7-40) If the position of the cart is $x(t)$ and the positions of the upper and lower masses are $\vec{x}_u(t)$ and $\vec{x}_\ell(t)$ then

$$T = (2m)\frac{\dot{x}^2}{2} + \frac{m}{2}\left(\dot{\vec{x}}_u^{\,2} + \dot{\vec{x}}_\ell^{\,2}\right) \qquad \underline{\text{where}} \begin{cases} x_u = x + b\sin\theta_u \\ y_u = y_0 - b\cos\theta_u \end{cases}$$

and

$$\begin{cases} x_\ell = x + b\sin\theta_u + b\sin\theta_\ell \\ y_\ell = y_0 - b\cos\theta_u - b\cos\theta_\ell \end{cases}$$

$$U = mgy_u + mgy_\ell$$

Thus,

$$T = m\dot{x}^2 + \frac{m}{2}\Big(\dot{x}^2 + b^2\dot{\theta}_u^2 + 2b\dot{x}\cos\theta_u\dot{\theta}_u + \dot{x}^2 + b^2\dot{\theta}_u^2 + b^2\dot{\theta}_\ell^2 + 2\dot{x}b(\dot{\theta}_u\cos\theta_u + \dot{\theta}_\ell\cos\theta_\ell) + 2b^2\dot{\theta}_u\dot{\theta}_\ell\cos\theta_u\cos\theta_\ell + 2b^2\sin\theta_u\sin\theta_\ell\,\dot{\theta}_u\dot{\theta}_\ell\Big)$$

$$U = mg\left\{2y_0 - 2b\cos\theta_u - b\cos\theta_\ell\right\}$$

Thus, $L = T - U$ i.e.,

$$L = 2m\dot{x}^2 + mb^2\dot{\theta}_u^2 + \frac{mb^2}{2}\dot{\theta}_\ell^2 + 2mb\dot{x}\dot{\theta}_u\cos\theta_u + mb\dot{x}\dot{\theta}_\ell\cos\theta_\ell + mb^2\dot{\theta}_u\dot{\theta}_\ell\cos(\theta_\ell - \theta_u) - 2mgy_0 + 2mgb\cos\theta_u + mgb\cos\theta_\ell .$$

The Euler-Lagrange equations for x, θ_u, and θ_ℓ are

$$\frac{d}{dt}\left(4m\dot{x} + 2mb\dot{\theta}_u\cos\theta_u + mb\dot{\theta}_\ell\cos\theta_\ell\right) = 0$$

which is conservation of x-direction momentum, and

$$\frac{d}{dt}\left(2mb^2\dot{\theta}_u + 2mb\dot{x}\cos\theta_u + mb^2\dot{\theta}_\ell\cos(\theta_\ell - \theta_u)\right) = -2mb\dot{x}\dot{\theta}_u\sin\theta_u\dot{\theta}_u + mb^2\dot{\theta}_u\dot{\theta}_\ell\sin(\theta_\ell - \theta_u) - 2mgb\sin\theta_u$$

and

$$\frac{d}{dt}\left(mb^2\dot{\theta}_\ell + mb\dot{x}\cos\theta_\ell + mb^2\dot{\theta}_u\cos(\theta_\ell - \theta_u)\right) = -mb\dot{x}\dot{\theta}_\ell^2\sin\theta_\ell - mb^2\dot{\theta}_u\dot{\theta}_\ell\sin(\theta_\ell - \theta_u) - mgb\sin\theta_\ell$$