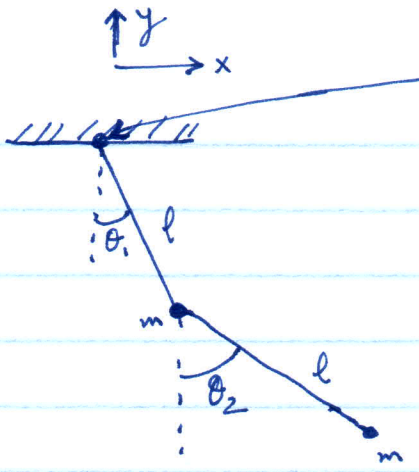
7-7)



$x_1 = l\sin\theta_1$
$y_1 = -l\cos\theta_1$

$x_2 = x_1 + l\sin\theta_2$
$y_2 = y_1 - l\cos\theta_2$

$$T = \frac{m}{2}(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2)$$

$$= \frac{m}{2}\left\{ l^2\dot{\theta}_1^2 + l^2(\cos\theta_1\dot{\theta}_1 + \cos\theta_2\dot{\theta}_2)^2 + l^2(\sin\theta_1\dot{\theta}_1 + \sin\theta_2\dot{\theta}_2)^2 \right\}$$

$$= \frac{ml^2}{2}\left\{ \dot{\theta}_1^2 + \dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2 \right\}$$

$$U = mgy_1 + mgy_2 = -2mgl\cos\theta_1 - mgl\cos\theta_2$$

$$L = T - U = \frac{ml^2}{2}\left\{ 2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2 \right\} + 2mgl\cos\theta_1 + mgl\cos\theta_2$$

θ_1 eqn $\quad 2ml^2\ddot{\theta}_1 + ml^2\cos(\theta_1-\theta_2)\ddot{\theta}_2 - ml^2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\dot{\theta}_2$

$$= -ml^2\sin(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2 - 2mgl\sin\theta_1$$

θ_2 eqn $\quad ml^2\ddot{\theta}_2 + ml^2\cos(\theta_1-\theta_2)\ddot{\theta}_1 - ml^2\sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\dot{\theta}_1$

$$= ml^2\sin(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2 - mgl\sin\theta_2$$

In both equations there is a cancellation of terms (or sum), and we can divide throughout by ml^2 to get

$$2\ddot{\theta}_1 + \cos(\theta_1-\theta_2)\ddot{\theta}_2 + \sin(\theta_1-\theta_2)\dot{\theta}_2^2 + 2\frac{g}{l}\sin\theta_1 = 0$$

and $\quad \ddot{\theta}_2 + \cos(\theta_1-\theta_2)\ddot{\theta}_1 - \sin(\theta_1-\theta_2)\dot{\theta}_1^2 + \frac{g}{l}\sin\theta_2 = 0$