Problem 4.18) As stated in the text, the power and compression steps are adiabatic ($Q=0$) while the ignition and exhaust steps occur at constant volume ($W=0$). Thus, the efficiency

$$\epsilon = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h}.$$

$$Q_h\ (2\to3) = \Delta U_{2\to3} = \frac{f}{2} nR\,(T_3 - T_2) = \frac{f}{2} V_2 (P_3 - P_2)$$
$$Q_c\ (4\to1) = -\Delta U_{4\to1} = \frac{f}{2} nR\,(T_4 - T_1) = \frac{f}{2} V_1 (P_4 - P_1).$$

For the adiabats $P_3 V_3^\gamma = P_4 V_4^\gamma$.
$P_1 V_1^\gamma = P_2 V_2^\gamma$

But $V_2 = V_3$ and $V_1 = V_4$.

Thus, $\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^\gamma = \left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}$, and so $P_3 P_1 = P_2 P_4$, thus $\frac{P_3}{P_2} = \frac{P_4}{P_1}$.

Thus, $$\epsilon = \frac{Q_h - Q_c}{Q_c} = \frac{V_2 (P_3 - P_2) - V_1 (P_4 - P_1)}{V_2 (P_3 - P_2)}$$

$$= 1 - \frac{V_1}{V_2}\frac{P_4\,(1 - P_1/P_4)}{P_3\,(1 - P_2/P_3)} = 1 - \frac{V_1 P_4}{V_2 P_3} = 1 - \frac{V_1}{V_2}\cdot\left(\frac{V_2}{V_1}\right)^\gamma$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}.$$