7-5) This is like problem 7-4 except that now the plane of motion is "vertical" and has a gravitational field, so $U = \frac{Ar^\alpha}{\alpha} + mgz = \frac{Ar^\alpha}{\alpha} + mgr\cos\theta$ if the angle that $\vec{r}$ makes with $\hat{z}$ is θ.

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{Ar^\alpha}{\alpha} - mgr\cos\theta .$$

The θ-equation is $\frac{d}{dt}(mr^2\dot{\theta}) = mgr\sin\theta$ and thus $mr^2\dot{\theta}$ is no longer a constant.

[The r-equation is $m\ddot{r} = -Ar^{\alpha-1} - mg\cos\theta + mr\dot{\theta}^2$.]

Since the component of $\vec{L}$ perpendicular to the plane is non-zero, there must be a torque acting on the object; this torque $(\vec{r}\times\vec{F})$ comes from the non-central part of $\vec{F}$, i.e., the part of $\vec{F}$ not along $\hat{r}$. The gravitational force $mg\hat{z}$ is such a force that supplies a component along $\hat{\theta}$.