7-4) $f = -Ar^{\alpha-1} \Rightarrow U = \dfrac{Ar^\alpha}{\alpha}$

$$L = T - U = \frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) - \frac{Ar^\alpha}{\alpha} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - \frac{Ar^\alpha}{\alpha}$$

Since the motion is planar, we take that plane to be $z=0$.
Now $L = \dfrac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \dfrac{Ar^\alpha}{\alpha}$.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = \frac{\partial L}{\partial r} \Leftrightarrow m\ddot{r} = -Ar^{\alpha-1} + mr\dot{\theta}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta} \Leftrightarrow \frac{d}{dt}(mr^2\dot{\theta}) = 0 \Rightarrow mr^2\dot{\theta} = \text{constant} = \ell \text{ (say)} \quad -(1)$$

Thus, the r-equation is

$$m\ddot{r} = -Ar^{\alpha-1} + mr\left(\frac{\ell}{mr^2}\right)^2 = -Ar^{\alpha-1} + \frac{\ell^2}{mr^3} \quad -(2)$$

The angular momentum about the origin is $\vec{r} \times \vec{p}$
$= \vec{r} \times m(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) = mr^2\dot{\theta}\,\hat{z} = \ell\hat{z}$.

Since this is a constant vector yes, it is conserved.
Multiplying eqn. (2) by $\dot{r}$ and integrating w.r.t. time immediately gives $\dfrac{m\dot{r}^2}{2} = -\dfrac{Ar^\alpha}{\alpha} - \dfrac{\ell^2}{2mr^2} + \text{constant}$

Using (1) we get

$$\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{Ar^\alpha}{\alpha} = \text{constant}$$

i.e., $E = T + U = \text{constant}$.