Test 3 Solutions

#1) Before transfer: $E = -\frac{k}{2a} = -\frac{k}{2r_1} = \frac{\mu v^2}{2} - \frac{k}{r_1} \Rightarrow \frac{\mu v^2}{2} = \frac{k}{2r_1}$

$\Rightarrow \quad v = \sqrt{k/\mu r_1}$.

After transfer $E = -\frac{k}{2a} = -\frac{k}{(r_1+r_2)} = \frac{\mu v_{t1}^2}{2} - \frac{k}{r_1} \Rightarrow v_{t1} = \sqrt{\frac{2kr_2}{\mu (r_1)(r_1+r_2)}}$

#2) Consider a rotating coordinate system at the center of the earth whose axes are parallel to the local axes as follows: x: East, y: North, z: up.

Then $\vec{\omega} = \omega(\cos\lambda\,\hat{y} + \sin\lambda\,\hat{z})$, $\vec{r} \approx R\hat{z}$, $\vec{v} \approx -gt\hat{z}$

Without corrections the motion is described by $z = (h - g\frac{t^2}{2})$ and $x = y = 0$. The small accelerations are

$\vec{a}_{cen.} = -\vec{\omega}\times(\vec{\omega}\times\vec{r}) = -\omega^2\hat{\omega}\times(R\cos\lambda\,\hat{x}) = \omega^2 R(\cos\lambda\,\hat{z} - \sin\lambda\,\hat{y})$, and

$\vec{a}_{cor.} = -2\vec{\omega}\times\vec{v} \cong +2\omega\cos(\lambda)gt\,\hat{x}$

Neglecting the z-component of $\vec{a}_{cen.}$ we get for the deflection two significant contributions:

Centrifugal $\quad \vec{d}_{cen} \approx -\omega^2 R\sin\lambda\,\frac{T^2}{2}\hat{y} = \frac{\omega^2 R h}{g}\sin\lambda$ (towards south)

Coriolis $\quad \vec{d}_{cor.} \approx 2\omega\cos\lambda\, g\frac{T^3}{6}\hat{x} = \frac{\omega\cos\lambda}{3}\left(\frac{8h^3}{g}\right)^{1/2}$ (towards east)

Which of these is larger in magnitude depends on h and λ: the Coriolis effect dominates if $\cos\lambda\sqrt{h} \gg \frac{\omega R\sin\lambda}{\sqrt{g}}$

i.e., if $\sqrt{gh} \gg \omega R\tan\lambda$.

#3) (a) $I_{ij} = \sum_\alpha m_\alpha (r_\alpha^2 \delta_{ij} - r_{\alpha,i} r_{\alpha,j})$.

For the O-atom all coordinate components are zero.

Thus, we need only include the H-atoms:

$$I_{ij} = m\{(1\, \delta_{ij} - (1,0,0)(1,0,0)) + (1\,\delta_{ij} - (-0.25,0.97,0)(-0.25,0.97,0))\}$$

$$= m \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{Å}^2 + m \begin{bmatrix} 15/16 & 0.24 & 0 \\ 0.24 & 1/16 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{Å}^2$$

$$= m \begin{bmatrix} 0.94 & 0.24 & 0. \\ 0.24 & 1.06 & 0. \\ 0. & 0. & 2 \end{bmatrix} \text{Å}^2$$

(b) $\vec{R}_{CM} = \sum_\alpha \frac{m_\alpha \vec{r}_\alpha}{M} = \frac{(1,0,0)}{18} + \frac{(-0.25, 0.97, 0.)}{18} = \frac{(0.75, 0.97, 0.)}{18}$

$$= (0.04, 0.055, 0.)\ \text{Å} .$$

Thus, $J_{ij} = I_{ij} - 18m(R^2\delta_{ij} - R_i R_j)$

Notice that the O-atom is so heavy that up to the second decimal place in Å^2 the correction terms $\sim R^2$ are all zero, and $J_{ij} \approx I_{ij}$.