8-25) At the perigee and apogee $\dot{r} = 0$ and

$$T = \frac{m r^2 \dot{\theta}^2}{2} = \frac{m v^2}{2}.$$

$$U = -\frac{GMm}{r}.$$

Conservation of E & ℓ at $r = r_p$ and $r = r_a$ give

$$m v_p r_p = m v_a r_a \Rightarrow v_a = v_p \cdot \left(\frac{r_p}{r_a}\right).$$

$$\frac{m v_p^2}{2} - \frac{GMm}{r_p} = \frac{m v_a^2}{2} - \frac{GMm}{r_a} = \frac{m}{2} v_p^2 \left(\frac{r_p^2}{r_a^2}\right) - \frac{GMm}{r_a}.$$

$$\Leftrightarrow v_p^2 - \frac{2GM}{r_p} = v_p^2 \frac{r_p^2}{r_a^2} - \frac{2GM}{r_a}$$

Thus, $v_p^2 \left\{ 1 - \frac{r_p^2}{r_a^2} \right\} = 2GM \left\{ \frac{1}{r_p} - \frac{1}{r_a} \right\} = 2GM \frac{(r_a - r_p)}{r_a r_p}$

$$\Leftrightarrow v_p^2 \frac{(r_a^2 - r_p^2)}{r_a^2} = 2GM \frac{(r_a - r_p)}{r_a r_p}$$

$$\Leftrightarrow v_p^2 \frac{(r_a + r_p)}{r_a} = \frac{2GM}{r_p}$$

$$v_p^2 \left(1 + \frac{r_p}{r_a}\right) = \frac{2GM}{r_p} \Leftrightarrow 1 + \frac{r_p}{r_a} = \frac{2GM}{v_p^2 r_p}$$

$$\Leftrightarrow \frac{r_p}{r_a} = \frac{2GM}{v_p^2 r_p} - 1 \Rightarrow r_a = \frac{r_p}{\left\{ \frac{2GM}{v_p^2 r_p} - 1 \right\}}$$

8-25) (Contd.)

In our case $V_p = 28070$ kph $= 7797.22$ m/s

Now, for earth $2GM = 2 \times 6.67 \times 10^{-11} \times 5.976 \times 10^{24}$

$= 7.972 \times 10^{14}$ m^3/s^2.

$$v_p^2 r_p = (7797.22)^2 \times 6.59 \times 10^6 = 4.0065 \times 10^{14} \text{ m}^3/\text{s}^2.$$

Thus, $r_a = \dfrac{r_p}{1.98977 - 1} = 1.01034\, r_p = 6658$ km, which is 288 km above the earth's surface.

$$V_a = V_p \cdot \frac{r_p}{r_a} = \frac{28070 \text{ kph}}{1.01034} = 27,783 \text{ kph}$$

Finally, $$\tau = \frac{2\pi a^{3/2}}{\sqrt{GM}} = \frac{\pi\, 2\sqrt{2}\, a^{3/2}}{\sqrt{2GM}} = \frac{\pi (2a)^{3/2}}{\sqrt{2GM}} = \frac{\pi (r_a + r_p)^{3/2}}{\sqrt{2GM}}$$

$$= \frac{\pi (1524844) \times 10^{4.5}}{2.8235 \times 10^7} = 5365.22\text{s}$$

$= 1.49$h