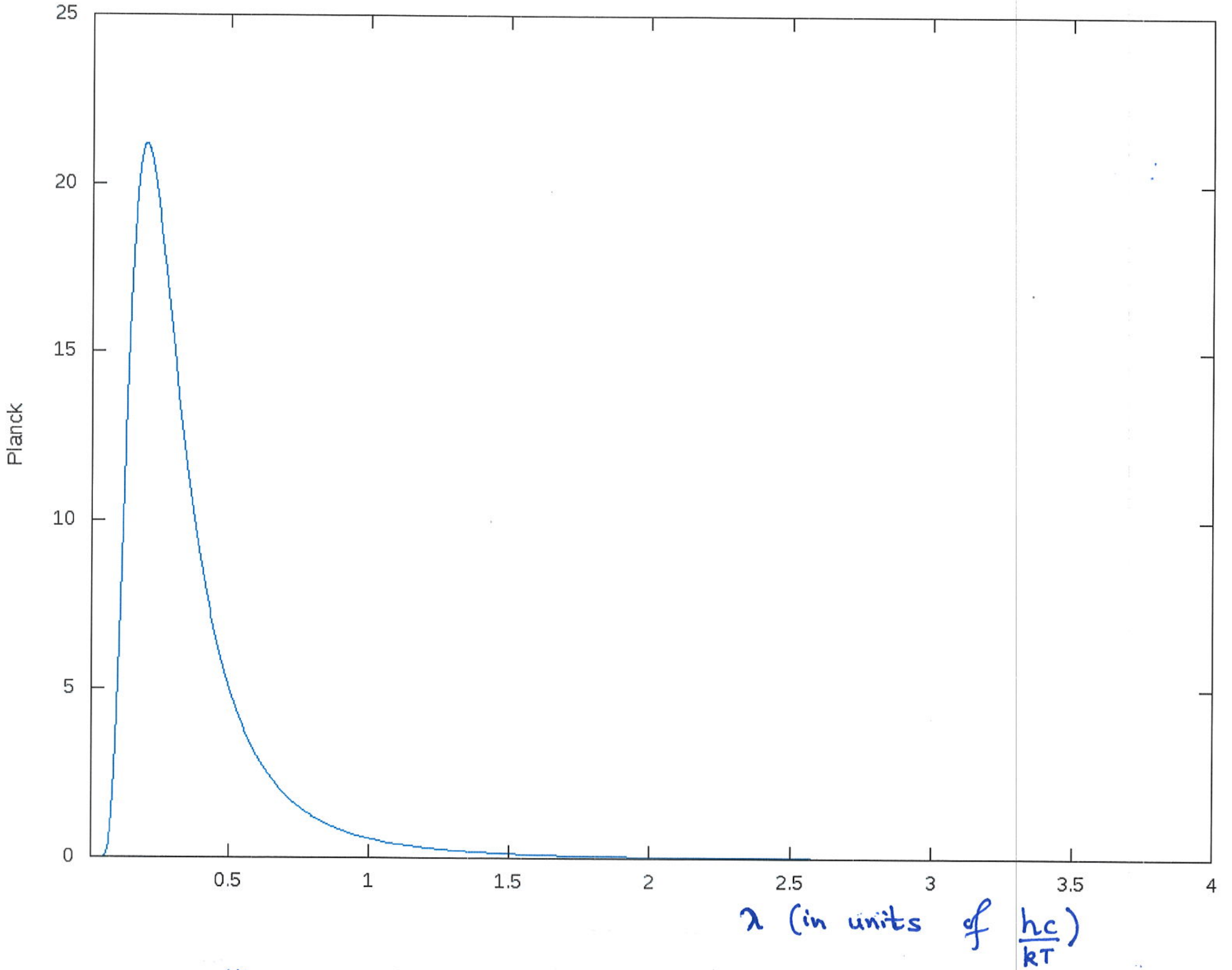
Thus, $\dfrac{5}{\lambda}(e^{b/\lambda} - 1) = \dfrac{b e^{b/\lambda}}{\lambda^2}$. Let $\dfrac{b}{\lambda} = x$.

$$e^x - 1 = \frac{x}{5}e^x \quad , \quad \left(1 - \frac{x}{5}\right)e^x = 1$$

Solved for $x = 0$ and $x = 4.965111$

Thus, the peak occurs when $\lambda_{max} = \dfrac{b}{x} = \dfrac{hc}{4.965111\,kT}$ (2)

This peak is not where $u(\nu)$ peaks because they are different densities; one does <u>not</u> obtain $u(\lambda)$ from $u(\nu)$ by simply substituting $\frac{c}{\nu}$ for λ. Note that (2) may be written as $\lambda_{max}T = 0.29$ mm-K.



25
20
15
10
5
0
Planck
0.5
1
1.5
2
2.5
3
3.5
4
λ (in units of hc/kT)