8-23) Since $E = \frac{\mu \dot{r}^2}{2} + \frac{\ell^2}{2\mu r^2} - \frac{k}{r}$ we see that the radial velocity $\dot{r}$ is maximized when

$$V(r) = \frac{\ell^2}{2\mu r^2} - \frac{k}{r} \quad \text{is minimized.}$$

This happens when $\frac{dV}{dr} = 0$ (since $\ell, k, \mu$ are constants).
Setting $\frac{-2\ell^2}{2\mu r^3} + \frac{k}{r^2} = 0$ we get $r(\dot{r}\,max) = \frac{\ell^2}{\mu k}$.

Then, $E = \frac{\mu}{2}\dot{r}^2_{max} + \frac{\ell^2}{2\mu(\ell^2/\mu k)^2} - \frac{\mu k}{\ell^2} \cdot k$

$$= \frac{\mu}{2}\dot{r}^2_{max} + \frac{\mu}{2}\frac{k^2}{\ell^2} - \frac{\mu k^2}{\ell^2} = \frac{\mu}{2}\dot{r}^2_{max} - \frac{\mu}{2}\frac{k^2}{\ell^2}.$$

Thus, $\dot{r}_{max} = \sqrt{\frac{2E}{\mu} + \frac{k^2}{\ell^2}}$

Using $\epsilon^2 = 1 + \frac{2E}{\mu}\frac{\ell^2}{k^2}$, we get

$$\dot{r}_{max} = \frac{k}{\ell}\sqrt{\frac{2E\ell^2}{\mu k^2} + 1} = \frac{k\epsilon}{\ell} = \frac{\epsilon}{\ell} \cdot \frac{\ell^2}{\mu\alpha} \quad (\text{from } (7.40)) \quad = \frac{\ell\epsilon}{\mu\alpha}.$$

But we know that $\tau = \frac{2\mu}{\ell} A = \frac{2\mu}{\ell} \cdot \pi ab = \frac{2\pi\mu b^2}{\ell\sqrt{1-\epsilon^2}}$

(where we used $a = b/\sqrt{1-\epsilon^2}$).
Thus, we can plug in $\ell = \left(\frac{\tau\sqrt{1-\epsilon^2}}{2\pi\mu b^2}\right)^{-1}$ and $\alpha = \frac{b^2}{a}$

to get $\dot{r}_{max} = \frac{\epsilon}{\mu} \cdot \frac{2\pi\mu b^2}{\tau\sqrt{1-\epsilon^2}} \cdot \frac{a}{b^2} = \frac{2\pi a\epsilon}{\tau\sqrt{1-\epsilon^2}}$